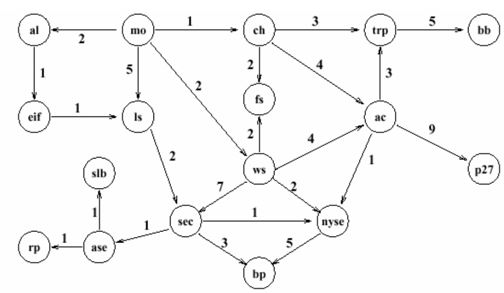
CSC384: Lecture 5

- Last time
  - search, DFS & BrFS; cycle checking & MPC
- Today
  - arc costs; heuristics; LCFS, BeFS, A*
  - misc: iterative deepening, etc.
- Readings:
  - Today: Ch.4.5, 4.6
  - Next Weds: class notes (no text reading)

Manhattan Bike Courier (Acyclic)



Arc Costs

- DFS/BrFS make sense when no arc costs
  - e.g., BrFS ensures shortest path (fewest arcs)
- If arc costs & **aim of finding least-cost path**, BFS in not suitable
  - e.g., goal=ls, start=mo: BrFS finds shortest path [ls,mo] with cost 5; but least-cost path is [ls,eif,al,mo] with cost 4 (even though it has more arcs)
- **Least-cost first search (LCFS)** : least cost path
  - works much like BrFS, except paths are ordered according to cost, rather than “length”

Least-cost First Search

- Implementing LCFS is straightforward
- Let cost of any path *p* to node *n* be denoted *g(n)*
  - note: this notation is misleading but conventional
- Organize frontier as a priority queue
  - with each path on frontier, attach cost *g(n)*
  - paths with lower cost are at the head of the frontier
  - new paths (nbrs) are inserted in order of cost
  - so *add_to_f* is just priority queue insertion
- Selecting a path from the head of the frontier
  - thus, you always get least cost path from the frontier

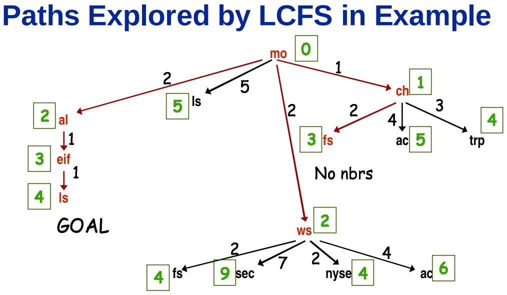
Trace of LCFS (with paths: *mo* to *ls*)

Frontier evolution:

1. [mo]:0
2. [ch,mo]:1 [al,mo]:2 [ws,mo]:2 [ls,mo]:5
3. [al,mo]:2 [ws,mo]:2 [fs,ch,mo]:3 [trp,ch,mo]:4 [ac,ch,mo]:5 [ls,mo]:5
4. [ws,mo]:2 [eif,al,mo]:3 [fs,ch,mo]:3 [trp,ch,mo]:4 [ac,ch,mo]:5 [ls,mo]:5
5. [eif,al,mo]:3 [fs,ch,mo]:3 [fs,ws,mo]:4 [myse,ws,mo]:4 [trp,ch,mo]:4 [ac,ch,mo]:5 [ls,mo]:5 [ac,ws,mo]:6 [sec,ws,mo]:9
6. [fs,ch,mo]:3 [ls,eif,al,mo]:4 [fs,ws,mo]:4 [myse,ws,mo]:4 [trp,ch,mo]:4 [ac,ch,mo]:5 [ls,mo]:5 [ac,ws,mo]:6 [sec,ws,mo]:9
7. **[ls,eif,al,mo]:4** [fs,ws,mo]:4 [myse,ws,mo]:4 [trp,ch,mo]:4 [ac,ch,mo]:5 [ls,mo]:5 [ac,ws,mo]:6 [sec,ws,mo]:9

