Tutorial Quiz #8 — Solutions

Using induction, prove that $2^n \leq 3^n$ for all integers $n \geq 1$. (You must use the format given in the course handbook.)

Answer:

Base Case: $(n = 1)$ $2^1 = 2 \leq 3 = 3^1$

Let $k \geq 1$ be an arbitrary integer.

Induction Hypothesis: Assume that $2^k \leq 3^k$.

Induction Step: We have to show that $2^{k+1} \leq 3^{k+1}$.

$$\begin{aligned} 2^{k+1} &= 2 \times 2^k && \text{(by definition of exponentiation)} \\ &\leq 3 \times 2^k && \text{(since } 2 \leq 3\text{)} \\ &\leq 3 \times 3^k && \text{(by the Induction Hypothesis)} \\ &\leq 3^{k+1} && \text{(by definition of exponentiation)} \end{aligned}$$

Conclusion: Hence, $2^n \leq 3^n$ for all integers $n \geq 1$.

Marking Scheme:

A. 1 mark for a correct base case

B. 1 mark for correctly stating the induction hypothesis (including the phrase "let $k \geq 1$ be an arbitrary integer" or something similar)

C. 1 mark for using the induction hypothesis correctly in the proof of the induction step

D. 1 mark for a correct proof of the induction step

E. 1 mark for stating the conclusion of the proof

F. -1 mark for using vague or undefined notation (*e.g.*, "show $k + 1$" or using $S(k)$ without defining it)

Common Errors: