

# Toward More Efficient Motion Planning with Differential Constraints

Maciej Kalisiak

Final Oral Exam  
December 14<sup>th</sup>, 2007

# Outline

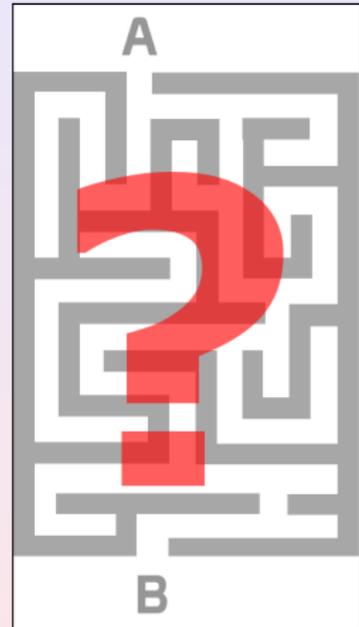
- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

# Outline

- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

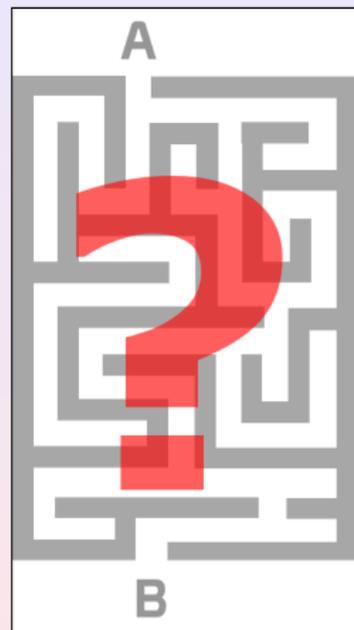
# What is Motion Planning (MP)?

- in a nutshell:  
    *“how to get from A to B?”*
- sometimes also:  
    *“... optimally?”*
- example problems



# What is Motion Planning (MP)?

- in a nutshell:  
    *“how to get from A to B?”*
- sometimes also:  
    *“... optimally?”*
- example problems



# What is Motion Planning (MP)?

- in a nutshell:  
    *“how to get from A to B?”*
- sometimes also:  
    *“... optimally?”*
- example problems



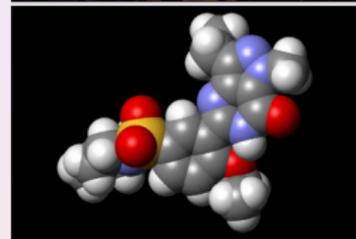
# What is Motion Planning (MP)?

- in a nutshell:  
    *“how to get from A to B?”*
- sometimes also:  
    *“... optimally?”*
- example problems



# What is Motion Planning (MP)?

- in a nutshell:  
    *“how to get from A to B?”*
- sometimes also:  
    *“... optimally?”*
- example problems

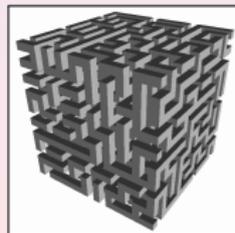
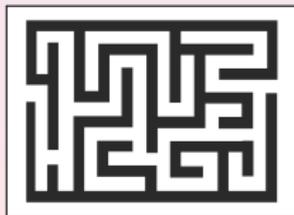


# Approach

- solved by converting to dual problem (agent  $\rightarrow$  point)
- complication: often cannot manipulate agent directly



$$\mathbf{x} = (x, y, \theta)$$



$$\mathcal{X} = x \times y \times \theta$$

$$\mathbf{x} \in \mathcal{X}$$

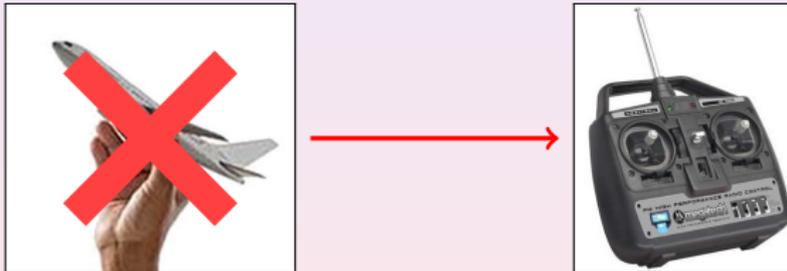
# Approach

- solved by converting to dual problem (agent  $\rightarrow$  point)
- complication: often cannot manipulate agent directly



# Approach

- solved by converting to dual problem (agent  $\rightarrow$  point)
- complication: often cannot manipulate agent directly



# Types of MP problems

## common types:

- kinematic
- nonholonomic
- kinodynamic



e.g., "Piano Mover's Problem"

# Types of MP problems

## common types:

- kinematic
- nonholonomic
- kinodynamic



e.g., agents w/rolling contacts

# Types of MP problems

## common types:

- kinematic
- nonholonomic
- **kinodynamic**



e.g., inertia & balance play big role

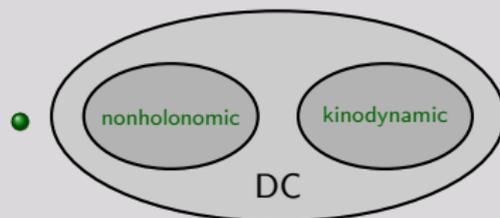
# Types of MP problems

## common types:

- kinematic
- nonholonomic
- kinodynamic

## Differential Constraints (DC)

- DC: constraints on  $q'$   
( $\frac{d}{dt}$  of agent configuration)



- DCs very common,  
but make MP more difficult

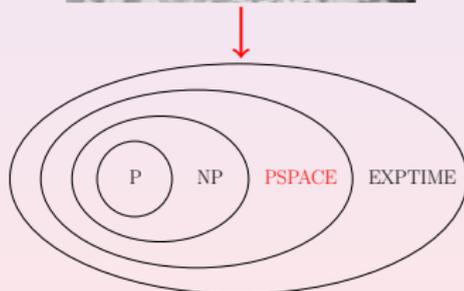
# MP is hard

## hardness

- Piano Mover's Problem:  
→ **PSPACE-complete**
- MP problems w/DC: at least as hard

## why?

- “curse of dimensionality”
- real world problems often high-D
- DCs complicate search space further



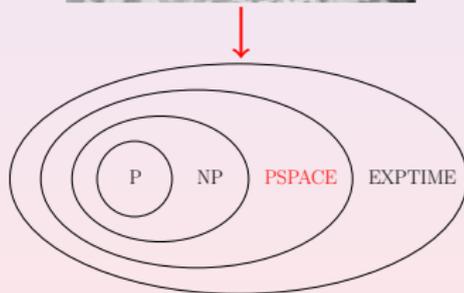
# MP is hard

## hardness

- Piano Mover's Problem:  
→ **PSPACE-complete**
- MP problems w/DC: at least as hard

## why?

- “curse of dimensionality”
- real world problems often high-D
- DCs complicate search space further



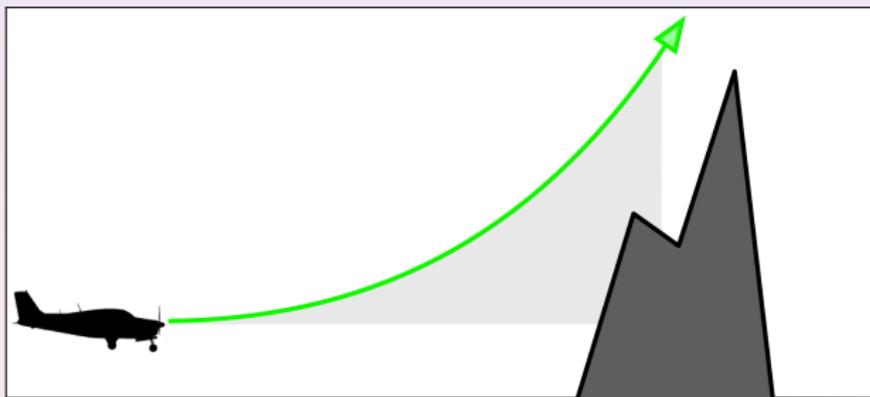
# Outline

- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

# What is Viability?

## “definition”

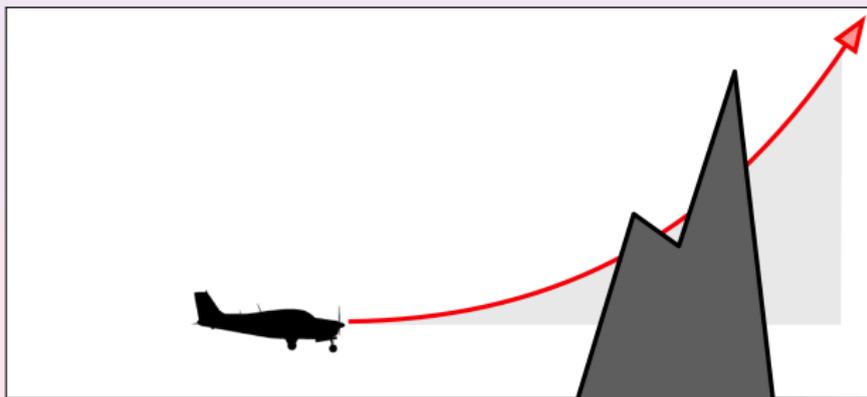
- **viable** state:  $\exists$  an evasive action
- nonviable state: constraint violation unavoidable



