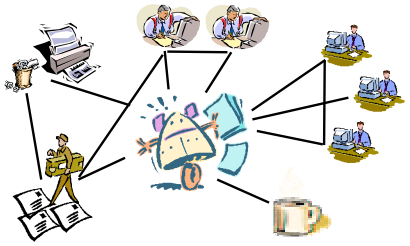
CSC384: Lecture 4

- Last time
  - done with DCL (except for the fact that we'll **use** it!)
- Today
  - quick summary of uses of DCL (from last time)
  - Intro to search; generic search procedure; BrFS, DFS, path extraction, cycle and mult.path checking
- Readings:
  - Today: Ch.4.1 – 4.4, 4.6
  - Next week: Ch.4.5/4.6

A Planning Problem

- A planning problem: we want robot to decide what to do; how to act to achieve our goals



A Planning Problem

- How to *change* the world to suit our needs
- Critical issue: we need to reason about ***how the world will be*** after doing a few actions, not *just* what it is like now

GOAL: Craig has coffee
CURRENTLY: robot in mailroom, has no coffee, coffee not made, Craig in office, etc.
TO DO: goto lounge, make coffee,...

Planning

- So far, we've seen DCL used to reason about *static environments* (what the world *is like*)
  - what is correct treatment for symptom X?
  - is this region water or land?
- Want to use DCL to reason about *dynamic environments*
  - A,B,C are true: will they still be true after doing X?
  - A,B,C are true: what do I need to do to make D true?
- The heart of decision making!
  - complexities: uncertainty (where is craig? navigate stairs?); *many actions to choose from (what is right sequence?);* exogenous events (battery loses charge)

Graph Search

- We'll abstract away complexities of planning
  - focus on **finding the right sequence of actions** only
  - going to ignore the fact that at any point in time many things are true and false
- Treat planning as the *search for a path* from one state (of the world) to a desired goal state
- **Informally**: We have a set of states, and a set of moves/actions that take us from one state to another. Given an initial state (current) and target state (goal), find a sequence of moves that gets me from initial state to goal
  - or *shortest* sequence, or *cheapest* sequence, or …

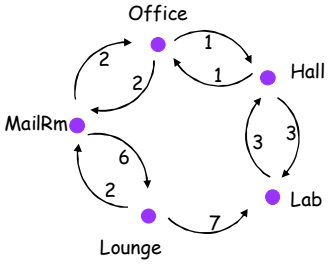
Graph Search is Very General

- This viewpoint applies to a wide variety of tasks
  - **RoboCof** – find plan that gives Craig coffee; route across floor plan; etc. [states? moves? objective?]
  - **Games** – 8-puzzle; backgammon; chess; Doom; [complications: other players making moves]
  - Scheduling, logistics, planning, most optimization problems
  - **Medical diagnosis**
  - **Scene interpretation** – find a consistent labeling
  - **Finding a derivation** in DCL/Prolog [what are states? moves? goal?]
  - **Finding a good travel package** [states? moves?]
  - Almost every problem in AI can be viewed this way!

Graph-based Search Formalization

- A directed graph: set of nodes N, set of directed edges $E \subseteq N \times N$ (these are *ordered* pairs)
  - nodes correspond to states we can move among
- If $\langle n_1, n_2 \rangle \in E$, then n_2 is a *neighbor* of n_1
  - written $n_1 \rightarrow n_2$ (relation is not symmetric)
  - edges correspond to possible moves we can make
- A *node labeling* is a function $L_n: N \rightarrow Nlabels$
  - denote properties of states (e.g., a value)
- An *edge labeling* is a function $L_e: E \rightarrow Elabels$
  - denote properties of edges (e.g., a move cost)

An Very Simple Search Graph



Nodes (states) are locations robot can move among

Edges are routes among these locations

Edge labels denote costs (e.g., expected travel time)

Paths and Cycles

- A *path* in graph G =(N,E) is a sequence of nodes $\langle n_1, n_2, \ldots n_k \rangle$ s.t. each $\langle n_i, n_{i+1} \rangle \in E$
  - $n_1 \rightarrow n_2 \rightarrow \ldots \rightarrow n_k$ is a path from n_1 to n_k
- A *cycle* is a path $\langle n_1, n_2, \ldots n_k \rangle$ with $n_1 = n_k$ $(k > 1)$
  - A graph G is acyclic if no path in G is a cycle
  - A path is acyclic of no subpath is a cycle

