Bend-a-rule: a fabrication-based workflow for 3D planar contour acquisition

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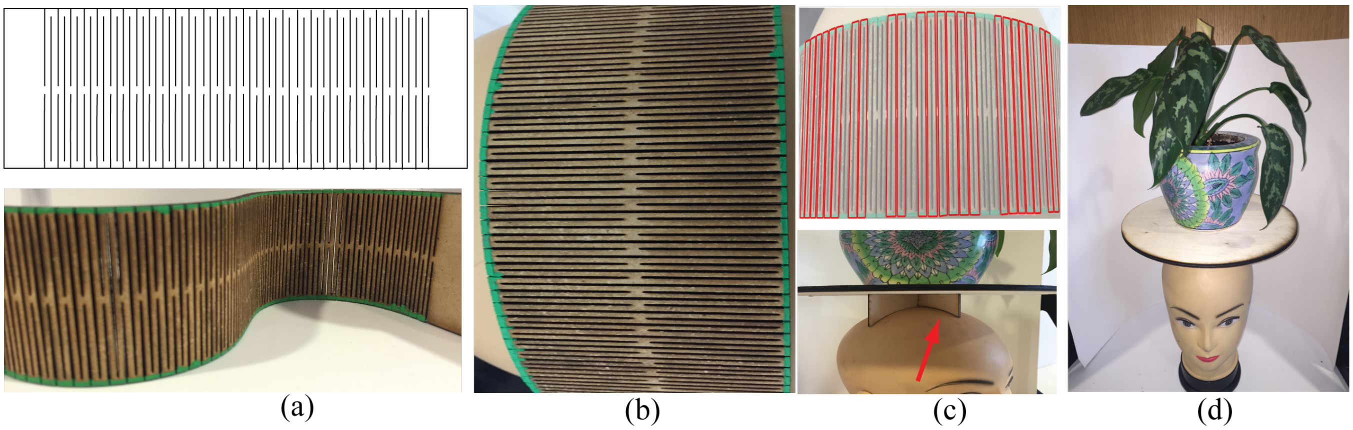

Figure 1: Bend-a-rule is a tool and workflow for planar 3D curve acquisition: physical rulers of custom size and material are laser-cut using a lattice structure that allows uni-directional bending (a); the ruler bent around a physical 3D object (b), is used to extract a planar 3D curve from a single image of the ruler using image gradients, homography and curve fitting (c); the 3D curve is subsequently used to design and fabricate laser-cut variations and accessories of the physical 3D object (d).

ABSTRACT

Bend-a-rule is a physical tool and workflow that enables the robust acquisition of planar contours of 3D shape. Our work exemplifies the design of physical artifacts that subsequently aid in digital design and fabrication. Bend-a-rule is a ruler, fabricated by laser-cutting a periodic pattern on a rigid board. The ruler has uni-directional flexibility, and readily bends to conform to the shape of curved planar contours on physical 3D objects. We present a novel workflow, by which this curved planar contour can be digitally acquired from a single image of the physical ruler. The acquired contour is then used to design laser-cut accessory shapes that attach to physcial 3D objects along the digitally acquired contour. We describe the construction of Bend-a-rule, propose an automatic algorithm for the extraction of a to-scale, planar 3D curve from a Bend-a-rule image, evaluate the resulting curves in comparison to ground truth data, and show example physical 3D objects augmented using Bend-a-rule.

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CCS CONCEPTS

• **Computing methodologies → 3D imaging**; **Interest point and salient region detections**; Camera calibration; Object detection;

KEYWORDS

fabrication, laser-cutting, planar curves, shape acquisition.

