Radiosity

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2003

Greatly inspired by: **- Radiosity & Global Illumination, F.X. Sillon & C. Puech**
- SIGGRAPH'93 Education Slide Set
- ...

Lecture outline

- ➡ **Introduction**
- **Preliminaries**
- **Radiosity equation**
- **Solving the equation**
- **Optimization**
- **Extensions**
- **Conclusion**

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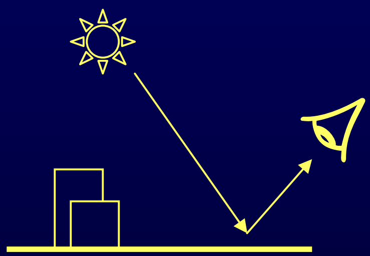
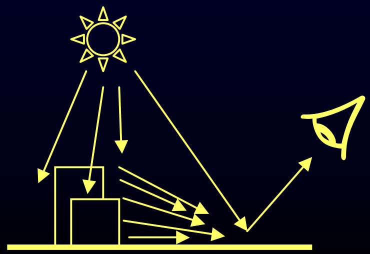
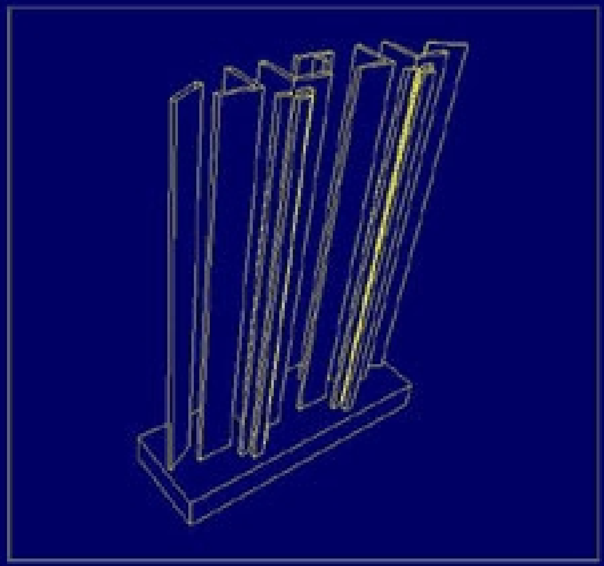
Introduction

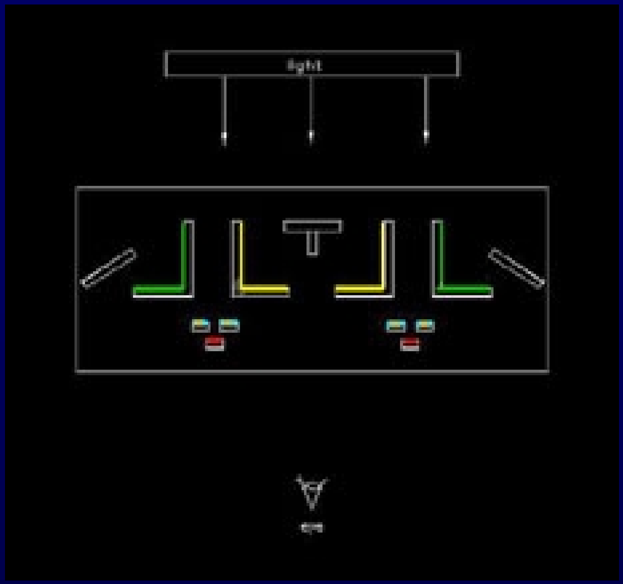
- **Local illumination:**
  - **source of illumination ≡ light**

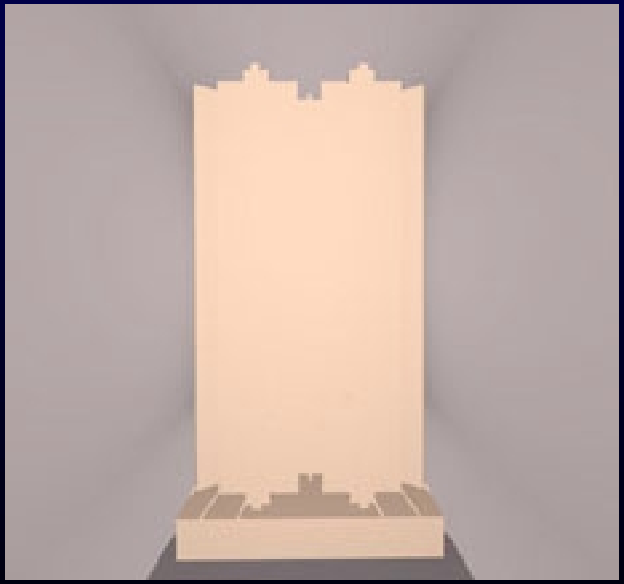


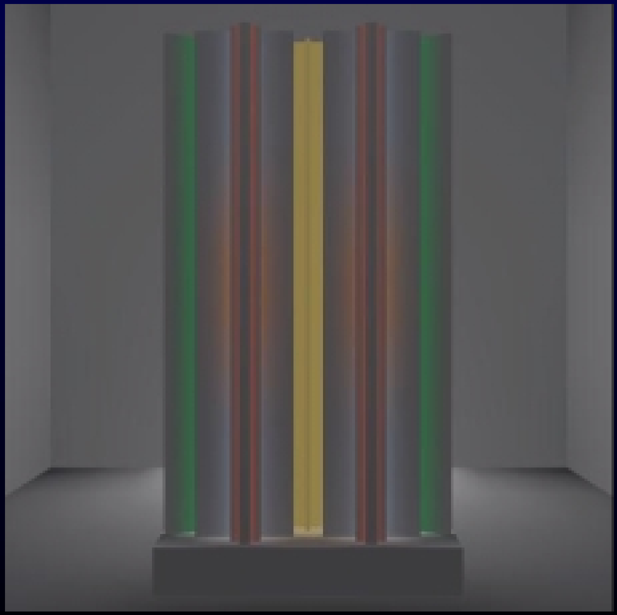
- **Global illumination:**
  - **source of illumination ≡ any object**











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Lecture outline

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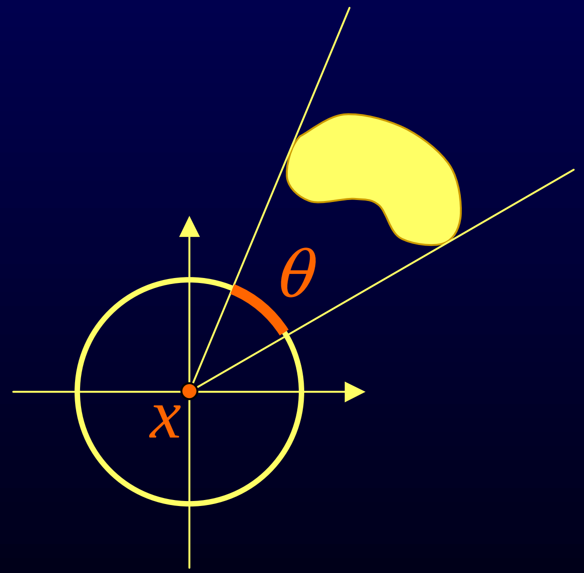
What quantities are we dealing with?

Solid angle

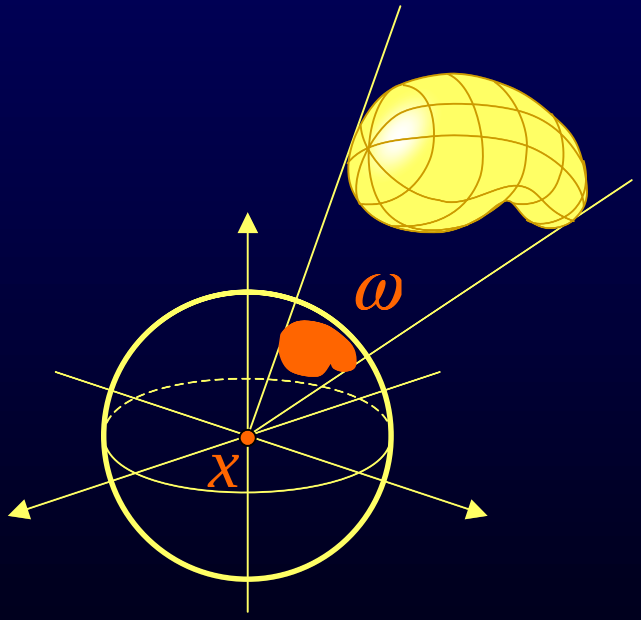
What is the 'size' of an object from a point?

