

## CSC384: Lecture 3

- Last time
  - DCL: syntax, semantics, proofs
  - bottom-up proof procedure
- Today
  - top-down proof procedure (SLD-resolution)
  - perhaps start on uses of DCL
- Readings:
  - Today: 2.7; 2.8 (details in tutorial),
    - perhaps Ch.3 (excl. 3.7); we'll discuss only part
  - Next week: wrap Ch.3; start on Ch.4: 4.1-4.4/4.6

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## Top-Down Proof Procedure

- BUPP is data-driven
  - not influenced by query  $q$ , just facts and rules in  $KB$
  - wasteful: proves things unneeded to prove  $q$
- **Top-down proof** procedure is query-driven:
  - focussed on deriving a specific query
- We'll describe a TDPP called **SLD-resolution**
  - Basically, the strategy implemented within Prolog
  - stands for **selected linear, definite-clause** resolution

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## SLD-Resolution (No vars)

- Basic intuitions:
  - suppose we have query  $?q_1 \& q_2$
  - suppose we have rule  $q_1 \leftarrow a \& b \& c$ .
  - if we prove **subgoal query**  $?a \& b \& c \& q_2$  then we know that original query must be true
- SLD a form of backchaining or subgoaling:
  - to prove  $q$ , we look for a rule with the head  $q$ , and then attempt to prove the body of that rule; if proven, we know  $q$  must be a consequence of KB
  - Progress: when subgoals are facts!
- **Defn:** An **answer clause**:  $yes \leftarrow q_1 \& \dots \& q_m$
- **Defn:** An **answer**:  $yes \leftarrow$ .

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## SLD-Resolution: Algorithm (no vars)

Given query  $?q_1 \& \dots \& q_m$  and a KB

1. Construct answer clause  $yes \leftarrow q_1 \& \dots \& q_m$
2. Until no KB-clause **choosable** or AC is an **answer**
  - (a) Select an atom  $a_i$  from the current AC  
 $yes \leftarrow a_1 \& \dots \& a_k$
  - (b) Choose a clause  $a_i \leftarrow b_1 \& \dots \& b_n$  from KB whose head matches selected atom
  - (c) Replace  $a_i$  in AC with body to obtain new AC  
 $yes \leftarrow a_1 \& \dots \& a_{i-1} \& b_1 \& \dots \& b_n \& a_{i+1} \& \dots \& a_k$

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## SLD-Resolution

- If we reach an answer, return YES
  - query is a logical consequence of KB
- If we find no choosable clauses, return NO
  - query not a consequence (but not necessarily false)
- A sequence of answer clauses that culminates in an answer is an **SLD-derivation of the query**
- Our algorithm **attempts** to find a derivation:
  - if it chooses incorrectly at Step 2, it may fail
  - see text for distinction between **choice** and **selection**
  - we say derivation attempt **fails** if we get stuck
  - how does Prolog deal with failure?

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## SLD: Example

KB: (1)  $a \leftarrow b \& c$ .  
(2)  $b \leftarrow d \& e$ .  
(2')  $b \leftarrow c$ .  
(3)  $b \leftarrow g \& e$ .  
(4)  $c \leftarrow e$ .  
(5)  $d$ .  
(6)  $e$ .  
(7)  $f \leftarrow a \& g$ .  
Query:  $?a$

### Derivation Attempt #1

$yes \leftarrow a$ .  
 $yes \leftarrow b \& c$ .      Select a; choose (1)  
 $yes \leftarrow g \& e \& c$ .      Select b; choose (3)  
 $yes \leftarrow g \& c$ .      Select e; choose (6)  
Select g: FAIL! no choosable clause

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## SLD: Example

KB: (1)  $a \leftarrow b \ \& \ c.$   
 (2)  $b \leftarrow d \ \& \ e.$   
 (2')  $b \leftarrow c.$   
 (3)  $b \leftarrow g \ \& \ e.$   
 (4)  $c \leftarrow e.$   
 (5)  $d.$   
 (6)  $e.$   
 (7)  $f \leftarrow a \ \& \ g.$