Goal found after 7 node expansions; least-cost path to ls

Paths Explored by LCFS in Example



Red paths: expanded
Black paths: added to frontier, but not expanded

Properties of LCFS

- Guaranteed to find least-cost path under certain circumstances
- If all arc costs are greater than 0 (assume a solution exists)
  - exercise: prove it will find least-cost path
  - what can happen if we have negative arc costs?
- Space and time complexity similar to BrFS
  - note: BrFS is a special case of LCFS when all arc costs are “uniform” (e.g., all arc costs are 1)

Uninformed Search Strategies

- For any search strategy so far (DFS, BFS, LCFS) suppose I give you goal *g1* and ask you to trace the paths explored. Then I change the goal to *g2* and ask you to repeat the process.
- Both traces will look the same (up to the point that the goal is found)
- These search strategies are *blind* or *uninformed*
  - search process in uninfluenced by the goal
  - e.g., in LCFS (goal=ls), first step is toward ch
  - e.g., Craig often turns right at red lights no matter what direction he’s heading

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Heuristics

- **Heuristics** generally refer to any rules of thumb that provide some help when solving a problem
  - e.g., an estimate/guess as to best way to proceed
  - generally guidance is not perfect
- In graph search, a *heuristic function* h(n) is an estimate of cost to goal g from node n
  - Why an estimate? What if h(n) were perfect?
  - Exercise: prove that if h(n) is true cost to goal for each n, you can find best path without backtracking
  - Note: h(n) will vary with goal g; so we sometimes write $h(n,g_1)$, $h(n,g_2)$, etc. for emphasis

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Good Heuristics

- Where do heuristics come from?
  - depends on the problem we’re trying to solve
  - planning? we’ll look at some
  - chess? rules of thumb about board position (vulnerability, number of pieces, etc.)
  - Manhattan bike courier? see handout of “grid”
- Features of a good heuristic function
  - should be somewhat accurate
  - should be easy to compute (e.g., if it requires lots of search, that defeats the purpose!)
  - should underestimate true cost (for reasons we’ll see)

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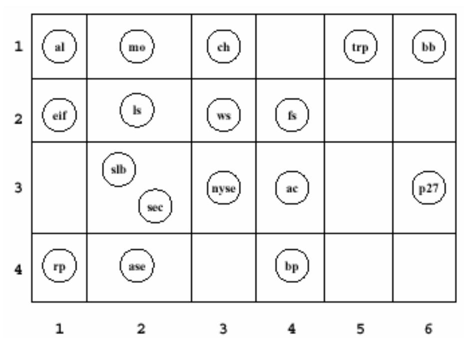
Heuristic for MBC



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Heuristic for MBC (see handout)

For instance, if our Goal location was `slb`, we could represent our heuristic function directly as follows:

```
h(mo, 2).  h(slb, 0).  h(trp, 5).  h(sec, 0).  h(fs, 3).  h(ch, 3).
h(bb, 6).  h(ws, 2).   h(eif, 2).  h(nyse, 1). h(ac, 2).  h(rp, 2).
h(al, 3).  h(p27, 4).  h(ase, 1).  h(ls, 1).   h(bp, 3).
```

A generic heuristic for arbitrary goals h(n,g):

```
md(Loc,G,D) :- coord(G,X1,Y1), coord(Loc,X2,Y2), dist(X1,Y1,X2,Y2,D).

dist(X1,Y1,X2,Y2,D) :- dist2(X1,X2,X), dist2(Y1,Y2,Y), D is X+Y.

dist2(X1,X2,Z) :- X1 >= X2, Z is X1-X2.
dist2(X1,X2,Z) :- X1 < X2, Z is X2-X1.

coord(al,1,1).  coord(mo,1,2).  coord(ch,1,3).  coord(trp,1,5). etc...
```

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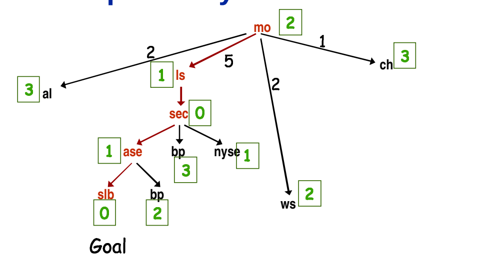
Best-first Search (BeFS)