- **Angle**



unit circle ≡ 2π **radians**

- **Solid angle**

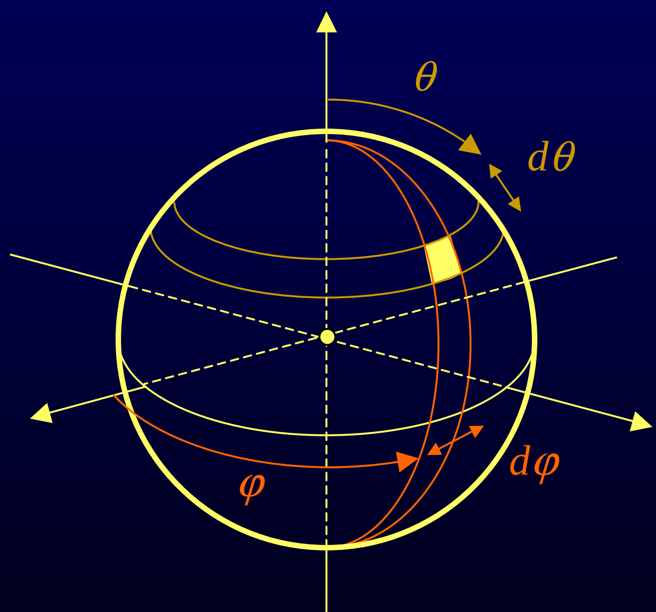


unit sphere ≡ 4π **steradians (sr)**

Solid angle

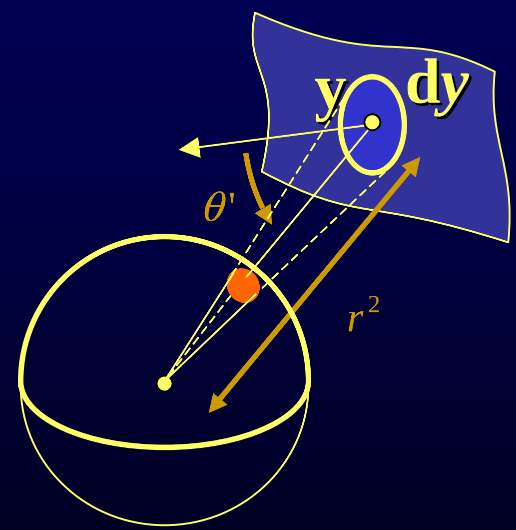
How is it related to other units?

- **Solid angle in spherical coordinates**



$$d\omega = \sin\theta\, d\varphi\, d\theta$$

- **Solid angle subtended by area d*y***

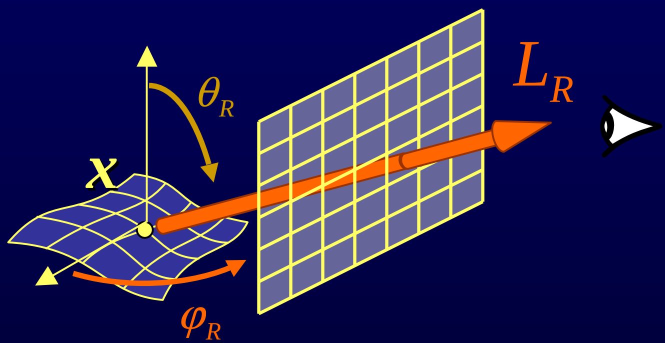


$$d\omega = \frac{dy \cos\theta'}{r^2}$$

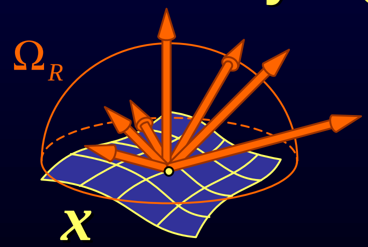
Radiance & Radiosity

What is the relevant quantity for light?

- **Radiance $L_R\,(x,\ \theta_R,\ \varphi_R)$**



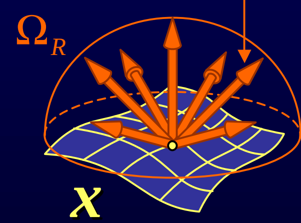
- **Radiosity $B\,(x)$**



$$B(x) = \int_{\Omega_R} L_R(x, \theta_R, \varphi_R) \cos\theta_R \, d\omega_R$$

- **Ideal diffuse reflector case:**

$$L_R(x, \theta_R, \varphi_R) \equiv L_R(x) = \frac{1}{\pi} B(x)$$

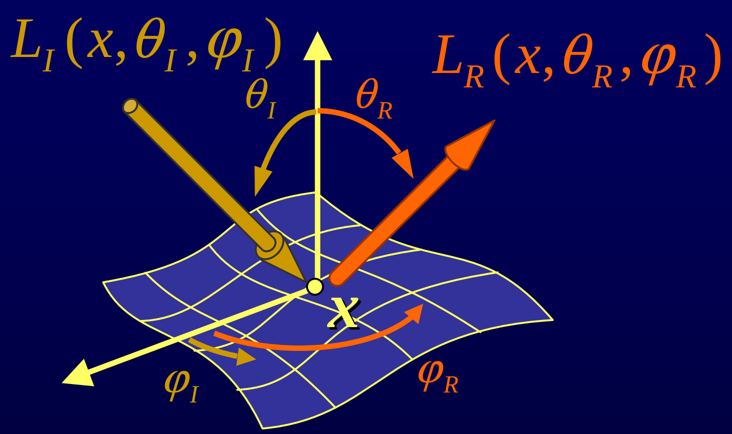


$L_R(x)$
$L_G(x)$ $\equiv$ **'color of light'**
$L_B(x)$

BRDF & DHRF

How are described surfaces?

- **BRDF** **(Bidirectional Reflectance Distribution Function)**



$$\rho_{bd}(x,\theta_R,\varphi_R,\theta_I,\varphi_I) = \frac{L_R(x,\theta_R,\varphi_R)}{L_I(x,\theta_I,\varphi_I)\cos\theta_I d\omega_I}$$

- **Ideal diffuse reflector case:**

$$\rho_{bd}(x,\theta_R,\varphi_R,\theta_I,\varphi_I) \equiv \rho_{bd}(x) = \frac{1}{\pi}\rho_{dh}(x)$$

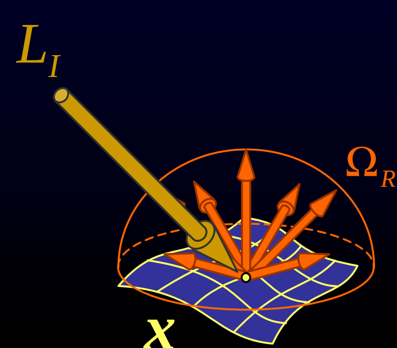
$\rho_{dh}^{R}(x) \in [0,1]$

$\rho_{dh}^{G}(x) \in [0,1]$ **≡ 'color of surface'**

$\rho_{dh}^{B}(x) \in [0,1]$

- **DHRF** **(Directional Hemispherical Reflectance Function)**

$$\rho_{dh}(x,\theta_R,\varphi_R) = \frac{\int_{\Omega_R} L_R(x,\theta_R,\varphi_R)\cos\theta_R d\omega_R}{L_I(x,\theta_I,\varphi_I)\cos\theta_I d\omega_I}$$



Lecture outline

- Introduction
- Preliminaries
- ➡ **Radiosity equation**
- Solving the equation
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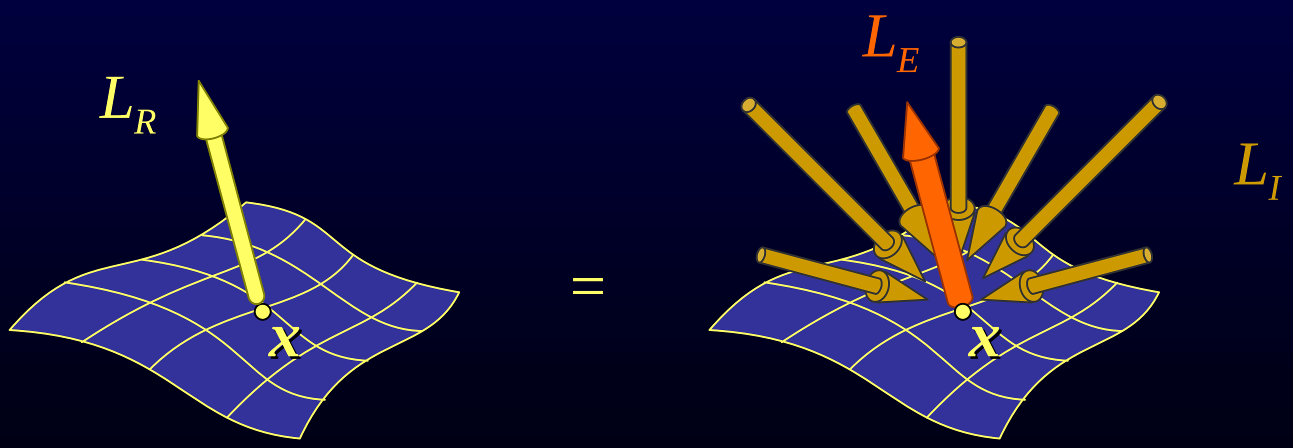
Global Illumination Equation

How can it lead to the Radiosity Equation?