Derivation Attempt #2

yes  $\leftarrow a.$   
 yes  $\leftarrow b \ \& \ c.$  Select a; choose (1)  
 yes  $\leftarrow d \ \& \ e \ \& \ c.$  Select b; choose (2)  
 yes  $\leftarrow e \ \& \ c.$  Select d; choose (5)  
 yes  $\leftarrow c.$  Select e; choose (6)  
 yes  $\leftarrow e.$  Select c; choose (4)  
 yes  $\leftarrow .$  Select e; choose (6)

Query: ?a

QUERY IS TRUE: obtained answer

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## SLD Notes

- Does atom selected to resolve away matter?
  - No: all must be "proven" eventually
- Does KB clause chosen to resolve with matter?
  - Yes: wrong choice can lead to failure
  - We'll talk later about backtracking/search for a proof
- Soundness:** should be fairly obvious
  - Exercise: prove that if any body in any answer clause is a consequence of KB, then so is query (soundness follows: if we derive an answer, query holds)
- Completeness:** if  $KB \models q$ , there is a derivation
  - can we find it? Yes, if we make correct choices
  - How? Might have to try all options (watch for cycles)

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## Aside: Resolution

$\frac{a \vee b, \neg b \vee c}{a \vee c}$

Resolution  
Proof Rule

Query  $\text{yes} \leftarrow g \ \& \ h$  equivalent to  $\neg g \vee \neg h \vee \text{yes}$   
 Rule  $h \leftarrow a \ \& \ b \ \& \ c$  equivalent to  $h \vee \neg a \vee \neg b \vee \neg c$

$\frac{\neg g \vee \neg h \vee \text{yes}, h \vee \neg a \vee \neg b \vee \neg c}{\neg g \vee \neg a \vee \neg b \vee \neg c \vee \text{yes}}$

Resolvent  $\neg g \vee \neg a \vee \neg b \vee \neg c \vee \text{yes}$   
 equiv. to  $\text{yes} \leftarrow g \ \& \ a \ \& \ b \ \& \ c$

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## Variables in SLD (no functions)

- Recall query  $q(X)$  is interpreted existentially:
  - is there some  $X$  s.t.  $q(X)$  is a consequence?
  - return a ground instance/term  $t$  (or all  $t$ ) s.t.  $q(t)$  holds
  - with no functions, terms are just constants

Example:

(1)  $\text{rich}(\text{joan}).$   
 (2)  $\text{mother}(\text{linda}, \text{joan}).$   
 (3)  $\text{mother}(\text{mary}, \text{linda}).$   
 (4)  $\text{rich}(X) \leftarrow \text{mother}(X, Y) \ \& \ \text{rich}(Y).$

Query:

?  $\text{rich}(\text{linda}).$   
 yes  
 ?  $\text{rich}(X).$   
 joan, linda, mary  
 &  $\text{rich}(Y).$

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## SLD: Queries with no vars

- Query: ? $\text{rich}(\text{linda})$ 
  - set up answer clause:  $\text{yes} \leftarrow \text{rich}(\text{linda})$
  - but body matches no heads in KB! How to start??
- Intuitively,  $\text{rich}(\text{linda})$  **does** match the head of the rule  $\text{rich}(X) \leftarrow \text{mother}(X, Y) \ \& \ \text{rich}(Y).$ 
  - just need to substitute constant  $\text{linda}$  for var  $X$
  - result:  $\text{yes} \leftarrow \text{mother}(\text{linda}, Y) \ \& \ \text{rich}(Y).$
- Applying constant substitution  $\{X/\text{linda}\}$  to rule (4) gives us an **instance** of rule (4):
  - $\text{rich}(\text{linda}) \leftarrow \text{mother}(\text{linda}, Y) \ \& \ \text{rich}(Y).$
  - Note: this instance is clearly entailed by KB

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## Example: SLD with vars in KB

KB:

(1)  $\text{rich}(\text{joan}).$   
 (2)  $\text{mother}(\text{linda}, \text{joan}).$   
 (3)  $\text{mother}(\text{mary}, \text{linda}).$   
 (4)  $\text{rich}(X) \leftarrow \text{mother}(X, Y) \ \& \ \text{rich}(Y).$

Query:

?  $\text{rich}(\text{linda}).$

Derivation:

$\text{yes} \leftarrow \text{rich}(\text{linda}).$   
 $\text{yes} \leftarrow \text{mother}(\text{linda}, Y) \ \& \ \text{rich}(Y).$   
 How: Select  $\text{rich}(\text{linda})$ ; resolve with (4) using  $\{X/\text{linda}\}$   
 $\text{yes} \leftarrow \text{rich}(\text{joan}).$   
 How: Select  $\text{mother}(\text{linda}, Y)$ ; resolve with (2) using  $\{Y/\text{joan}\}$   
 $\text{yes} \leftarrow .$   
 How: Select  $\text{rich}(\text{joan})$ ; resolve with (1) using  $\{\}$

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## SLD: Queries with vars

- Query:  $?rich(Z)$ 
  - set up answer clause:  $yes(Z) \leftarrow rich(Z)$
  - once derivation reaches an answer, this allows us to extract an "individual" for which query holds
  - can't just say yes: must say "for who"
- Intuitively,  $rich(Z)$  **does** match the head of the rule  $rich(X) \leftarrow mother(X, Y) \& rich(Y)$ .
  - just need to substitute var  $Z$  for var  $X$
  - result:  $yes(Z) \leftarrow mother(Z, Y) \& rich(Z)$ .
- Applying substitution  $\{X/Z\}$  to rule (4) gives:
  - $rich(Z) \leftarrow mother(Z, Y) \& rich(Y)$ .

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## Example: SLD with vars in query

KB:

```
(1) rich(joan).
(2) mother(linda,joan).
(3) mother(mary,linda).
(4) rich(X) <- mother(X,Y) & rich(Y).
```

Query:

```
? rich(Z).
```

Derivation:

```
yes(Z) <- rich(Z).
yes(Z) <- mother(Z,Y) & rich(Y).
  Select rich(Z): resolve with (4) using {X/Z}
yes(Z) <- mother(Z,joan).
  Select rich(Y): resolve with (1) using {Y/joan}
yes(linda) <- .
  Select mother(Z,joan): resolve with (2) using {Z/linda}
```

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## Example: SLD with vars in query

KB:

```
(1) rich(joan).
(2) mother(linda,joan).
(3) mother(mary,linda).
(4) rich(X) <- mother(X,Y) & rich(Y).
```

Query:

```
? rich(Z).
```

A Different Derivation:

```
yes(Z) <- rich(Z).
yes(joan) <- .
  Select rich(Z): resolve with (1) using {Z/joan}
```

Different derivations can give different answers;  
Exercise: construct derivation that gives the answer "mary".

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## SLD with Variables

- To recap, we've seen SLD with:
  - variables in KB, but ground queries
  - variables in KB and variables in query
- Basic idea: we need to make appropriate substitutions of our variables in order to make atoms in answer clause match heads of KB rules
- Let's look at one more example, sticking with the "intuitive" definition of a substitution
- Then we'll formalize unifiers and MGUs

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## Example Derivation #1

KB:

```
1. busy(Z) <- teaches(Z,X) &
   teaches(Z,Y) & distinct(X,Y).
2. busy(Z) <- teaches(Z,148).
3. teaches(craig, 384).
4. teaches(craig, 2534).
5. teaches(kyros, 384).
6. teaches(kyros, 2501).
7. teaches(suzanne, 148).
8. distinct(2534,384).
9. distinct(2501,384).
   distinct...
```