Graph Search Problems

- A **graph search problem**: given graph (N,E), a start node $s \in N$, and a set of goal nodes $G \subseteq N$, find a path P from s to some $g \in G$ satisfying property X.
- Possible properties X:
  - X = "null": Any path to any node in G will do
    - *Find me any path to the office*
  - X = "P is the *best* path to goal G": this means we have some optimization criterion we need to satisfy
    - possible criteria: shortest (# edges); least cost; etc.
    - *Find me shortest/fastest path to office*
  - X = "P has quality $\geq q$" (a satisficing problem)
    - *Find me a path that gets me to the office by 10AM*

A Couple Notes

- If arcs are labeled with actions, we often want to return the sequence of actions, not just the path
- Most interesting search problems involve *implicit search graphs*
  - e.g., consider chess: nobody constructs a graph of all 10^{30} board positions explicitly
  - e.g., Prolog: answer clauses generated as derived
  - when solving search problem, we generate neighbors as we need them
  - define states, neighborhood relation…or define moves and how to generate neighboring state

Generic Search Procedure

- Many graph search techniques share the following common structure
  - let the **frontier** refer to set of nodes we already know how to reach from the start node *s*

1. Let F = {s} ;; (initial frontier is start node)
2. Loop until frontier is empty
   (a) Choose some node *n* on frontier; remove *n* from F
   (b) If $n \in G$ then stop; report success
   (c) Otherwise add each neighbor of *n* to F

A Generic DCL Implementation

```
search( F ) :- select(Node, F, RemF), is_goal(Node).

search( F ) :- select(Node, F, RemF),
               nbs(Node, NbList),
               add_to_frontier(RemF, NbList, NewF),
               search( NewF ).
```

1. nbs(Node, NbList) defines the search graph
   - for each node, we assert its list of nbs in KB
   - if implicit search graph, nbs will generate NBList
2. is_goal(N) defines the set of goal nodes
   - for each goal node, we assert this in KB

Generic Search Procedure

- Assume frontier represented as a list of nodes
  - *search(F)* returns *yes* there is some path to *G* from *F*
  - so call with start node *s* on frontier: *search([s])*
- *select(N,F,RemF):* true if selecting node *N* from frontier *F* leaves remaining frontier *RemF*
- *add_to_frontier(RemF,NbList,NewF):* true if adding nodes in *NbList* to *RemF* results in *NewF*

One Instantiation of Search Proc.

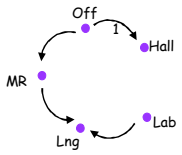
```
search( F ) :- select(Node, F, RemF), is_goal(Node).
search( F ) :- select(Node, F, RemF), nbs(Node, NbList),
               add_to_frontier(RemF, NbList, NewF),
               search( NewF ).

select(Node, [Node|RemF], RemF).

add_tf (RemF, NbList, NewF) :-
                append(NbList, RemF, NewF).
```

- Simple implementation
  - you can only select the first node on the frontier
  - you add neighbors of selected node to the beginning of the frontier (just append them to front)

A Modified Example



Graph:

```
nb(off,[mr,h]).
nb(h, []).
nb(mr, [lng]).
nb(lng, []).
nb(lab, [lng]).
```

Goal Node:

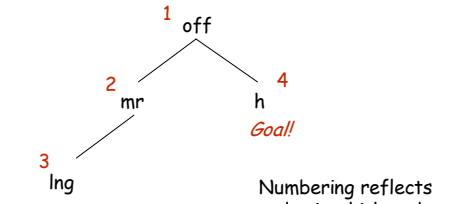
```
isgoal(h).
```

Start Node:

```
office
```

```
search([off]).
  select off, RF = []
  not a goal, NF = [mr, h]
search([mr, h]).
  select mr, RF = [h]
  not a goal, NF = [lng, h]
search([lng]).
  select off, RF = [h]
  not a goal, NF = [h]
search([h]).
  select off, RF = []
  is a goal, NF = [mr, h]
Return yes!
```


Search Tree for Modified Example



Numbering reflects order in which nodes are examined (isgoal?)