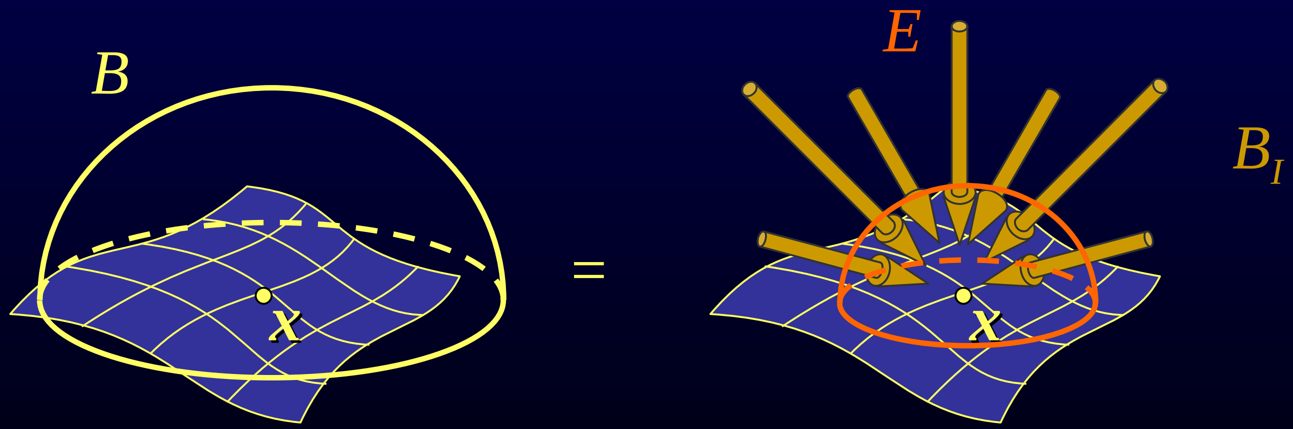
$$L_R(x,\theta_R,\varphi_R) = L_E(x,\theta_R,\varphi_R) + \int_{\Omega_I} \rho_{bd}(x,\theta_R,\varphi_R,\theta_I,\varphi_I) L_I(x,\theta_I,\varphi_I)\cos\theta_I \, d\omega_I$$



Ideal diffuse reflectors Equation

$$L_R(x,\theta_R,\varphi_R) = L_E(x,\theta_R,\varphi_R) + \int_{\Omega_I} \rho_{bd}(x,\theta_R,\varphi_R,\theta_I,\varphi_I) L_I(x,\theta_I,\varphi_I) \cos\theta_I \, d\omega_I$$

$$B(x) = E(x) + \rho_{dh}(x) \int_{\Omega_I} \frac{1}{\pi} B_I(x,\theta_I,\varphi_I) \cos\theta_I \, d\omega_I$$



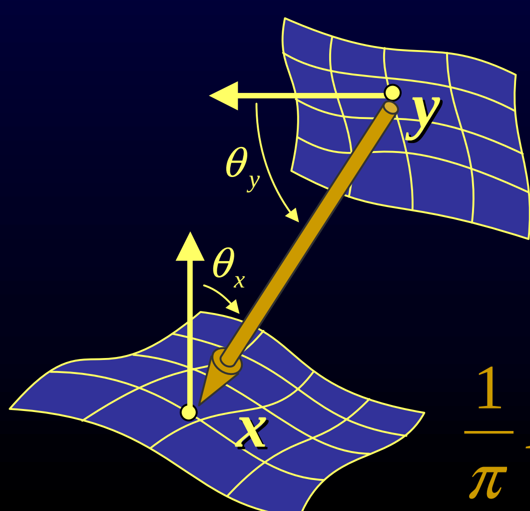
Ideal diffuse : $L_R(x,\theta_R,\varphi_R) \equiv \frac{1}{\pi} B(x)$ $\rho_{bd}(x,\theta_R,\varphi_R,\theta_I,\varphi_I) \equiv \frac{1}{\pi} \rho_{dh}(x)$

Introducing Visibility

Can we replace the 'incoming radiosity' terms?

$$B(x) = E(x) + \rho_{dh}(x)\int_{\Omega_I} \frac{1}{\pi} B_I(x,\theta_I,\varphi_I)\cos\theta_I d\omega_I$$

$$B(x) = E(x) + \rho_{dh}(x)\int_{y\in S} \frac{1}{\pi} B(y)V(x,y)\cos\theta_x d\omega_y$$

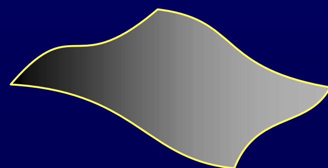


$$\frac{1}{\pi}B(y)V(x,y)$$

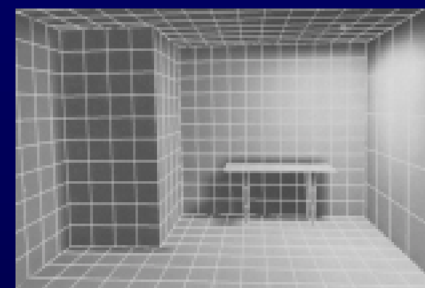
$$||$$

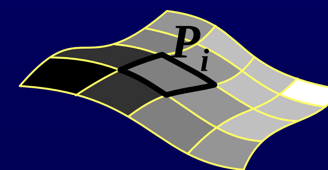
$$\frac{1}{\pi}B_I(x,\theta_I,\varphi_I)$$

Discrete Radiosity Equation



DISCRETIZATION





$$B(x) = E(x) + \rho_{dh}(x) \int_{y \in S} \frac{1}{\pi} B(y) V(x,y) \cos\theta_x \, d\omega_y$$

$$B(x) = E(x) + \rho_i \sum_{j=1}^{N} B_j \int_{y \in P_j} \frac{1}{\pi} V(x,y) \cos\theta_x \, d\omega_y$$

$$\frac{1}{A_i} \int_{x \in P_i}$$

$$B_i = \frac{1}{A_i} \int_{x \in P_i} B(x) \qquad E_i = \frac{1}{A_i} \int_{x \in P_i} E(x)$$

Radiosity Equation

- **Discrete formulation**

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j \frac{1}{A_i} \int_{x \in P_i} \int_{y \in P_j} \frac{V(x,y)\cos\theta_x d\omega_y}{\pi}$$

Radiosity Equation

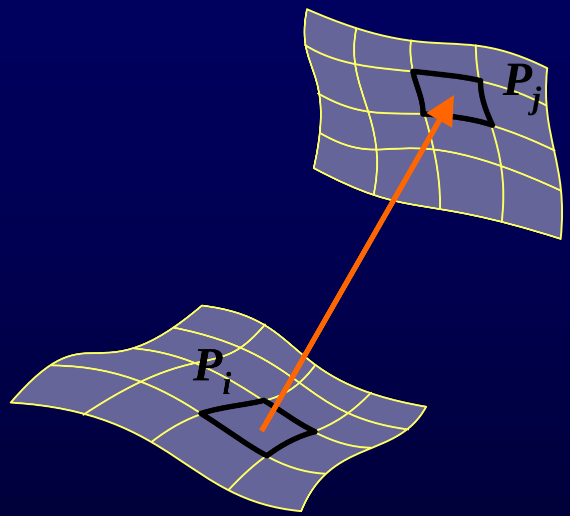
$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

- **Solving radiosity :**
  - **compute form factors**
  - **solve N equations**

Form Factors

- **Amount of energy**

$$F_{ij}$$

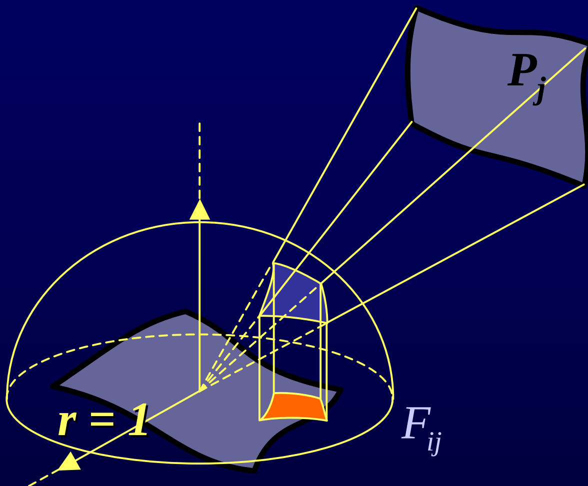


- **Property**

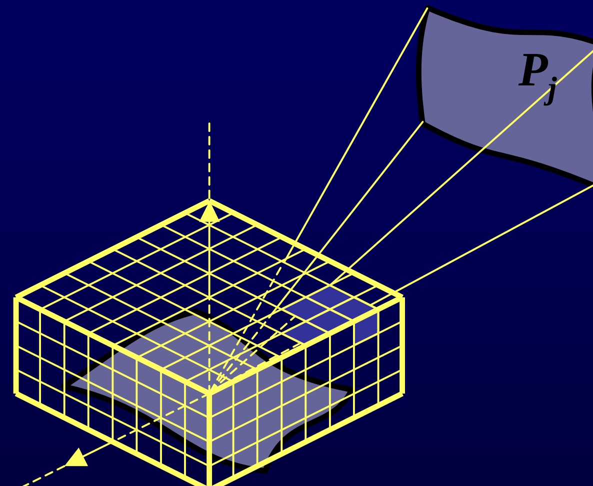
$$A_i F_{ij} = A_j F_{ji}$$

Form Factors

- **Nusselt's analogy**



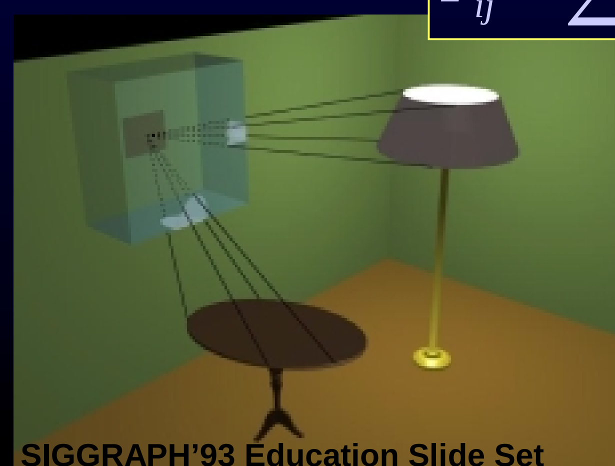
- **Hemi-cube**



$$F_{ij} = \sum \Delta F$$

$$\frac{1}{A_i} \int_{x \in P_i} \int_{y \in P_j} \frac{V(x,y) \cos\theta_x \, d\omega_y}{\pi}$$

$$F_{ij} \approx \int_{y \in P_j} \frac{V(x,y) \cos\theta_x \, d\omega_y}{\pi}$$



Lecture outline

- Introduction
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- Radiosity equation
- ➡ **Solving the equation**
- Optimization
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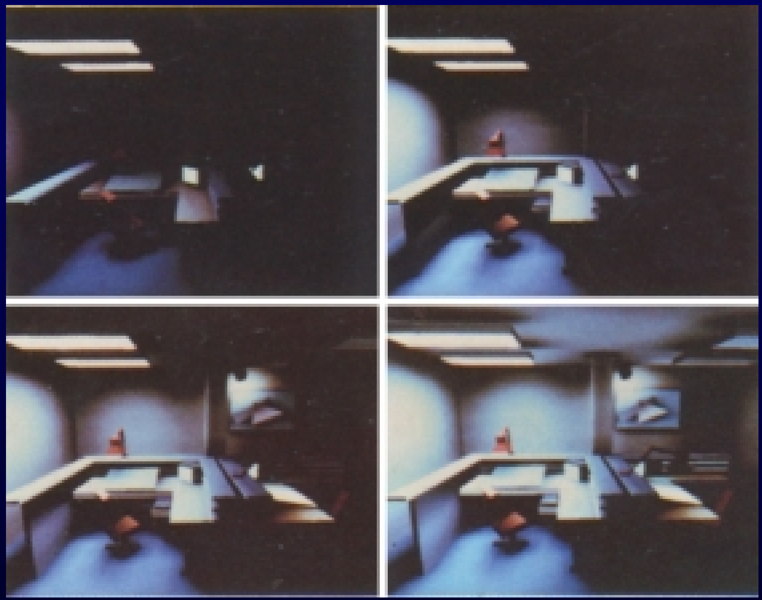
- Conclusion



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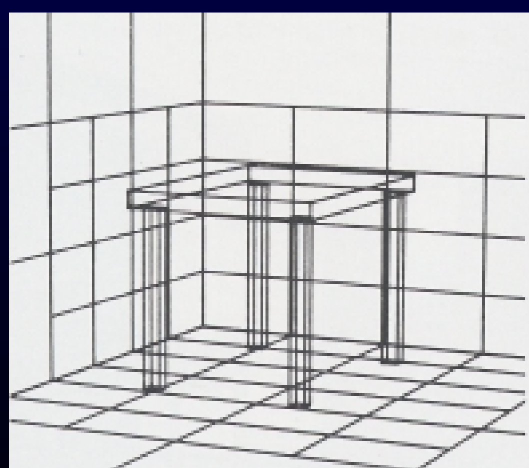
Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

Solving the equation

4 famous methods for solving the ‘classic’ radiosity

- **Matrix inversion**
- **Jacobi relaxation**
- **Gauss-Seidel relaxation (gathering)**
- **Southwell relaxation (shooting)**



Matrix inversion

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i \qquad \boldsymbol{i \in [1, N]}$$

$$\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ -\rho_N F_{NN} & \cdots & \cdots & 1-\rho_N F_{NN} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix}$$

$$MB = E$$

Invert M

Drawbacks: all form factors required / no intermediate solution

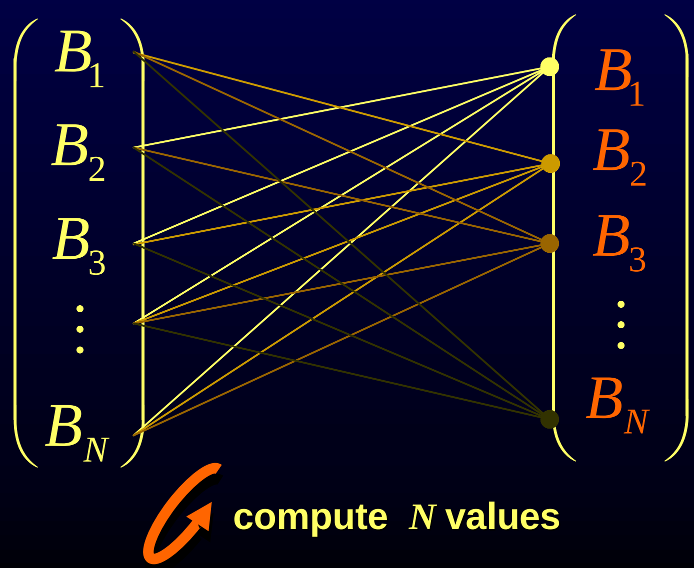
Jacobi relaxation

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i = \frac{E_i}{1-\rho_i F_{ii}} + \rho_i \sum_{\substack{j=1 \\ j \neq i}}^{N} B_j \frac{F_{ij}}{1-\rho_i F_{ii}}$$

vector B^{t} **vector B^{t+1}**



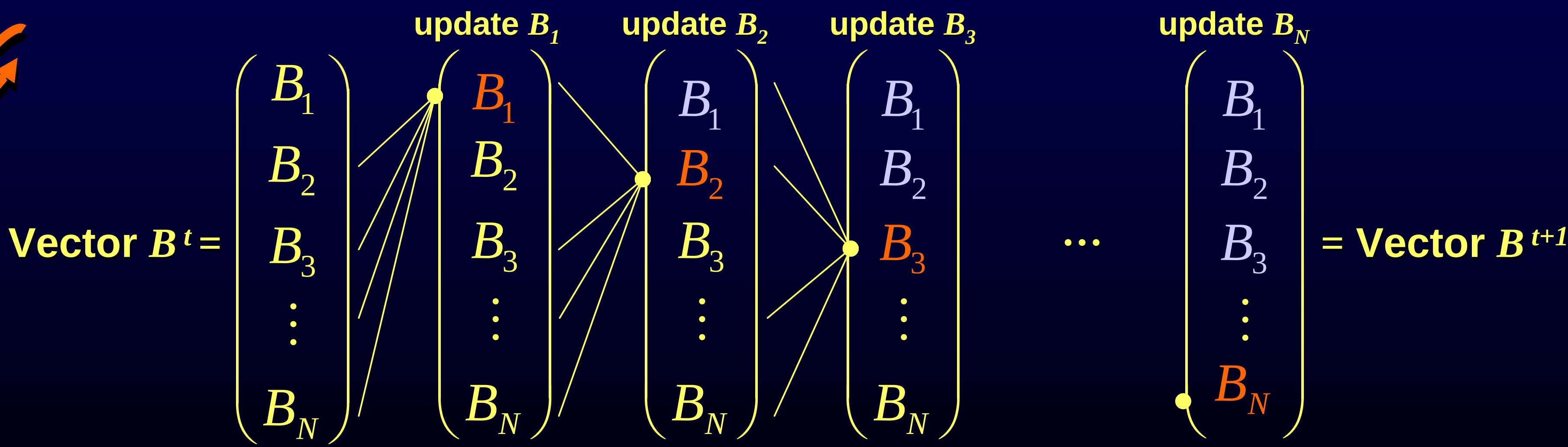
Drawbacks: all form factors required / two vectors B^{t} and B^{t+1} required

Gauss-Seidel relaxation

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i = \frac{E_i}{1-\rho_i F_{ii}} + \rho_i \sum_{\substack{j=1 \\ j \neq i}}^{N} B_j \frac{F_{ij}}{1-\rho_i F_{ii}}$$