Query:

```
?busy(P).
```

Answer Clause:

```
yes(P) <- busy(P).
```

Derivation:

```
yes(P) <- busy(P).
yes(P) <- teaches(P,148).
  Select busy(P): resolve with
  (2) using {P/Z}
yes(suzanne) <- .
  Select t(Z,148): resolve with
  (2) using {Z/P}
Answer: suzanne
(others: craig, kyros... show!)
```

Could have used {Z/P} instead: as long as vars match

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## SLD-Resolution: Algorithm (w/ vars)

Given query  $?q_1 \& \dots \& q_m$  with vars  $x_1 \dots x_n$  and a KB

- Construct answer clause  $yes(x_1 \dots x_n) \leftarrow q_1 \& \dots \& q_m$ .
- Until no KB-clause choosable or AC is an answer
  - Select an atom  $a_i$  from the current AC  $yes \leftarrow a_1 \& \dots \& a_k$
  - Choose a clause  $h_i \leftarrow b_1 \& \dots \& b_n$  from KB  
and a substitution  $\sigma$  that **unifies** the head  $h_i$  of the KB clause with the selected atom  $a_i$  (i.e., that when applied to  $h_i$  and  $a_i$  makes them the same)
  - apply  $\sigma$  to AC and KB clause to obtain  $AC\sigma$ ,  $KB\sigma$
  - Replace  $a_i\sigma$  in  $AC\sigma$  with body of  $KB\sigma$  to obtain new AC  
 $(yes(x_1 \dots x_n) \leftarrow a_1 \& \dots \& a_{i-1} \& b_1 \& \dots \& b_n \& a_{i+1} \& \dots \& a_k) \sigma$

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## Example Derivation #2

KB:

1.  $\text{busy}(Z) \leftarrow \text{teaches}(Z, X) \ \& \ \text{teaches}(Z, Y) \ \& \ \text{distinct}(X, Y)$ .
2.  $\text{busy}(Z) \leftarrow \text{teaches}(Z, 148)$ .
3.  $\text{teaches}(\text{craig}, 384)$ .
4.  $\text{teaches}(\text{craig}, 2534)$ .
5.  $\text{teaches}(\text{kyros}, 384)$ .
6.  $\text{teaches}(\text{kyros}, 2501)$ .
7.  $\text{teaches}(\text{suzanne}, 148)$ .
8.  $\text{distinct}(2534, 384)$ .
9.  $\text{distinct}(2501, 384)$ .
10.  $\text{d}(148, 384), \text{d}(2534, 2501), \text{d}(2534, 148), \text{d}(2501, 148)$ .

Same query:  $? \text{busy}(P)$ .

Derivation:

```
yes(P) <- busy(P).
yes(P) <- t(P,X) & t(P,Y) & d(X,Y).
      busy(P): (1); {Z/P}
yes(craig) <- t(craig,Y) & d(384,Y).
      t(P,X): (3); {P/craig, X/384}
yes(craig) <- d(384,2534).
      t(c,Y): (4); {X/2534}
```

FAILS! Nothing will unify with  $\text{d}(384, 2534)$ .

Problem lies in KB. We didn't axiomatize domain correctly. Add  $\text{distinct}(384, 2534)$ , etc... or add rule:  $\text{distinct}(C,D) \leftarrow \text{distinct}(D,C)$ .

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## Example Derivation #3

Assume KB fixed with rule: 12.  $\text{distinct}(C,D) \leftarrow \text{distinct}(D,C)$ .