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1 INTRODUCTION

The onset of consumer grade fabrication technology and a growing body of research on digital design hold the promise of rapid prototyping and customization of household 3D objects. In this paper we shift focus from the digital design of physical 3D objects, to the design of physical tools and workflows that support subsequent design and fabrication of 3D artifacts. In particular, we look at the problem of augmenting existing 3D objects using assemblies of planar parts cut using a laser-cutter (see Figure 1). While design and fabrication using 3D planar sections are well studied [McCrae et al. 2014], the sections are either designed in a void or in reference to a precise digital 3D model. Augmented objects as seen in Figure 1d essentially require the design of planar sections with contours that conform precisely to planar curves on the physical

3D object. Our problem is to provide lay users with the ability to digitally acquire these planar curves with the simple hardware and efficient workflow of traditional household construction tools like a compass and ruler.

Despite great research strides in 3D shape acquisition, 3D models from consumer photographs are still coarse and of ambiguous scale [Autodesk 2009]; laser scanning by contrast is precise but expensive and requires operational set-up and expertise. Most scanning techniques are also highly sensitive to light, texture and material properties of the object, specular and transparent objects being particularly problematic. Arguably, a complete scan of the 3D object, simply to extract a few planar curves is overkill. Given the importance of curves in geometric modelling, devices like Shapetape [Grossman et al. 2003], or props with motion capture markers, that provide a digital curve proxy, have existed for over a decade now. Such specialized technology however, is not as ubiquitous, economical, or portable as smartphones and physical tools like rulers. Note that while we employ a laser-cutter as a cutting device, designs may be easily printed and cut manually.

We observe that a fine lattice-like structure (Figure 1a) cut into a rigid material like wood or acrylic, renders it flexible orthogonal to the direction of cuts, as each thin segment is able to minimally bend relative to its neighbour. We further note that the cells of this anisotropic lattice structure are amenable to robust segmentation in a single image, from which they can then be reconstructed to-scale in 3D as simple homographies. We thus propose Bend-a-rule, a physical ruler cut from a rigid material as shown in Figure 1a, and a largely automatic workflow, where a single image of the ruler bent along a desired curve, is used to algorithmically reconstruct the curve in 3D. We evaluate the quality of the curve reconstructed using our approach via a comparison to ground truth curves, and show 3D artifacts created using Bend-a-rule.

Our contribution is thus a novel ruler+camera prototype that uses digital fabrication to create physical tools and workflows that mimic traditional construction tools in the creation of physical 3D objects.

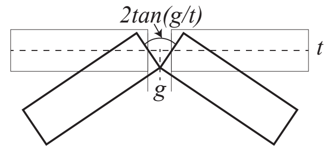
2 RELATED WORK

Our approach relates to work on object augmentation, planar-section models, flexible structures, shape acquisition, and shape fabrication:

Object Augmentation: there has been recent work [Chen et al. 2015] done on augmenting 3D printed objects by 3D printing on top of these objects attachments such as grips and hooks. However, the attachments are either permanently printed onto the object or affixed by means of zip ties where the fit is not tight. Although it only measures planar sections, Bend-a-rule allows the creation of attachments that tightly conform to the shape of objects, extending the capabilities of shape augmentation software.

Planar-section modelling: is an important area of ongoing research as it pertains to the design of artifacts that can be cut out of flat materials using aqua-jet, laser-cutters, routers, or even manually. A large body of work deals with the creation of objects using assemblies of planar-sections, a good overview of which can be found in [McCrae et al. 2014]. These techniques all assume planar sections to be created from scratch or as a proxy to a given digital 3D model. Our work is complementary to this thread of research in that we reconstruct planar curves that conform to physical 3D objects, which can form part of the contours of digital planar-sections.

Flexible structures: kerf-bent wood is an old technique [Woodworking 1985], where wood is bent by cutting saw-width (kerf) notches into a thin piece of wood. The links bend to fill in kerf-space.



The maximum curvature of the bend, number of cuts, kerf width and material thickness are all linked by simple geometric formulas, as seen inset. The kerf pattern we use (Figure 1a) is known to have good pliability and strength, but any number of lattice hinge patterns can be used [Breuer 2012].