What if we added an edge from lng to lab?

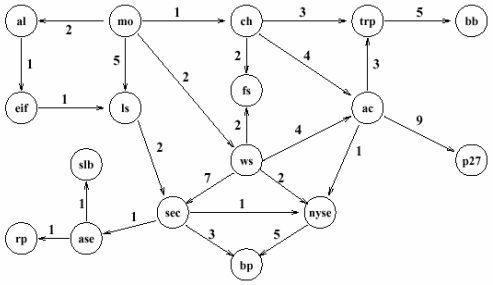
Depth-First Search

- The specific instantiation we've seen is simply **depth-first search** (which you've seen previously, no doubt)
  - in the search tree, we work deep into the tree until we reach a dead-end, then we "backtrack"
- In generic search algorithm, this is achieved by:
  - always selecting first node on the frontier
  - always inserting the new neighbors at head of frontier
- Note: all "bookkeeping" required for backtracking is taken care of by organization of the frontier

Defn of Search Tree

- Given a search graph, start node *s*
- The *search tree rooted at s* is a tree such that:
  - root node is *s*
  - each node has all its neighbors for children
- So at level *k* of the tree are all states that you can reach in exactly *k* moves (where root = 0)
  - each path in search graph (including cyclic paths) is a path through search tree
  - nodes in graph can appear *many* times in tree
- Frontier moves "down" the tree

Manhattan Bike Courier (Acyclic)

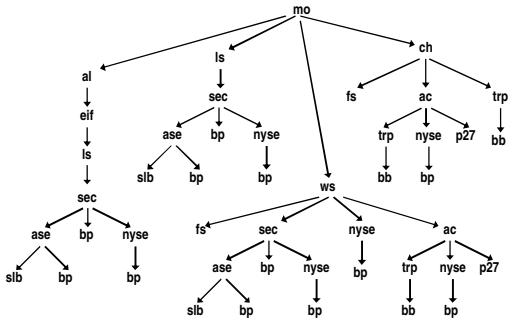


The MBC Rep'n (No Costs)

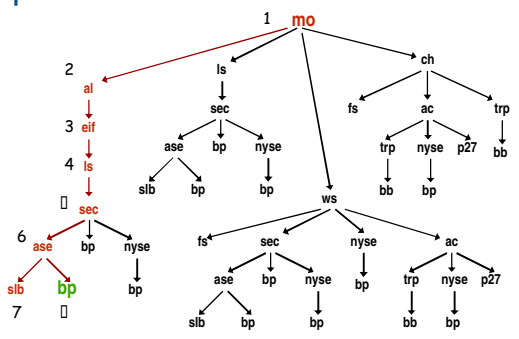
mo:	Main Office	ch:	City Hall	trp:	T.Rowe Price
al:	Alley	fs:	Fulton Street	bb:	Brooklyn Bridge
eif:	Ellis Island Ferry	ws:	Wall Street	ac:	Anderson Consulting
ls:	Loeb Securities	sec:	Securities Exch. Comm.	p27:	Pier 27
slb:	Shearson-Lehmann Bros.	bp:	Battery Park	nyse:	NY Stock Exch.
rp:	Rector Park	ase:	Amer. Stock Exch.		

```
nb(mo, [al, ls, ws, ch]).     nb(slb, []).
nb(sec, [ase, bp, nyse]).     nb(fs, []).
nb(ws, [fs, sec, nyse, ac]).  nb(eif, [ls]).
nb(ac, [trp, nyse, p27]).     nb(al, [eif]).
nb(ase, [slb, rp]).           nb(ls, [sec]).
nb(ch, [fs, ac, trp]).        nb(rp, []).
nb(trp, [bb]).                nb(p27, []).
nb(bb, []).                   nb(bp, []).
nb(nyse, [bp]).
```


Search Tree: MBC Acyclic; Start *mo*



DepthFirst Search: Start *mo;*Goal *bp*



Example Summary

- DFS *expands* eight nodes in this example
  - it examines eight nodes and tests if they are the goal (if they are not, it add neighbors to frontier)
- DFS does not find the shortest path to bp
  - the order in which it is required to examine nodes doesn't allow it to find the shortest path