Derivation:

```
yes(P) <- busy(P).
yes(P) <- t(P,X) & t(P,Y) & d(X,Y).
      busy(P): (1); {Z/P}
yes(craig) <- t(craig,Y) & d(384,Y).
      t(P,X): (3); {P/craig, X/384}
yes(craig) <- d(384,2534).
      t(c,Y): (4); {X/2534}
yes(craig) <- d(2534,384).
      d(384,2534): (12); {C/384, D/2534}
yes(craig) <- .
      d(2534,384): (8); {}
```

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## Substitutions

- Defn: A **substitution**  $\sigma$  is any assignment of terms to variables
  - we write it like  $\sigma = \{X/t1, Y/t2, \dots\}$
  - constant substitution is a special case; terms can be any terms (nonground included)
    - without functions, only terms are constants, vars
  - e.g.,  $\sigma = \{X/\text{craig}, Y/\text{father}(\text{craig}), Z/P, W/\text{father}(X)\}$
- A substitution is applied to an expression by **uniformly** and **simultaneously** substituting each term for the corresponding variable
  - e.g. using subst. above on  $\text{related}(\text{mother}(X), W)$  gives  $\text{related}(\text{mother}(\text{craig}), \text{father}(X))$

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## Unifiers

- Defn: A substitution **unifies** two expressions  $e_1$  and  $e_2$  iff  $e_1\sigma$  is identical to  $e_2\sigma$
- E.g.,  $p(X, f(a))$  and  $p(Y, f(Z))$  are unified by:
  - $\{X/b, Y/b, Z/a\}$ : gives  $p(b, f(a))$  for both expressions
  - $\{X/Y, Z/a\}$ : gives  $p(Y, f(a))$  for both expressions
  - $\{X/Z, Y/Z, Z/a\}$ : gives  $p(Z, f(a))$  for both expressions
- Unifier  $\sigma$  is a **most general unifier (MGU)** of  $e_1$  and  $e_2$  iff  $e_1\sigma'$  is an *instance of* (unifies with)  $e_1\sigma$  for any other unifier  $\sigma'$ 
  - An MGU gives the most general instance of an expression; any other unifier gives a result that would unify with that given by the MGU

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## MGUs: Examples

- Let  $e_1 = \text{busy}(X)$ ,  $e_2 = \text{busy}(Y)$
- Unifier  $\sigma_1: \{X/\text{kyros}, Y/\text{kyros}\}$ 
  - result:  $e_1\sigma_1 = e_2\sigma_1 = \text{busy}(\text{kyros})$
- Unifier  $\sigma_2: \{X/\text{craig}, Y/\text{craig}\}$ 
  - result:  $e_1\sigma_2 = e_2\sigma_2 = \text{busy}(\text{craig})$
- Unifier  $\sigma_3: \{Y/X\}$ 
  - result:  $e_1\sigma_3 = e_2\sigma_3 = \text{busy}(X)$
- Unifier  $\sigma_3$  an MGU of expressions; not  $\sigma_1, \sigma_2$ 
  - $e_1\sigma_3$  unifies with result of any other unifier
  - $e_1\sigma_1 = \text{busy}(\text{kyros})$  **cannot** (e.g., cannot unify  $e_1\sigma_1$  with  $e_2\sigma_1 = \text{busy}(\text{craig})$ )

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## Notes on General SLD Resolution

- Generally insist that you only use **MGUs** in SLD resolution to match a body atom with a KB head
  - ensures we don't make too specific a choice and force us into failure unnecessarily
- To obtain all answers:
  - once we derive an answer, we pretend the derivation failed and backtrack to find other derivations
  - **we only reconsider KB-clause choices**, not atom selections, or unifier choice

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## Notes on General SLD Resolution

- Prolog (see Appendix B, Ch3.2, Ch3.3)
  - based on SLD-resolution
  - searches for derivations using a specific strategy: (a) always **selects** atoms from answer clause in left-to-right order; (b) always **chooses** KB clauses in top-to-bottom order (using first *unifiable* rule/fact)
  - records choices and tries alternatives if failure (essentially does depth-first search: why?)
  - provides a single answer for nonground queries; but you can force it to search for others (semicolon op)

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## Renaming of Variables: Example

KB:  
(1) rich(joan).  
(2) mother(linda,joan).  
(3) mother(mary,linda).  
(4) rich(X) <- mother(X,Y) & rich(Y).

Query:  
? rich(mary).

Derivation:  
yes <- rich(mary).  
yes <- mother(mary,Y) & rich(Y).  
rich(mary): (4): {X/mary}  
yes <- mother(mary,X) & mother(X,X) & rich(X).  
rich(Y): (4) using {Y/X}

Must fail! Nobody (in our KB) is their own mother!

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## Renaming of Variables

- When we add body of KB clause to answer clause, we may have accidental name conflicts
  - in example, Y in answer clause is not "same person" as Y in KB clause (yet both replaced by X)
- To prevent problems, we always rename vars in KB clause (uniformly) to prevent clashes
  - changing var names in KB clause cannot change meaning
- System: (a) each clause has diff. vars; (b) index KB vars, increase with each use of the clause
  - use rich(X<sub>i</sub>) <- mother(X<sub>i</sub>,Y<sub>i</sub>) & rich(Y<sub>i</sub>). i-th time you use this clause in a derivation

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## Renaming of Variables: Example

KB:  
(1) rich(joan).  
(2) mother(linda,joan).  
(3) mother(mary,linda).  
(4) rich(X) <- mother(X,Y) & rich(Y).

Query:  
? rich(mary).

Derivation:  
yes <- rich(mary).  
yes <- mother(mary,Y<sub>1</sub>) & rich(Y<sub>1</sub>).  
rich(mary): (4): {X<sub>1</sub>/mary}  
yes <- mother(mary,X<sub>2</sub>) & mother(X<sub>2</sub>,Y<sub>2</sub>) & rich(Y<sub>2</sub>).  
rich(Y<sub>1</sub>): (4) using {Y<sub>1</sub>/X<sub>2</sub>}  
etc... (no conflict now)

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## DCL: How can we use it?

- Query-answering system:
  - given KB representing a specific domain, use DCL (and suitable proof procedure) to answer questions
- A Deductive Database System
  - much like the above
- A Programming Language
  - Prolog (we've seen) is a dressed up DCL using SLD
  - Important to realize that as a programming language, we are still making logical assertions and proving logical consequences of these assertions

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## Prolog List Operations

- A distinguishing feature of Prolog is its built-in facilities for *list manipulation*
  - not hacks, but genuine logical assertions/derivations
- Consider the function *cons*, constant *el*:
  - *cons* accepts two args, returns pair containing them
  - e.g. *cons(a,b)*, *cons(a,cons(b,c))*
  - *el* is a constant denoting the empty list
- A proper list is either *el* or a pair whose second element is a proper list
  - *cons(a, cons(b, cons(c, el)))* = (a b c) or [a,b,c]

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## Prolog List Operations

- Prolog uses a more suggestive notation:
  - `[]` is a constant symbol (empty list)
  - `[ | ]` is a binary function symbol: infix notation (`cons`)
  - `[a,b,c]` shorthand for `[a | [b | [c | []]]]`
- But these are just terms in DCL
- Standard list manipulation operations correspond to logical assertions
  - e.g., the usual definition of `append(X,Y,Z)` simply defines what it means for `Z` to be the appending of `X` and `Y`

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## Defining Append

(A1) `append([], Z, Z).`

(A2) `append([E1 | R1], Y, [E1 | Rest]) <-  
append(R1, Y, Rest).`

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## Proving the Append Relation #1

Query: `? append([a,b], [c,d], [a,b,c,d]).`

(A1) `append([], Z, Z).`

(A2) `append([E1 | R1], Y, [E1 | Rest]) <-  
append(R1, Y, Rest).`

Derivation:

`yes <- append([a,b], [c,d], [a,b,c,d]).`

`yes <- append([b], [c,d], [b,c,d]).`

Resolve with (A2) using { `E1/a, R1/[b], Y/[c,d], Rest/[b,c,d]` }

`yes <- append([], [c,d], [c,d]).`

Resolve with (A2) using { `E1/b, R1/[], Y/[c,d], Rest/[c,d]` }

`yes <- .`

Resolve with (A1) using { `Z/[c,d]` }

Answer: `yes`

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## Proving the Append Relation #2