Shape Acquisition: numerous techniques deal with the problem of 3D surface shape acquisition, typically, exploiting image properties such as shape from shading, structure from motion, or stereo triangulation using lasers or structured light [Tyler 2011]. As input, we require not only the 3D shape of the design object at the correct scale, but a marking of the desired planar contour on the surface of the object. We solve both in one shot with Bend-a-rule: the desired planar curve is explicitly marked on the 3D object by the ruler, and the regular structure of the ruler lends itself to robust 3D shape reconstruction to precise scale, from a single image.

Shape Fabrication: while the bulk of research on fabrication is focused on 3D printing, a body of recent work [Beyer et al. 2015; Hildebrand et al. 2013; McCrae et al. 2014; Mueller et al. 2013; Umapathi et al. 2015] emphasize the importance and use of planar sections in geometric modelling. Our work is complementary to this research in that the input planar contours for the above cited research could be created by scanning physical contours using Bend-a-rule. Research on fabrication is also typically focused on the actual creation of 3D objects themselves, rather than object accessories such as connectors [Koyama et al. 2015], or tools and jigs that further support the fabrication process [Iarussi et al. 2015]. Spata [Weichel et al. 2015] comprises a pair of callipers and protractors which can provide their measurements as digital inputs to a CAD software. This can speed up the fabrication process as the measurements do not have to be input and taken separately. Foldio [Olberding et al. 2015] and Unimorph [Heibeck et al. 2015] are systems which allow users to fabricate objects with actuated components. Both of them have demonstrated that the actuators can help sense how the fabricated object is being deformed/warped. This can be used to also estimate planar curves but it is unclear how accurate these models can be.

3 DESIGN RATIONALE

Bend-a-rule is presented as an example of how useful household construction tools (like a common ruler) needed for measurement, design and assembly of laser-cut objects, can be conceived and cheaply fabricated using the same process.

Our problem is thus to devise a simple, inexpensive, to-scale, planar curve acquisition solution, robust to extreme surface appearance (texture, reflectance, transparency). Photogrammetry, structured light, and laser scanners can be expensive, surface sensitive, scale agnostic, and require calibration. Sensor instrumented tools like

ShapeTape, Unimorph, Foldio, similarly fall within the scope of custom hardware setups.

Within the space of common physical tools we could use as a curved ruler are artifacts like flexible wire, a string of beads, or paper printed with QR or similar codes. We believe Bend-a-rule is a superior solution to these alternatives:

Reconstructing a 3D curve from the 2D projection of a visually uniform wire is a highly unconstrained problem. The key to 3D reconstruction using Bend-a-rule is that its shape is piecewise planar. The 2D rectangles we extract by image processing can be precisely reconstructed in 3D (planarity reduces the 4x4 transformation to a 3x3 homography matrix in 2D, precisely solved using the 4 corners of the rectangle). We experimented with etching unique visual markers (like QR codes) as suggested on Bend-a-rule, but foreshortened markers etched on kerf widths of less than a cm, photograph too poorly for robust detection. The problem with printing the same patterns on bendable paper, is that unlike our wood ruler, paper bends continuously and any reconstruction using a piecewise planar pattern approximation will be inaccurate.

A string of beads is an interesting alternative. While a bead necklace allows arbitrary 3D curves to be captured, constraining the string to be planar, for use in laser-cut designs can be problematic. Designing a good 3D reconstruction algorithm is also difficult. Assuming we are able to robustly isolate 2D centers and radii of the beads in the 2D image, the imaged radii relative to the original bead size, can define a scale value for the link between two adjacent beads. The absolute depth difference between the two beads can then be computed by estimating the foreshortening of the scaled link to connect the centers of the adjacent beads. A locally concave or convex interpretation will further need to be disambiguated, as will issues relating to adjacent beads partially occluding each other. Bend-a-rule, thus naturally provides a planarity constraint and a reconstruction algorithm that is conceptually simpler and less brittle.

4 WORKFLOW AND ALGORITHM

Our workflow for 3D planar contour acquisition is shown in Figure 2. An edge of Bend-a-rule is bent and held along a desired planar contour on an object. Then the ruler is photographed as frontally as possible, with desired end-points marked either on the object prior to imaging, or digitally on the photo. The photo is processed using our algorithm below, and the output 3D curve is read into a program like Adobe Illustrator, where it can be used to form paths of designs that can be fabricated using various etch and cut devices.

The ruler itself can be procedurally fabricated with various lengths, widths and kerf settings by laser-cutting it out of a largely rigid board like wood or acrylic of varying thickness. We paint the two long edges of the ruler in bright green (Figure 1a) to aid in 2D image segmentation of the ruler's boundaries. The user wraps the ruler over the desired contour and takes a picture of the ruler (we use an iPhone 6), where the lattice cells are clearly imaged (Figure 1b). While it is possible to extract 3D planar curve silhouettes for some shapes from a side-view, this approach is both sensitive to small view changes and simply fails for most shapes in the presence of occlusions (Figure 3b). The user then interactively marks, in the image, the two end-points of the curve to be acquired (by default

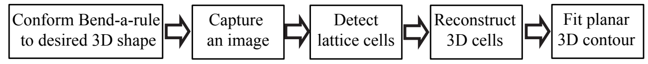
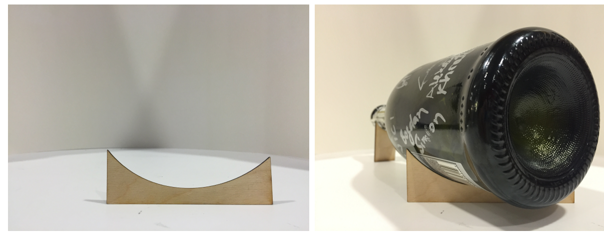


Figure 2: Bend-a-rule shape acquisition workflow.



(a) Shape of the planar section. **(b) Side-view silhouette.**

Figure 3: Insufficiency of side-view for shape acquisition. The bottle self-occludes when imaging the elliptic contour.

the entire visible ruler will be reconstructed). The marked curve is then reconstructed in 3D from the image automatically, using an algorithm that combines image processing and curve fitting (Figure 1c).

Our algorithm makes various assumptions and constraints on the curves that can be acquired by Bend-a-rule:

Physical limitations: being a physical ruler, there are constraints on the length and curvature of the acquired 3D curve. Both can be alleviated to some extent by fabricating a longer ruler with thinner cells and a larger kerf (at the cost of reduced durability). Curves longer than the length of the ruler, curves with sharp discontinuities, or curves with tangent variation more than 180 degrees, can be acquired in a piecewise fashion. Concavities on the object surface orthogonal to the desired curve can also collide with the ruler making it hard to wrap over the object, and this can also be alleviated by fabricating a ruler with narrower width.

Planarity: While the design of Bend-a-rule largely provides unidirectional flexibility, the ruler can be twisted somewhat. This allows us to capture near planar curves as well, but may be undesirable if precise planarity is desired. We address this by fitting a best-fit 3D plane to the reconstructed 3D curve, on which the curve is subsequently projected.

Elasticity: While the ruler is designed to bend as an articulated rigid body, it is able to minimally compress along the length into the gaps created by the kerf cut. This expansion and compression however, requires stretching the ruler by pulling it lengthwise, which is not a problem in practice when bending the ruler.

Accessibility: While locally concave objects can be captured as the edge of the ruler can be bent to contour the desired curve on surface, the width of the ruler does limit capture in small crevices for which narrower and thinner rulers can be printed.

Imaging constraints: We can only reconstruct curves that can be visually captured from a single image viewpoint. Closed curves, for example, can only be captured in a piecewise fashion. The approach is resilient to imaging conditions, such as using a flash.

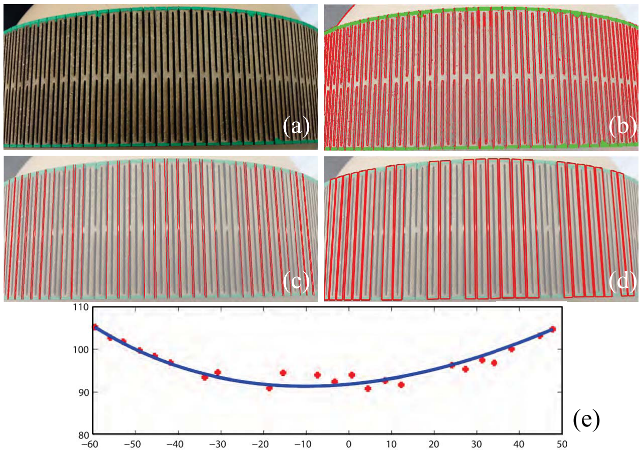


Figure 4: Reconstruction Algorithm: the original image (a), is processed to extract boundary curves as green pixels using mean-shift clustering and lattice cell edges as red pixels using image gradients (b); lattice edges are refined by RANSAC (c), and intersected with the boundary curves to define lattice cells (d). 2D projections of the homography reconstructed cell centers and a best-fit smooth curve (e).

4.1 Bend-a-rule image

In order to take an image of Bend-a-rule, the user must conform Bend-a-rule to the shape of the desired curve. A timer can be placed on the camera so that the user can take an image while using both hands to conform the shape of Bend-a-rule. We show some examples of images taken in Figure 5. These images were taken using an iPhone 6 in timer portrait mode of Bend-a-rule conforming to different objects. Alternatively, we can use scotch tape to affix the Bend-a-rule to the object. Once this is done, we can take an image of Bend-a-rule without needing a timer or tripod.

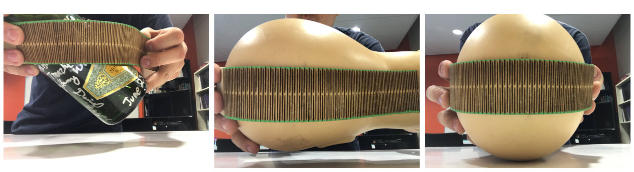


Figure 5: Images of Bend-a-rule wrapped around 3D objects.

4.2 Reconstruction steps

Given a Bend-a-rule image, the kerf cuts along the width of the ruler are extracted as dominant edges by processing image gradients (Figure 4b). The boundary curves are segmented using mean-shift clustering for pre-computed samples for green, as well as the colours of any interactively selected curve end-points (Figure 4b). The selected clusters for the boundary curves are thinned and then used to detect the 2D corners of lattice cells as the intersection between the boundary curves and kerf edges. Each lattice cell comprising of 4 2D corners (Figure 4d) can then be reconstructed in 3D as a homography of a canonical lattice cell. The 3D curve points are then filtered for outliers and fit using a smooth spline (Figure4e).

4.3 Lattice cell corner detection

The corners of lattice cells lie at the intersection of two strong perpendicular edges in the image. We tried using the Harris Corner detector [Harris and Stephens 1988] and SURF features [Bay et al. 2006] to detect these corners. The former found too many false positives (Figure 6 green) and the latter missed a number of perceptually strong corners (Figure 6 blue). We thus decided to find corners by intersecting extracted kerf cuts and ruler boundaries.

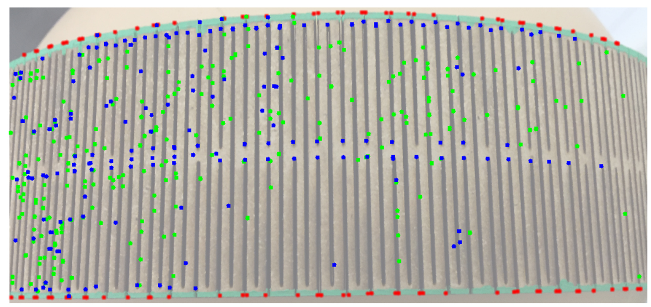


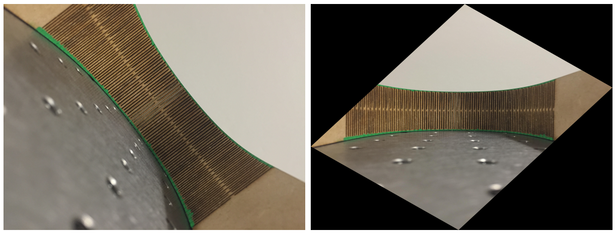
Figure 6: Corner detection methods: ours (red), Harris (green), and SURF features (blue).

Kerf edge detection. We leverage the uni-directional bending of Bend-a-rule to extract the sides of each cell. Since we require images to be taken from the front view, the image can be rotated (Figure 7) to where the uni-directionality constrains the sides of the ruler to be near vertical (Figure 4c). Note that we cannot use vertical gradients to compute the boundary curves.

In order to find nearly vertical lines, we extract *edgels*, pixels with high horizontal gradients, from the normalized image . We threshold pixels from the left and right edges separately using threshold values 0.1 and 0.05 respectively, turning the images into binary images. Then we use RANSAC [Fischler and Bolles 1981] to find vertical lines that have strong support, edgels inlier to the line (Figure 4c). Lines with angular deviation larger than 10° from a vertical line are rejected automatically. We modify RANSAC by using 101 by 7 (H x W) local neighbourhoods (to bias the proposed lines towards vertical ones) centred around edgels to generate candidate lines and the support of each line is computed using all the edgels in the image. Although not all the cell edges are detected by this process, the detected cells are sufficient for estimating the curve shape.

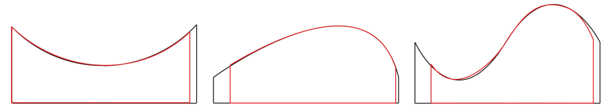
As Bend-a-rule input, images can have any orientation, we can find the 2D image rotation that yields the most near vertical edgels by simple linear search (Figure 7).

Boundary curve detection. Mean shift [Comaniciu and Meer 2002] has been shown to be effective in segmenting the image into clusters of pixels with similar colour. We therefore use mean shift with bandwidth 0.1 to extract the boundary of the curves (Figure 4b). We downsampled the original image by 10 in both directions to speed



(a) Original image. (b) Best rotation.

Figure 7: Rotation with the highest number of edgels.



(a) Circular. (b) Oval. (c) Sine wave.

Figure 8: Curves acquired using Bend-a-rule (red) are shown superposed over example ground truth curves (black).

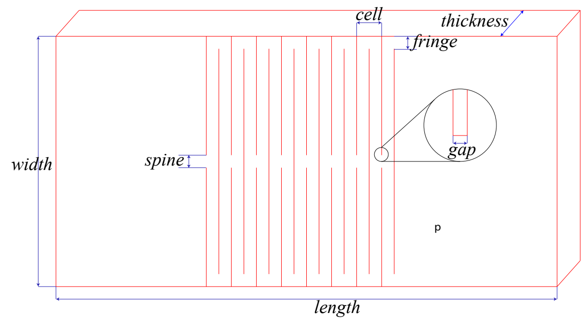


Figure 9: Bend-a-rule construction parameters.

up mean shift. In general, we found that the green painted on the ruler is distinct enough in colour from the colour of the ruler that it can be used effectively to find the painted region. We extract the cluster of green pixels when the user chooses the endpoints along the ruler to measure.

4.4 Lattice cell Homography

In order to compute the homography, we use solvePNP from the OpenCV library [Bradski 2000]. This process can be done using either a calibrated or uncalibrated camera system. The uncalibrated case requires an additional step of using the homography to compute some calibration settings. We calibrated the iPhone 6 [Bouguet 2008; Zhang 2000] for the examples shown in this paper. Note that the calibration step is done offline and only needs to be done once. The homography for each cell provides its physical 3D location relative to the camera. This property allows us to create to-scale curves. Since Bend-a-rule is largely uni-directional, the centres of the cells lie close to a plane. We then project the centres onto their best fit plane (Figure 4e).

4.5 Curve fitting

Once we have the 2D projections of the centres, we export the coordinates of the curve into Adobe Illustrator and use the simplify path function to smoothly fit the curve (Figure 4e).

5 RESULTS

We show the quality of the estimated curves by comparing it in two settings.