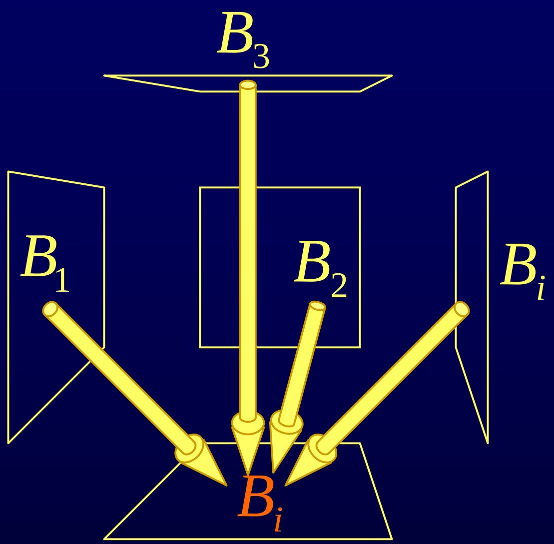
Drawbacks: all form factors are required for one complete iteration

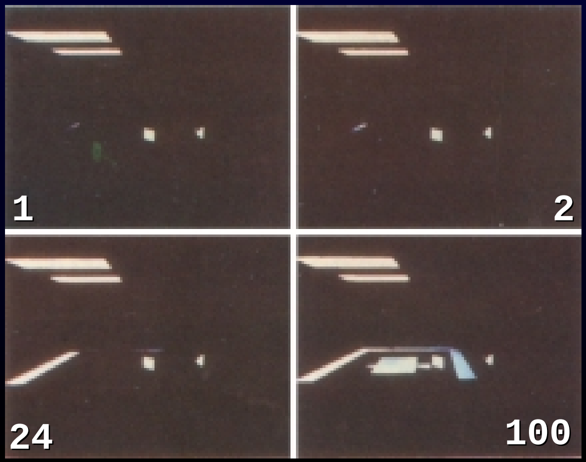
Advantages: only one storage vector required / faster convergence than Jacobi

Gauss-Seidel relaxation

- **Why is it called <u>Gathering</u>?**

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_i = \dfrac{E_i}{1-\rho_i F_{ii}} + \rho_i \sum_{j=1}^{N} B_j \dfrac{F_{ij}}{1-\rho_i F_{ii}} \\ \vdots \\ B_N \end{pmatrix}$$





Southwell relaxation

- **Idea: distribute the residual energy of the patch of most residual energy, P_i**

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{N} A_j B_j F_{ji}$$

Energy Equation

$$\beta_i = \varepsilon_i + \rho_i \sum_{j=1}^{N} \beta_j F_{ji}$$

Residuals energy

$$r_i = \varepsilon_i - \beta_i + \rho_i \sum_{j=1}^{N} \beta_j F_{ji}$$

1. $k = \arg\max_k (r_k)$

2. $r_{i \neq k} \mathrel{+}= \dfrac{\rho_i F_{ik}}{1-\rho_k F_{kk}} r_k \,, \quad r_k = 0$

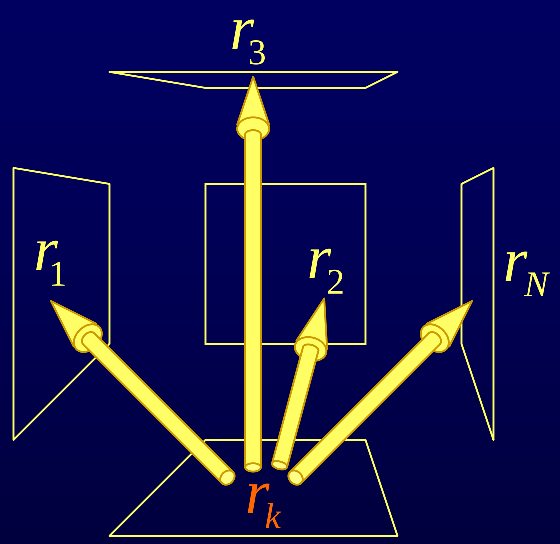
3. $\beta_k = \dfrac{\varepsilon_k}{1-\rho_k F_{kk}} + \rho_i \sum_{\substack{j=1 \\ j\neq k}}^{N} \beta_j \dfrac{F_{jk}}{1-\rho_k F_{kk}} \quad , \quad B_i = \dfrac{\beta_i + r_i}{A_i}$

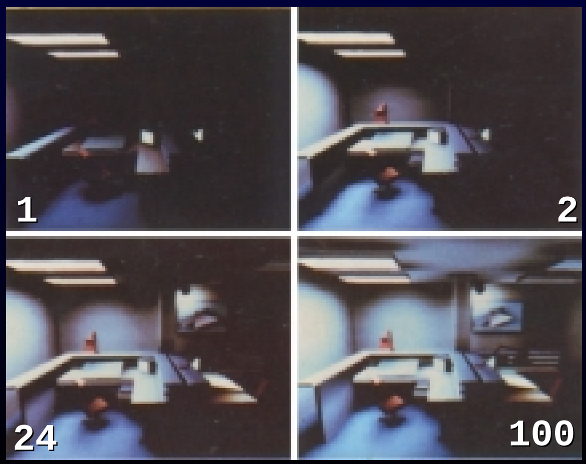
Southwell relaxation

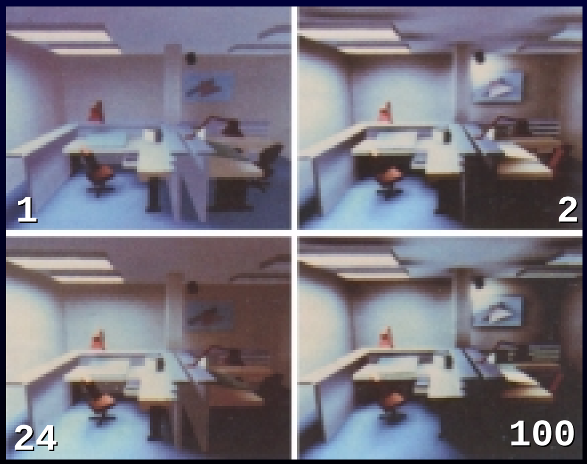
- **Why is it called <u>Shooting</u>?**

$$r_{i \neq k} += \frac{\rho_i F_{ik}}{1 - \rho_k F_{kk}} r_k$$

$$r_k = 0$$







Ambient correction

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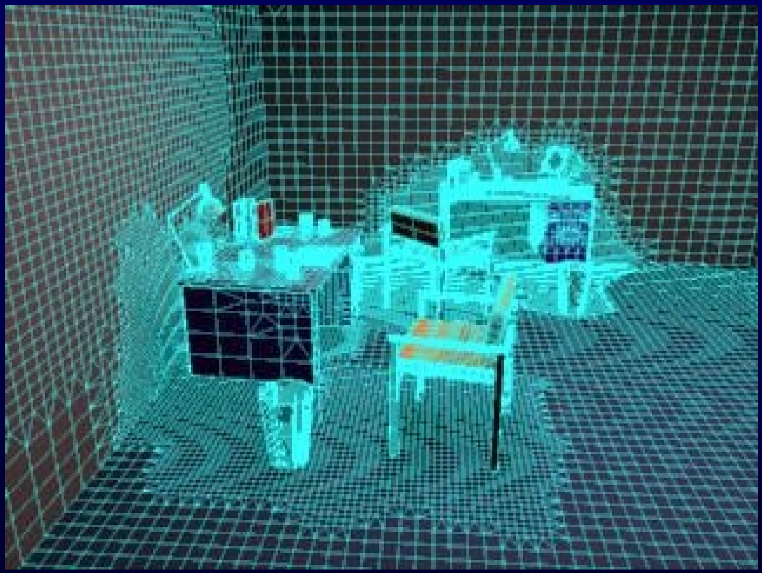
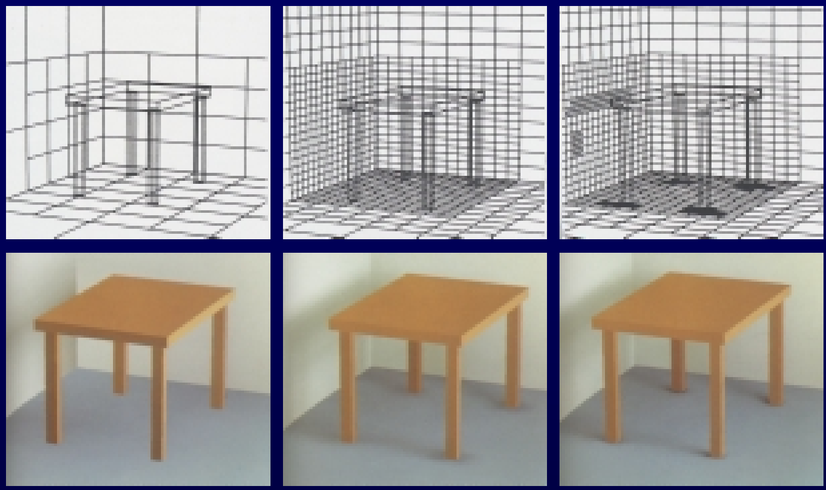