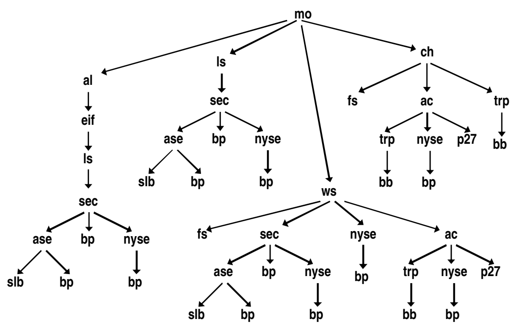
- We can use heuristics to guide search in heuristic DFS (see text), best-first search, A*
- **Best-first search** works just like LCFS except we attach h(n) to each path instead of g(n)
  - i.e., priority queue sorts paths based on h(n) value
  - we explore paths whose end points *appear to be closest to the goal* (according to h)

Paths Explored by BeFS: *mo* to *slb*



Red paths: expanded
Black paths: added to frontier, but not expanded

Search Tree: MBC Acyclic; Start *mo*



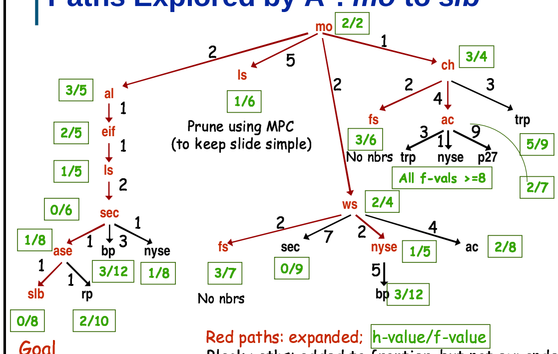
Problem with BeFS

- In previous example, BeFS guides us *very* directly to a path to slb (in fact, *no* backtracking)
- Unfortunately, not the least-cost path
- Indeed, BeFS ignores arc costs altogether!
  - chooses path to expand based only on estimated cost-to-go, h(n), and is uninfluenced by cost of path so far g(n)
  - makes sense if you've already "gone" to the node, but not if you're searching for the shortest path

A* Search

- A* search combines aspects of LCFS and BeFS
  - we use both h(n) and g(n) when choosing paths
- Quality of path on frontier is given by the **evaluation function**: f(n) = g(n) + h(n)
- Paths are ordered on the frontier according to f-value f(n)
  - if expanded path is not a soln, it is extended by its neighbors; which are inserted according to f-values
  - always select path from frontier with minimal f-value
  - Implementation: priority queue sorted on f-value

Paths Explored by A*: *mo* to *slb*

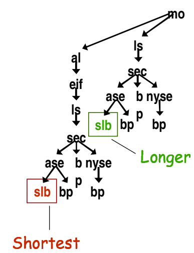


A* Analysis

- In this example, A* leads pretty directly to the goal *slb*
  - it expands six "false leads" and "prunes" one more
- A* also found the least-cost path to *slb*
- Seems to combine the best of LCFS (best path) and BeFS (goes fairly directly to the goal)
- Space and time complexity similar to BrFS
  - note: BrFS and LCFS are special cases of A* (under what conditions?)

Properties of A* (Informally)

- Will A* always find shortest path?
- Not necessarily:
  - suppose *h(al) = 17* in our example?
  - this very misleading (and pessimistic!) estimate of cost-to-go from *al* means it won't get expanded before *[ls, mo]*
  - will find longer path to *slb*