We printed the black curves on a piece of paper and used Bend-a-rule to reconstruct the measured curve (Figure 8). We superimposed the estimated curve in red onto the original curve in black. In general, Bend-a-rule suffers slightly from the reconstruction of curves with inflexion points (Figure 8c), because it is harder to conform Bend-a-rule to curves with arbitrary inflexion points. However, arbitrary curves with no inflexion points can be constructed with high accuracy (Figure 8a,8b). Some regions of the curve with large foreshortening in the image are not recovered well, as corners of lattice cells with strong forshortening are harder to detect.

We show some artifacts created using Bend-a-rule on everyday objects. First we show the ability to measure the inside of a bowl, allowing us to create dividers for the bowl (Figure 10a). In general, the 3D shape of the bowl is hard to acquire because of its specularity. Furthermore, due to the concavity of the bowl, there is no side view from which the estimated curve could be observed. Regardless, we are still able to measure the inside of the bowl. We also show the ability of Bend-a-rule to measure general planar contours (Figure 10c). In particular, for the wine bottle, we were able to measure a planar contour diagonal to the body. This is not a planar contour that is observable from any angle (Figure 3b). We also show for the case of the wine bottle, that we are able to acquire the shape of planar contours in transparent objects, another class of objects that 3D sensors have difficulty sensing. Lastly, we show how we can use Bend-a-rule to convert rounded surfaces into tabletops (Figure 1d).

Each kerf pattern has its own set of physical properties that trade offs durability and flexibility. Various parameters that impact these physical properties for Bend-a-rule are shown in Figure 9. While some properties such as bendability or curvature can be determined geometrically from the parameters, most physical properties are best tested empirically for a given ruler material. In Table 1, we show properties for eight rulers of varying parameters that we fabricated out of MDF. As seen in the Table, the small *cell* size of ruler-4, or the larger *gap* of ruler-2, allow the ruler to be bent to a small radius of curvature, at the cost of greater torsion or twistability of the ruler, as well as greater stretching in the case of ruler-4. Thinner and more bendable rulers also tend to be more fragile and break on extreme bending or stretching. We found ruler-1 to be a strong ruler that provides a balance of good bendability, and low stretching and torsion.

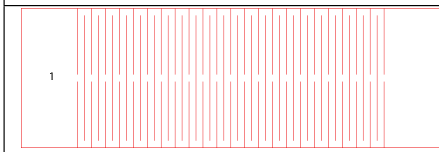
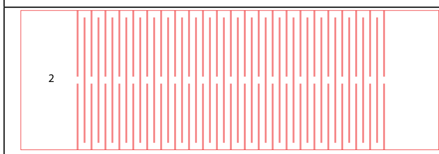
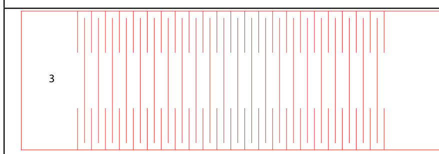
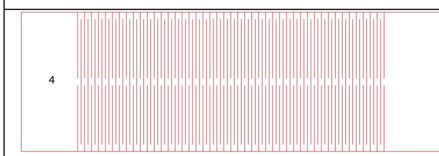
6" length and 2" width rulers	other ruler parameters				properties for rulers of thickness 3/32", 1/8"		
	cell	fringe	spine	gap	stretch	torsion	radius of curvature
	0.2"	0.1"	0.1"	0.013"	7.68" , 6.10"	35^o , 23^o	0.49" , 0.98"
	0.2"	0.1"	0.1"	0.026"	7.68" , 6.30"	51^o , 40^o	0.25" , 0.49"
	0.2"	0.1"	0.8"	0.013"	6.30" , 6.02"	10^o , 7^o	0.49" , 0.98"
	0.1"	0.1"	0.1"	0.013"	8.67" , 6.70"	$> 90^o$, 67^o	0.12" , 0.25"

Table 1: Physical properties from variable ruler parameters.