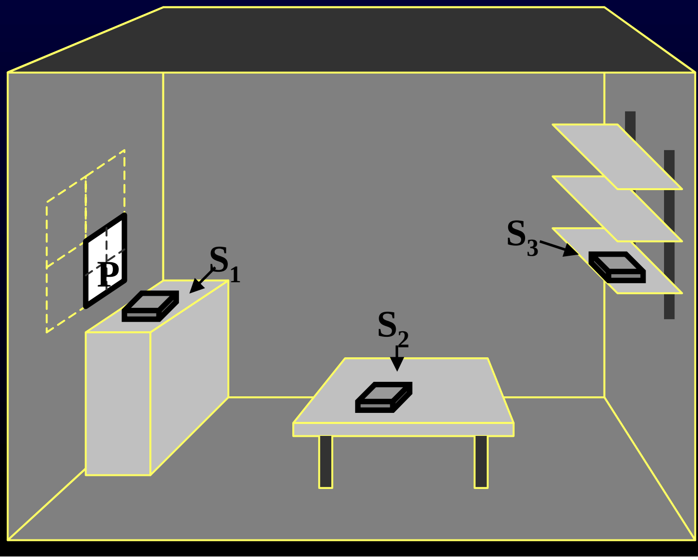
F. Sillion

Optimizing

- **Sampling tradeoff**
  - **too coarse: ugly solution**
  - **too fine: time consuming**



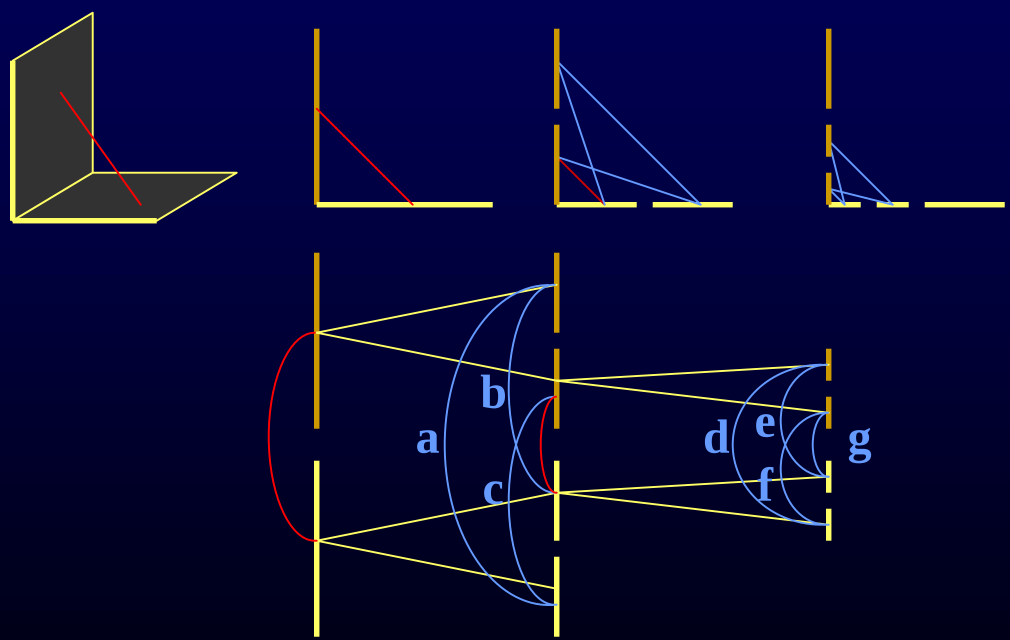
- **The necessity to emit energy at different levels:**

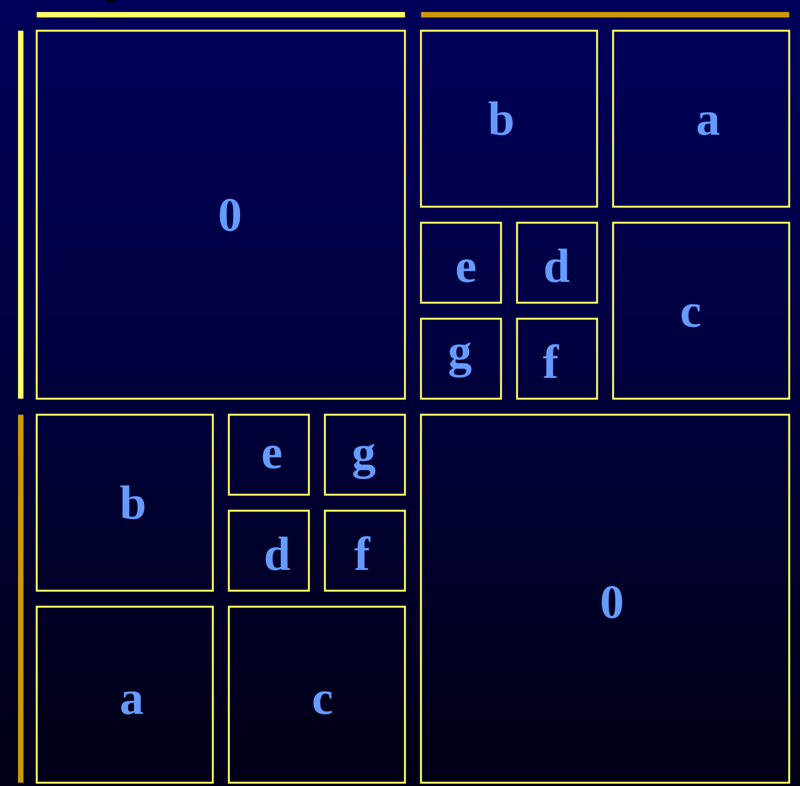


Hierarchical Radiosity

- **Allow exchanges at different levels**
- **Links between two hierarchical patches:**

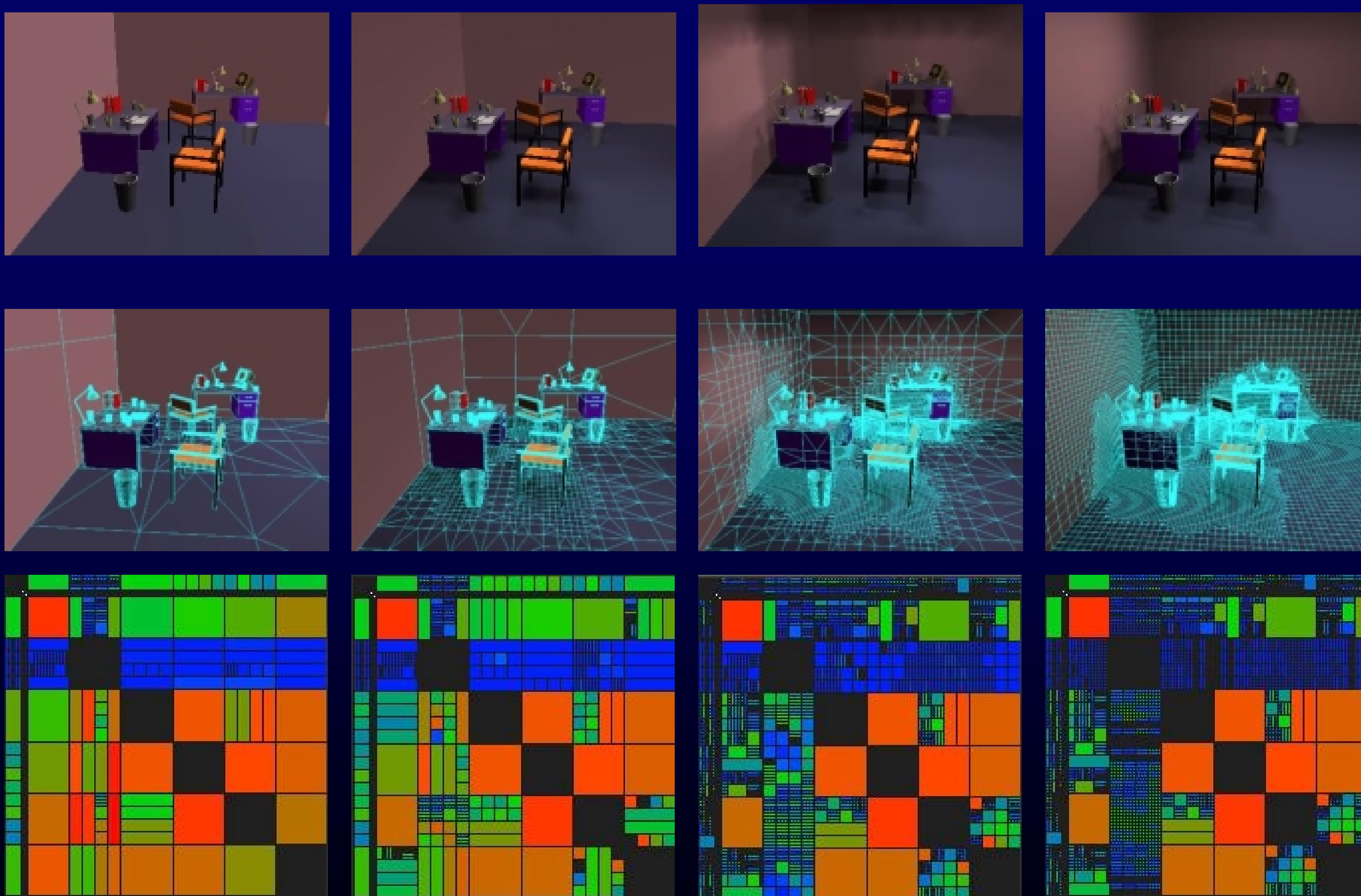
  $\Rightarrow$ threshold + splitting criterion