Path Extraction

```
search( F ) :- select(Node, F, RemF), is_goal(Node).
search( F ) :- select(Node, F, RemF), nbs(Node, NbList),
               add_to_frontier(RemF, NbList, NewF),
               search( NewF ).
```

- Generic search procedure behaves as follows:
  - assert *isgoal(bp)* ; ask *?search([mo]).*
  - algorithm says *yes*.
  - but what path does courier take??
  - says *yes* if there is a path, but doesn't tell us what it is
- We need a *path extraction mechanism*
  - simplest thing to do is store paths on frontier
  - when you select a node from frontier, you also have a specific path to that node attached
- *search(F,Path) :* true if Path is solution to search

Implementing Path Extraction

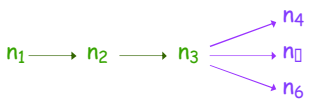
Assume a path is represented as a list of nodes in reverse order: so the path mo → al → eif is represented as the list: [eif, al, mo]

```
search( F, [Node | Rest] ) :-
    select( [Node | Rest] , F, RemF),
    is_goal(Node).

search( F, [Path] ) :-
    select( [Node | Rest] , F, RemF ),
    nbs(Node, NbList),
    extendpath(NbList, [Node | Rest], NewPaths ),
    add_to_frontier(RemF, NewPaths, NewF),
    search( NewF, [Path] ).
```


Notes on Path Extraction

- We call search from start node *mo* using *search([[mo]], Path).*
  - initial frontier consists of a length one *path* (not node)
- Only new predicate needed is *extendpath*
  - basic idea: given a path $[n_3, n_2, n_1]$ and neighbors of n_3 specified by $nbs(n_3, [n_4, n_5, n_6])$; it produces 3 new paths organized in a list:
    $[[n_4, n_3, n_2, n_1] , [n_5, n_3, n_2, n_1] , [n_6, n_3, n_2, n_1]]$



DFS with Path Extraction

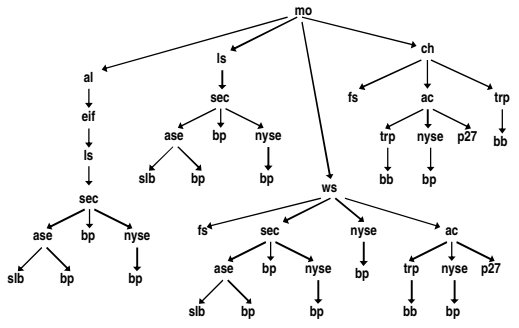
```
search( F, [Node | Rest] ) :-
    select([Node | Rest], F, RemF),
    is_goal(Node).

search( F, Path ) :-
    select( [Node | Rest], F, RemF ),
    nbs(Node, NbList),
    extendpath(NbList, [Node | Rest], NewPaths ),
    add_to_frontier(RemF, NewPaths, NewF),
    search( NewF, Path ).

select(Path, [Path|RestPaths], RestPaths).

add_tf (RemF, Paths, NewF) :-
                append(Paths, RemF, NewF).
```


Search Tree: MBC Acyclic; Start *mo*



Trace of DFS (with paths: *mo* to *fs*)

Frontier evolution:

1. [mo]
2. [al,mo] [ls,mo] [ws,mo] [ch,mo] (=)
3. [eif,al,mo] ..
4. [ls,eif,al,mo] ..
5. [sec,ls,eif,al,mo] ..
6. [ase,sec,..,mo] [bp,sec,..,mo] [nyse,sec,..,mo] .. .. (=)
7. [slb,ase,..,mo] [rp,ase,..,mo] .. .. ..
8. [rp,ase,..,mo] .. ..
9. [bp,sec,..,mo] [nyse,sec,..,mo] ..
10. [nyse,sec,..,mo] ..
11. [bp,nyse,sec,..,mo] .. ..
12. [ls,mo] [ws,mo] [ch,mo]
13. [sec,ls,mo] [ws,mo] [ch,mo]