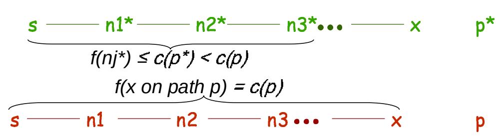


Admissible Heuristics

- Suppose h(n) never overestimates the true cost-to-goal from n?
  - A* will find least-cost path (assuming arcs costs > 0)
  - a heuristic s.t. $h(n) \leq mincost(n,g)$ is **admissible**
  - our example heuristic turns out to be admissible
- Special case: let *h(n) = 0* for all *n*
  - since *f(n) = h(n) + g(n) = g(n)*: reduces to LCFS
  - an admissible, but uninformative heuristic
- In general, the more "informative" *h(n)* is, the better A* will perform (more "direct" search)
  - Exercise: Prove that if *h(n) = mincost(n,g)* – that is, *h(n)* is perfect – A* will find optimal path directly (no backtracking)

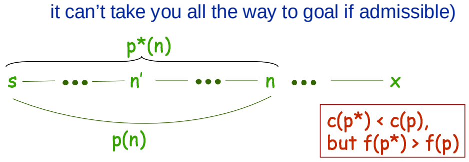
Optimality of A* (Intuitions)

- Assume admissible heuristic h
  - Let p be a nonoptimal path to goal x with cost c(p)
  - Let p* be optimal path to goal x with cost c(p*) < c(p)
  - Note: every subpath q of p* has f-value ≤ c(p*) < c(p) since h is admissible
  - So every such path—including p* -- will be expanded (removed from frontier) before p
  - Note: *some* subpaths of p can be expanded, but not p

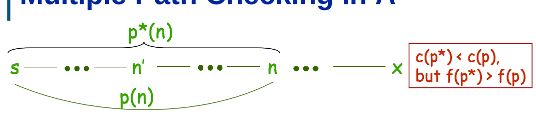


Multiple Path Checking in A*

- MPC: If you find a path to node n that you've already expanded, don't expand it again
  - was OK for BFS and LCFS, since first path expanded to any node n was assured to be shortest/cheapest
  - In A*, you can be misled by heuristic that takes you all the way to node *n* along an "expensive path" (though it can't take you all the way to goal if admissible)



Multiple Path Checking in A*



- In example, p expanded before p*, and MPC ignores shorter path p* to node n
  - MPC can destroy optimality of A*
- But this can only happen if:
  - some n' on p* is on frontier, with $f_{p*}(n') > f_p(n)$
- But $g_{p*}(n') + dist(n',n) < g_p(n)$
- So we must have h(n') > h(n) + dist(n,n')
  - thus h(n') makes n' look worse than n by more than the actual distance it takes to get from n' to n
  - this can happen even if h is admissible: basically it means heuristic is too optimistic about n relative to n'

The Monotone Restriction

- Can insist h satisfy the **monotone restriction:**

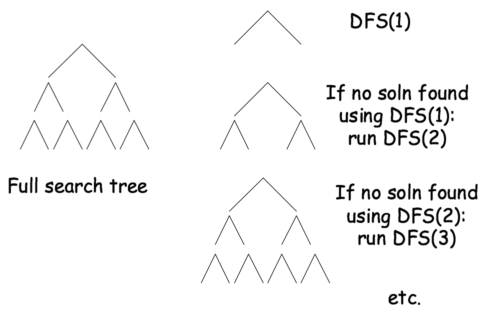
  $|h(n,) - h(n)| \le d(n',n)$ *for all nodes n, n'*

- This is enough to ensure that MPC can be performed safely with A* (i.e., MPC will preserve optimality)

Iterative Deepening (IDS)

- IDS is motivated by the following tension:
  - BFS guarantees optimal soln, requires expnt'l space
  - DFS requires linear space, can't guarantee optimality
  - How can we get best of both worlds?
- Trick: add a depth bound d to DFS
  - normal DFS, but never expand path with length > d
- How do I ensure I find solution if one exists?
  - if failure at depth bound d, increase bound and repeat
- How do I ensure shortest path is found first?
  - use the depth bounds: d=1, d=2, d=3, d=4, etc.

Iterative Deepening Graphically



Properties of IDS

- Guaranteed to find shortest solution
- Will only use linear space:
  - O(db) space with depth bound d, branching factor b
  - Important: do *not* "save" results from previous iteration
- How do we get this benefit?
  - we're repeating computation!
  - At depth bound d, we repeat all computation done at all earlier depth bounds. The only "new" steps are the expansion of leafs from previous iteration
- Why redo? Why not store previous tree?
  - requires exponential space

What Price do We Pay?