Gathering radiosity for a patch P_i:

1. **Compute gathered radiosities B_G for patch P_i and its subpatches**
2. **Update the radiosities B of the tree (bidirectionnal sweep)**



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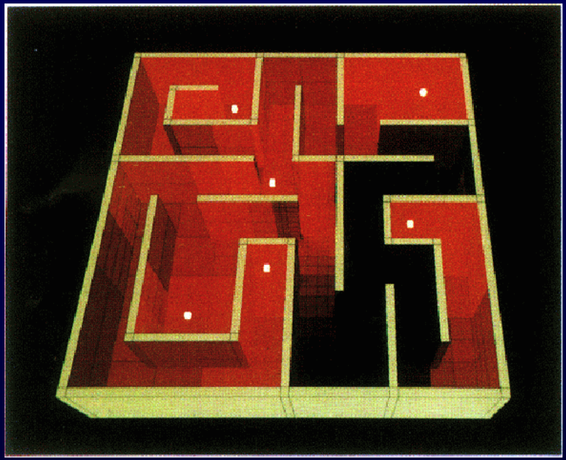
Decreasing tolerance

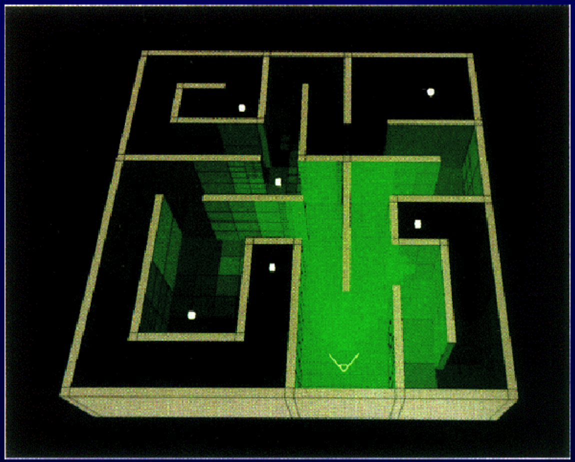
(Blocks are breaking)

Importance

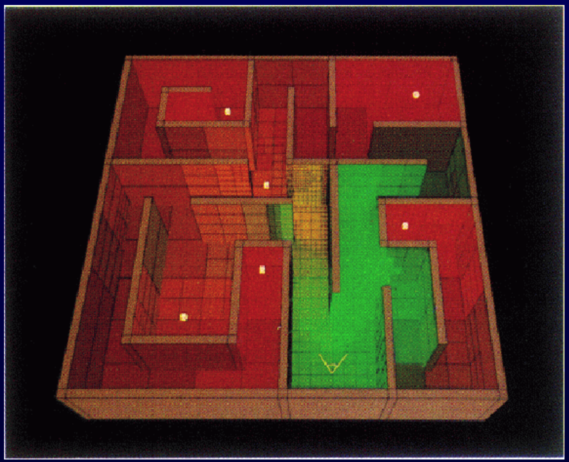
- **View dependent refinement**



Radiosity solution



Importance solution



Superposition, in yellow

Importance Equation

$$I_i = R_i + \sum_{j=1}^{N} \rho_j F_{ji} I_j$$

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- ➡ **Extensions**
- Conclusion

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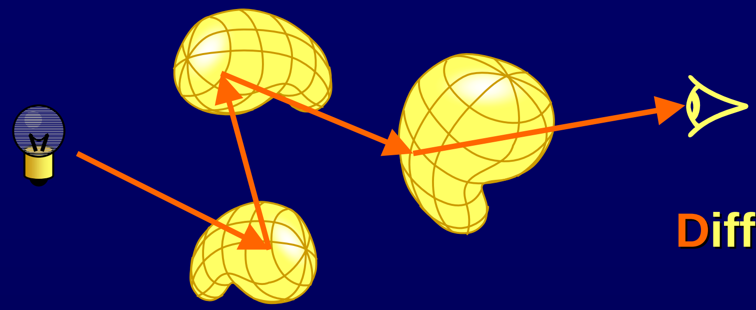
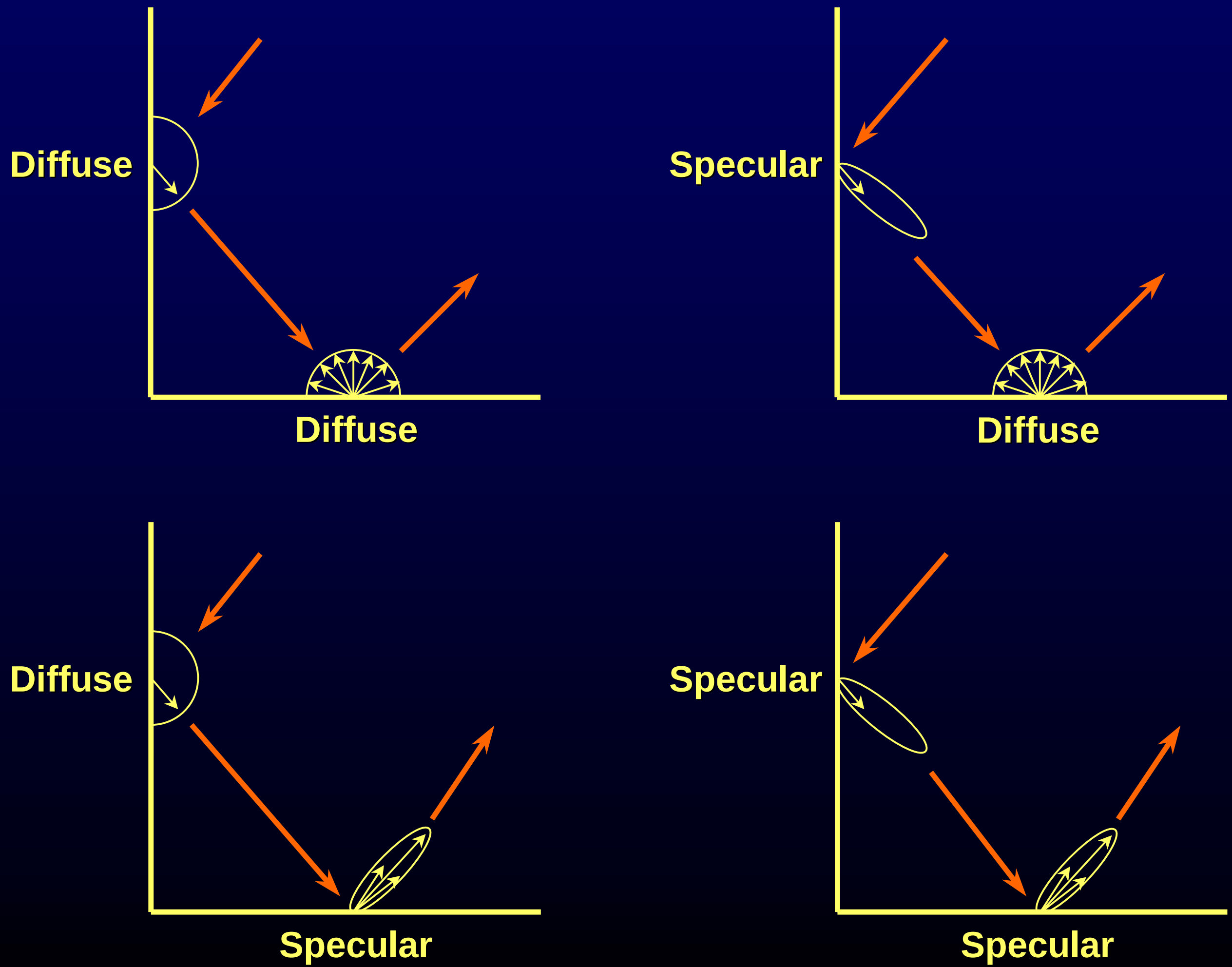