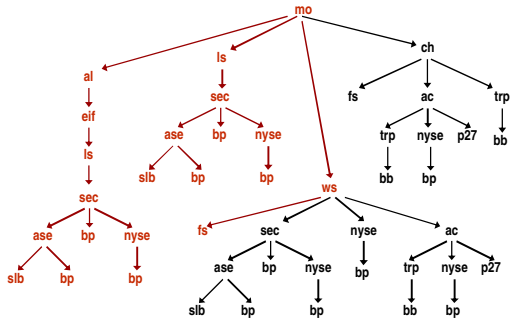
14-19. Exactly like expansion of [sec,ls,wtc,al,mo] in Steps 6-11

20. [ws,mo] [ch,mo]
21. [fs,ws,mo] [sec,ws,mo] [nyse,ws,mo] [ac,ws,mo] [ch,mo]

GOAL = fs is found

Total Search steps: 21 nodes expanded

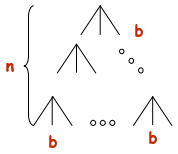
Paths Explored by DFS in Example



Properties of DFS: Time

- How long can DFS take?

(a) **Finite graph, no cycles**:
- Could explore each branch of search tree (until goal found or search fails).
- If branching factor bounded by b, depth bounded by n, then we explore $O(b^n)$ full paths (length n)
- note: $n \leq N$ (number of nodes in G)

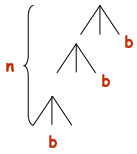


(b) **Finite graph with cycles**:
- may not terminate unless we perform cycle checking (later)
- Does it ever make sense to explore a cyclic path?

(c) **If we're lucky with the node ordering**:
- may find soln in n* steps (n* is length of shortest path to goal)

Properties of DFS: Space

- How many paths on frontier at any one time?
  - If current path length m (i.e., current node selected), then there are *bm* paths on frontier
  - If longest path is length *n*, then never more than *bn* paths
  - *b* paths have length 1, *b* length 2, etc. up to *b* paths of length *n*
  - Total space $b + 2b + 3b + \ldots nb$ which is $O(n^2b)$ space
- Is quadratic space required?
  - No: many paths have common substructure
  - Consider how to store frontier in linear space O(nb): use a tree!



Properties of DFS: Solution Quality

- Will DFS find the shortest (min # arcs) solution?
  - In general, no.
  - It can if you are lucky.
  - In Manhattan (acyclic) problem, with start mo and goal ls, it returns soln: mo → al → eif → ls, even though shorter solution mo → ls exists.
- So how can we find shortest path?

Breadth-First Search

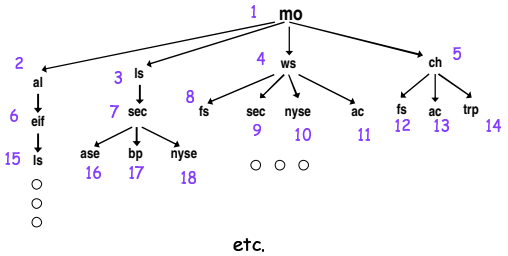
- One way to ensure shortest solution is found (wrt # of arcs) is to explore paths in order of length
- **Breadth-first search (BFS)** does exactly this
- It is implemented by selecting nodes/paths from the front of the frontier (like DFS), but inserting new neighbors/paths at the *end of the frontier*

```
select(Path, [Path|RestPaths], RestPaths).

add_tf (RemF, Paths, NewF) :-
        append( RemF, Paths, NewF).
```

differs from DFS only in *order* of arguments to append

Frontier Growth in BFS



etc.

Trace of BFS (with paths: *mo* to *fs*)

Frontier evolution:

1. Length 0 Paths

[mo]

2. Length 1 Paths (inserted after 1 len1 path) inserted at end of frontier in order shown

[al,mo] [ls,mo] [ws,mo] [ch,mo]

3. Length 2 Paths (inserted after 4 len1 paths) inserted at end of frontier in order shown

[eif,al,mo] [sec,ls,mo] [fs,ws,mo] [fs,ch,mo]
[sec,ws,mo] [ac,ch,mo]
[nyse,ws,mo] [trp,ch,mo]
[ac,ws,mo]