Query: `? append([a,b], [c,d], [g, b,c,d]).`

(A1) `append([], Z, Z).`

(A2) `append([E1 | R1], Y, [E1 | Rest]) <-  
append(R1, Y, Rest).`

Derivation:

`yes <- append([a,b], [c,d], [f,b,c,d]).`

No append rule can unify with this atom  
(convince yourself: look at `E1`)

Answer: `no`

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## Proving the Append Relation #3

Query: `? append(L, M, [a,b,c,d]).`

(A1) `append([], Z, Z).`

(A2) `append([E1 | R1], Y, [E1 | Rest]) <-  
append(R1, Y, Rest).`

Derivation:

`yes(L,M) <- append(L, M, [a,b,c,d]).`

`yes([a|R1], M) <- append(R1, M, [b,c,d]).`

Resolve with (A2) using { `L/[a|R1], Y/M, E1/a, Rest/[b,c,d]` }

`yes([a], [b,c,d]) <- .`

Resolve with (A1) using { `R1/[], M/[b,c,d], Z/[b,c,d]` }

Answer: `L = [a], M = [b,c,d]`

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## Proving the Append Relation

- Exercise: Give derivations for at least two other answers for the previous query:

Query: `? append(L, M, [a,b,c,d]).`

- `L = [], M = [a,b,c,d]`

- `L = [a], M = [b,c,d]`

- `L = [a,b], M = [c,d]`

- `L = [a,b,c], M = [d]`

- `L = [a,b,c,d], M = []`

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## DCL and Knowledge Representation

- DCL has obvious uses as a question answering system for complex knowledge
  - A key issue: how does one effectively represent knowledge of a specific domain for this purpose?
  - Unfortunately, there are generally many ways to represent a KB: some more useful (compact, natural, efficient) than others
- Let's go through a detailed example to see where choices need to be made, what the difficulties are, etc.

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## The Herbalist Domain

- Suppose we want to build a KB that answers queries about what sorts of homeopathic remedies we need to treat different symptoms
  - This "expert system" will underly a Web site where users can ask for advice on herbal remedies
- We need to build a KB that represents info we have about different clients, their symptoms, treatments, etc.

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## What Functionality is Needed?

- Before designing KB, we need to know what types of queries we'll ask; do we want:
  - a) `?treatment(john,T)`.
  - b) `?treatment(symptom,T)`.
  - c) `?treatment(combination-of-symptoms,T)`.
  - d) `?safe(combination-of-treatments)`.
  - e) `?medical_records(john,R)`.
  - f) `?paid_bills(john)`.
- and so on

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## What Individuals Do We Need?

- What constants/functions will I need?
- Clients (people), other entities:
  - constants: *joan, ming, gabrielle, greenshield...*
  - functions: *insurer(X)*, etc.
- Symptoms (constants): *fever, aches, chills, ...*
- Treatments:
  - constants: *echinacea, mudwort, feverfew, ...*
  - or maybe function: *tmt(feverfew,capsule), tmt(mudwort, tincture)*, where we have a treatment requires a substance and a preparation
  - then we need constants for substances, preparations

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## What Individuals Do We Need?

- Diseases: do we need diseases?
  - why? why not? (our treatment philosophy will be to apply treatments to symptoms: simplicity!)
- Combinations of symptoms? treatments?
- We'll consider combinations:
- symptomList is a list of symptoms:
  - e.g, function: *symList(symptom, SList)*
  - or using Prolog notation: *[aches, fever, chills]*
- treatmentList similar:
  - *[tmt(mudwort,tincture), tmt(echinacea,capsule)]*

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## What Relations?