- IDS seems silly: a lot of wasted effort it seems!
  - but how bad is it compared to BFS?
  - Assume shortest soln has length d
- BFS generates:

  $b^d + b^{d-1} + b^{d-2} + \ldots + b^0 = O(b^d)$ nodes

- IDS generates:

  $b^d + 2\, b^{d-1} + 3\, b^{d-2} + \ldots + d\, b^0$ nodes

  which is roughly $b^d (1-1/b)^{-2} = O(b^d)$ nodes

Benefit of IDS

- We pay a **constant** time overhead (compared to BFS) for **exponential** space savings!
- Note: constant factor $(1-1/b)^{-2}$ is pretty small
  - if b = 2, overhead factor is 4 (4 times as long as BFS)
  - if b = 4, overhead factor is 1.8
  - overhead factor decreases with b!
- Iterative Deepening can be used with A*: **IDA***
  - basically, do DFS, but let "depth bound" be maximum f-value you consider, and increase f-value-bound gradually

Implicit Search Graphs

- Most search problems are not specified with explicit search graphs; nbr predicate "creates" neighboring states *on the fly*
  - chess, SLD-derivations, planning robot activity, etc.
- Example: 8-puzzle
  - Each board position a state
  - 9! = 362880 states
  - each state has 2, 3, or 4 nbrs
  - nbrs correspond to possible moves
  - nbr predicate: returns list of states reachable
- State Representation? Neighbor implementation? Possible Heuristics? see assignment 2!

1	2	3
8		4
7	6	5

Other Issues

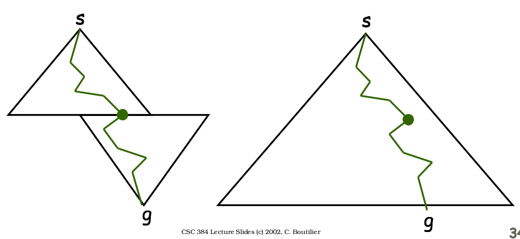
- Suppose list of neighbors is too large:
  - to add to frontier? to calculate all heuristic values?
  - What might one do? How could you use heuristic info to limit your attention?
  - One possibility: *generate* neighbors in heuristic order (only a subset of nbrs ever put on frontier)
  - can destroy optimality unless more nbrs added when backtracking
- Other things we can do to increase efficiency?
  - control the direction of search

Backward Search

- *Backward* branching factor is the (avg) set of moves that can be made *to* a specific node
  - if I have the inverse nbr relation available, I can search in the graph backwards from the goal to the start state
- Advantage: if backward BF *b-* less than forward BF *b+,* then search algth'm (any type) benefits
  - examples: planning (as we'll see later)
  - lower time and space complexity since optimal path length still the same
  - heuristic methods need a *backwards* heuristic, though

Bidirectional Search

- Search simultaneously in both directions
  - if two frontiers intersect, you can "join" forward and backward paths to node in intersection to get a sol'n
  - contrast # expansions for b-d BrFS vs. normal BrFS



Bidirectional Search

- Suppose we do BrFS
  - length of sol'n (shortest path) is k
  - branching factor (frwd/bkwd) is b
- Each component of the bidirectional search expands $O(b^{k/2})$ nodes
- Normal BrFS expands $O(b^k)$ nodes
- Bidirectional is exponential, but offers exponential savings
- Issues: need bkwd dynamics, need to test intersection, must choose search alg. carefully

Island Search

- Suppose you know that any (good) path to goal must pass through *island states* $i_1, i_2, \ldots i_k$
  - e.g., must pass through specific tunnels to deliver pkg
- Complexity can be cut significantly by searching for path from s to i_1, i_1 to i_2, …, i_{k-1} to i_k, i_k to g
  - what is potential savings (say) for BrFS using this strategy if avg subpath between islands has length m?