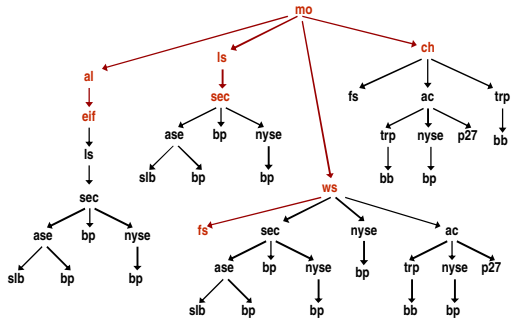
4. Length 3 Paths (inserted after 9 len2 paths) inserted at end of frontier in order shown

[ls,eif,al,mo] [ase,sec,ls,mo] (plus 14 more paths not added to frontier)
[bp,sec,ls,mo]
[nyse,sec,ls,mo]

On Third Step
(3rd Path in the Length 2 Frontier),
the goal FS will be found.

Total Search steps:
1+4+3 = 8
8 nodes expanded

Paths Explored by BFS in Example



Properties of BFS

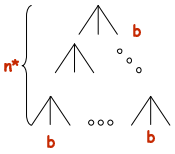
- All paths of length *k* occur on frontier after all length *k-1* paths
- No length *k* path is added to frontier until we have expanded all length *k-1* paths
- This last property ensures that we are guaranteed to find shortes path (if sol'n exists)
  - If we find a length *k* sol'n, since we've looked at all shorter paths, no shorter sol'n exists

Properties of BFS: Time

- How long can BFS take?

(a) **Finite graph, no cycles**:

- If branching factor bounded by b, and the shortest sol'n has length n*, then we'll explore $O(b^{n*})$ paths (length n)
- note: presence of cycles has no effect if a solution is present
- If no solution, will explore all paths: $O(b^{|N|})$

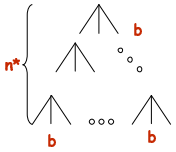


(b) **Finite graph with cycles**:

- if sol'n: same as above
- if no sol'n: may not terminate unless we perform cycle checking or multiple path checking

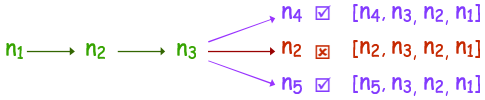
Properties of BFS: Space

- How many paths on frontier at any one time?
  - If current path length *m*, then there are between b^{m-1} and b^m paths on frontier
  - If shortest path has length *n**, then guaranteed to have frontier of size $O(b^{n*})$
  - Luck with node ordering plays no role: BFS is systematic
- Space cost is the price you pay for optimality



Cycle Checking (see text for more)

- Cycles can hurt DFS: can prevent termination
- **Cycle checking** in DFS: requires a simple test
  - when you select a path from frontier and extend it with its neighbors, we only add a new path to the frontier if the neighbor is not already on the path
- Test requires linear time in length of path
  - some tricks can be used to reduce this
- Cannot affect existence of soln, rule out best soln



Multiple Paths to Same Node

- In Manhattan example, with start = mo:
  - at depth 2, we have path P_1 = [ls, mo]
  - at depth 4, we have path P_2 = [ls, eif, al, mo]
- Why add P_2 to the frontier?
  - If there is a path from ls to goal, then extension of P_1 to the goal is shorter than the extension of P_2 (each extension of P_2 added to frontier is just wasted)
  - If there is no path from ls to goal, not adding P_2 to frontier cannot hurt

Multiple Path Checking in BFS

- Cycle checking can be applied to BFS too
  - saves some time (don't explore cyclic path)
  - ensures termination in cyclic graphs with no solution
- **Multiple path checking** is more general
  - Every time a (path to a) node is considered for addition to frontier, check list of *visited* nodes (those that have already been expanded).
  - If node is on list, do not add it to the frontier.
  - If node is not on list, add it to frontier and visited list.
- In BFS, need an extra argument: *VisitedList*
- MPC subsumes cycle checking

Notes on MPC

- In BFS, we need an extra argument, *VisitedList,* to maintain the list of visited nodes
  - what would you initialize *VisitedList* with on first call?
- MPC *subsumes* cycle checking
  - a cycle is just one type of "multiple path" to same node
- Why doesn't MPC make sense for DFS?
- Exercise: Sketch out revised clauses defining:
  - DFS with cycle checking
  - BFS with MPC