- Relations depend on functionality desired
- If we ask `?treatment(john,T)`, we need information about john in KB (e.g., symptoms)
  - e.g.: *symptom(john,fever), symptom(john,chills)*.
  - or: *symptoms(john, [fever,chills])*.
  - or maybe symptoms are relations themselves and not individuals: *fever(john), chills(john)*.
- Maybe we don't even discuss individual clients:
  - e.g., we only ask: `?treatment(SList,TList)`.
- Different choices influence how you express your knowledge: some make life easy, or difficult!

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## Facts and Rules

- Once we've decided on suitable relations we need to populate our KB with suitable facts and rules
  - facts/rules should be correct
  - facts/rules should cover all *relevant* cases (which depends on the task at hand)
  - try to keep facts/rule concise (only relevant facts)
- For example: we can often express a zillion facts using one or two simple rules

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## Some Example Facts/Rules

- Facts about individual patients

### Specific Visit Facts (enter into KB during exam):

```
musclepain(mary,shoulders).
slow_digestion(john).
fever(john).
```

### Semi-permanent Facts (persist in KB):

```
arthritis(ming).
hypertensive(john).
relaxed_disposition(mary).
```

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## Some Example Facts/Rules

- Rules relating treatments to symptoms

### We can relate treatments to symptoms directly:

```
remedy(X,echinacea) :- fever(X) & cough(X) & sniffles(X).
remedy(X,echinacea) :- chills(X) & cough(X) & sniffles(X).
```

### Or relate treatments to diseases, and diseases to symptoms:

```
remedy(X,echinacea) :- has_cold(X).
```

```
has_cold(X) :- fever(X) & cough(X) & sniffles(X).
has_cold(X) :- chills(X) & cough(X) & sniffles(X).
```

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## Some Example Facts/Rules

- We might even have more general rules

- Appropriate level of generality can make KB expression more concise

### We might have general problems:

```
general_dig_probs(X) :- slow_digestion(X).
general_dig_probs(X) :- heartburn(X) & relaxed_disposition(X).
general_dig_probs(X) :- gastritis(X).
```

### and relate treatments to such classes of problems:

```
remedy(X,clives) :- general_dig_probs(X).
remedy(X,meadowsweet) :- gastritis(X).
```

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## Some Example Facts/Rules

- Design choice for relations, individuals can have impact on ability to prove certain things (easily)
- Suppose we want to find a treatment list for *john*:
  - list should cover each symptom john exhibits (in KB)
  - but how do we "collect" all the facts from the KB of the form *fever(john)*, *slow\_digestion(john)*, etc.
  - (actually Prolog has some hacks, but SLD doesn't)
- Thus we make our lives easier by thinking of symptoms as individuals, and relating patients to a list of all symptoms
  - symptoms(john, [fever, aches, slow\_digestion])*.

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## Example Facts/Rules

- Let's attempt to define *treatment(S,T)*: treatment list T is satisfactory for symptom list S
  - Note: it suggests new relations to specify/define
- Is this definition correct? complete? efficient? for what types of queries will it work?

```
treatment([], []).
treatment([S1 | RestS], [T1 | RestT]) :-
    treats(T1,S1),
    treatment(RestS, RestT),
    safe([T1 | RestT]).
```

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## Example Facts/Rules

```
treatment([], []).
treatment([S1 | RestS], [T1 | RestT]) :-
  treats(T1, S1),
  treatment(RestS, RestT),
  safe([T1 | RestT]).
```

- *?treatment([aches, fever], T)*: is this defn OK?
- *?treatment([aches, fever], [ech, mudwort])*: OK?
  - what if *ech* treats *fever* and *mudwort* treats *aches*?
  - must rewrite to make order-independent
- Final Tlist is *safe* if no nasty interactions:
  - why is this definition inefficient?
  - why prove for each *sublist*? how would you rewrite it?
  - *could* proving it each time make sense (for Prolog)?
  - Exercise: define a version of the *safe* predicate

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## KB Design: The Moral

- There are many design choices
- The queries you plan to ask influence the way you break the world into individuals and relations
- Even with fixed functionality, there are often several ways to approach the problem
- Different approaches lead to more or less natural, efficient, and compact KBs

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