Basic algorithms fails at transitions

Diffuse → Specular ¦ Specular → Diffuse



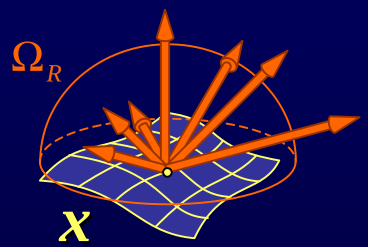
Basic Raytracing
LS*E

Basic Radiosity
LD*E

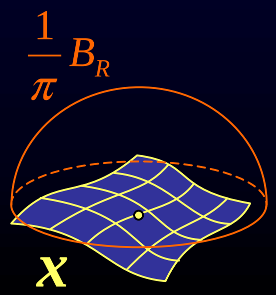
Global Illumination
L(D|S)*E

Directional radiosity

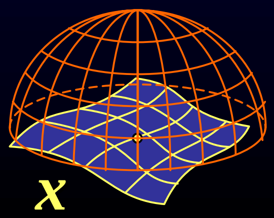
- **Global Illumination:**



- **Radiosity:**
  radiance is constant



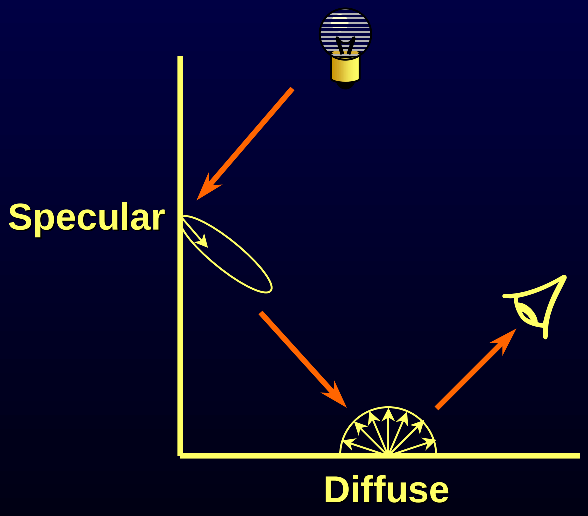
➡ **Directional radiosity :**
radiance is piecewise constant



Two-pass approach

- Two-pass approach
  1. Radiosity: LD*E
  2. Raytracing LDS*E

- Limitation: misses LS*DE

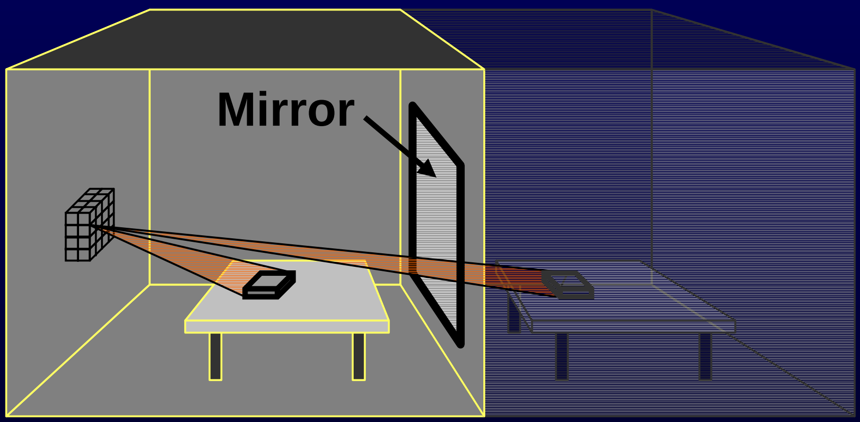


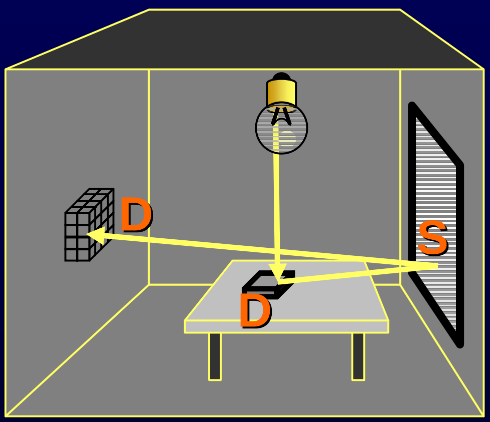
Complete* two-pass approach

(* Ideal diffuse & ideal specular surfaces)

- **Two-pass approach with extended form factors**

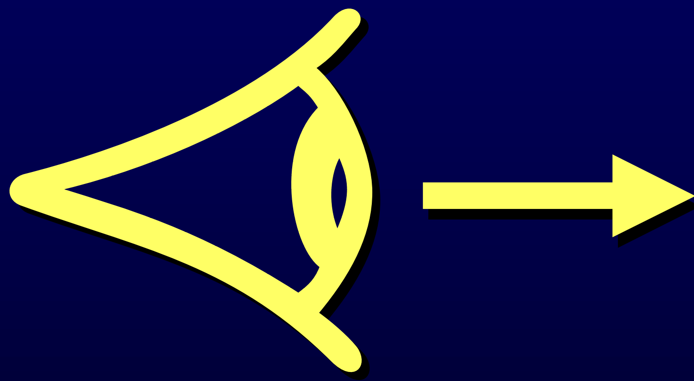
  Idea: Include D→ S* → D in the first pass





F. Sillion

For the eyes



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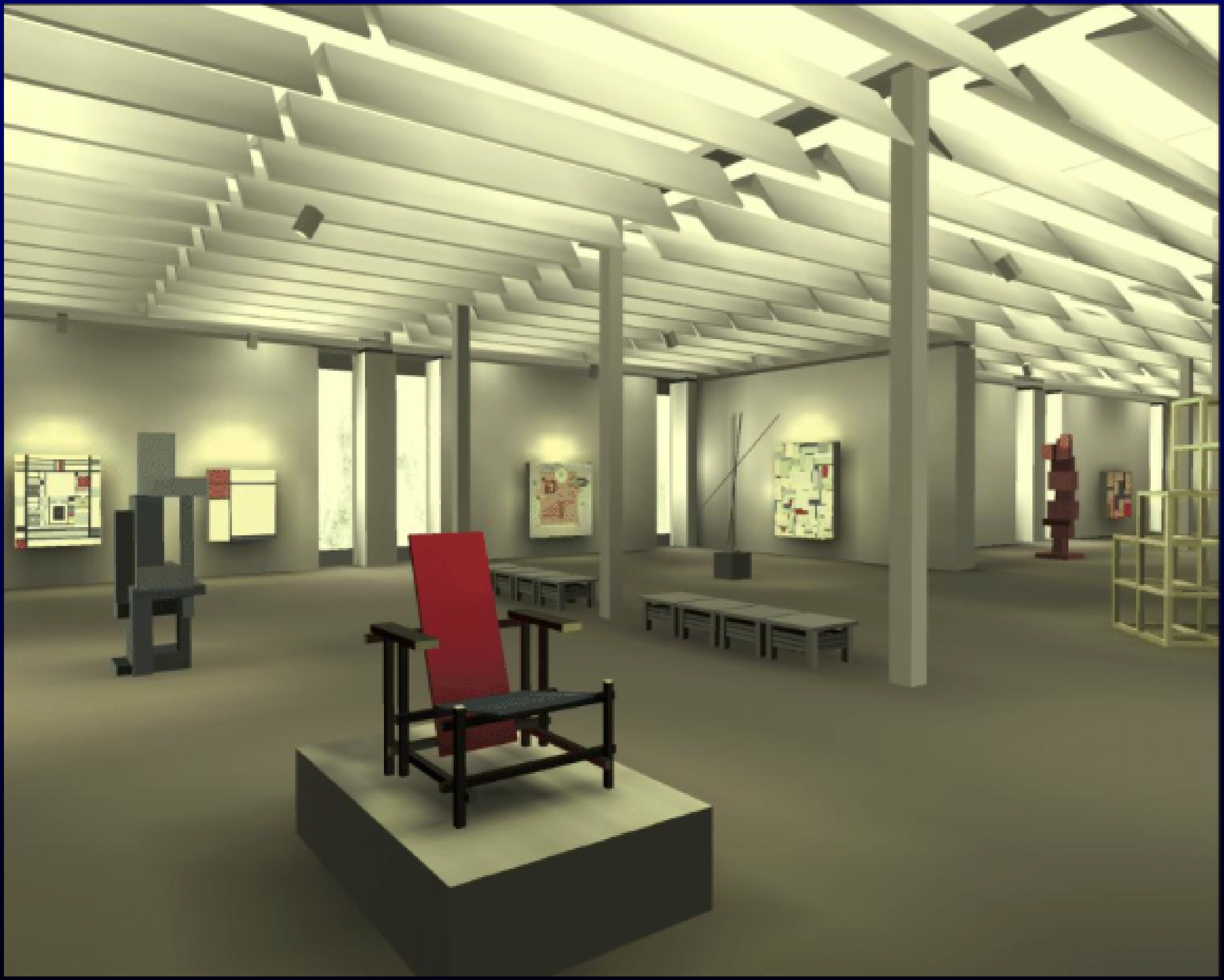

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