6 LIMITATIONS + FUTURE WORK

While we show the ability to acquire to-scale planar contours, we acknowledge some limitations to the workflow and propose solutions to these limitations which can be explored in future work.

6.1 Outlier removal

Outliers are currently present in the kerf edge detection process as well as the homography estimation step. At present, we eliminate outliers in the edge detection by examining how far they deviate from vertical. When computing the 3D locations of cell centres, we leverage the fact that Bend-a-rule approximates a smooth continuous curve to remove any centre points which would result in large curvatures in the planar contour. While these methods allow us to eliminate most outliers, false positives may still exist. In this case, we provide the user with the option to manually remove these outliers.

6.2 Inflexion points

Planar sections with inflexion points are more difficult for Bend-a-rule to measure as observed in the ground truth examples. This is attributed to both the difficulty in physically conforming Bend-a-rule to the planar sections and the limited depth of field of the cellphone camera. We can alleviate the physical limitation by using thinner kerf patterns. Blurred lattice cells result in weaker horizontal gradients causing these cells to be less perceptible to the edge detection method we use. This can be alleviated by changing the depth of field.

6.3 Corner detection

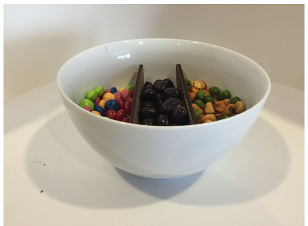
Corner detection is important to Bend-a-rule as it provides information about how the curve moves through 3D space. While we are able to fit the curve accurately, our current corner detection method is unable to find all the corners. The inclusion of more cell centres would be beneficial to the accuracy of the estimated curves, especially in the case where inflexion points are present. Currently we are unaware of any vision techniques that would allow us to find all the corners robustly; this would be an interesting area to explore as accurate corner detection is crucial to Bend-a-rule, especially corner detection methods with adaptive thresholds that are robust to light variation in the image.

6.4 Rectangular cells

Our algorithm assumes rigid rectangular lattice cells. As seen in Table 1, in practice the rulers can stretch by widening the cuts in the middle, giving rectangular lattice cells a hexagonal appearance. Our algorithm can be extended to strongly stretchable rulers by modelling lattice cell edges as hexagons instead of rectangles.

6.5 Curve stitching

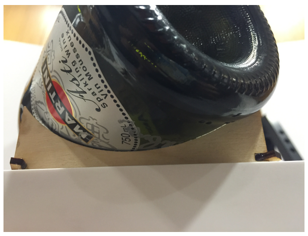
Only portions of a planar contour captured within a single unoccluded image of the ruler can be reconstructed. To capture long curves with complex undulations, multiple ruler images capturing overlapping portions of the curve must be taken and reconstructed independently. The pieces of the curve can then be stitched together by registering the segments of overlap either automatically or by manually marked end-points.



(a) Bowl with food dividers.

(b) Close-up of divider in bowl.

(c) Wine bottle on a stand.



(d) Close-up of the stand.

(e) Pencil holder cover.

(f) Side-view of the cover.

Figure 10: Artifacts created using Bend-a-rule.

6.6 Flexible Patches

Since it is possible to create kerf patterns that allow bidirectional bending, a natural extension to Bend-a-rule is flexible patches for measuring surfaces. Lattice cell corners on the flexible patches can be used to provide a discretization of the surface. The discrete points can then be interpolated to estimate the underlying surfaces of the objects.

7 CONCLUSION

In this paper, we presented Bend-a-rule, a workflow for automatically finding the to-scale shape of a planar section using a fabricated ruler and a smartphone camera. While a ruler can only be used to measure straight edges accurately, Bend-a-rule can be used in a more general setting to measure the shape of arbitrary planar sections. To this end, we showed how Bend-a-rule can be used in a household setting to capture the shapes of planar contours that cannot be easily measured using a straight edge or even acquired using existing 3D sensing methods. We believe that Bend-a-rule allows users to capture 3D curves quicker, cheaper and faster than previously possible, allowing them to fabricate objects that fit existing physical artifacts.

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