# What is Viability?

## “definition”

- viable state:  $\exists$  an evasive action
- **nonviable** state: constraint violation unavoidable



# What is Viability?

## “definition”

- viable state:  $\exists$  an evasive action
- nonviable state: constraint violation unavoidable

## why of interest?

- crops up in many contexts, useful
- exploited throughout thesis:
  - to expedite MP
  - to aid in user-control

# Outline

- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

# Overall goal of thesis

- **aim:** explore some novel ideas in MP
- **focus:** improving MP speed
- **grand vision:** MP with motion “macro-primitives”

# Outline

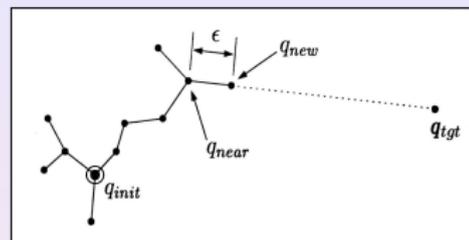
- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

# MP in highly constrained problems

**key idea:** “*any progress*” is better than “*no progress*”

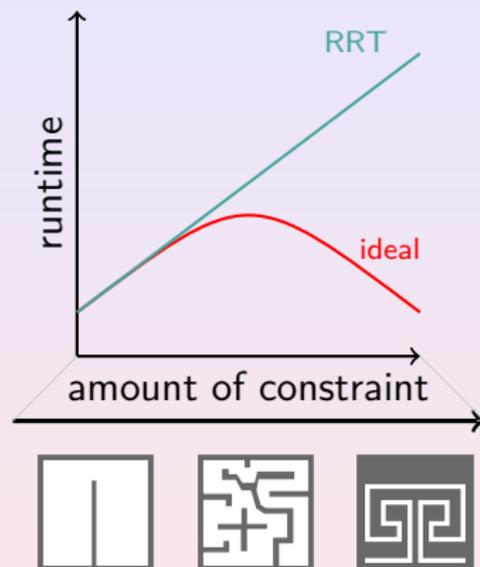
# MP in highly constrained problems

- improvement to **RRT** algorithm
- highly-constrained problems:  
poor performance
- proposed: **RRT-Blossom**
- result: big speed ups ( $>10x$ )



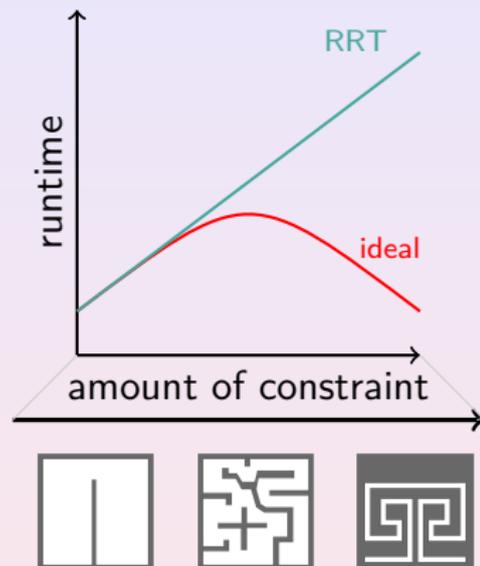
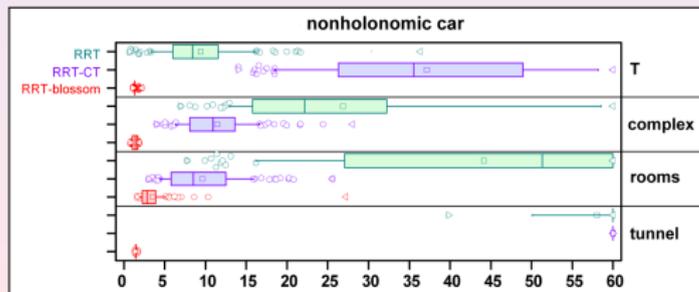
# MP in highly constrained problems

- improvement to RRT algorithm
- highly-constrained problems:  
poor performance
- proposed: RRT-Blossom
- result: big speed ups ( $>10x$ )



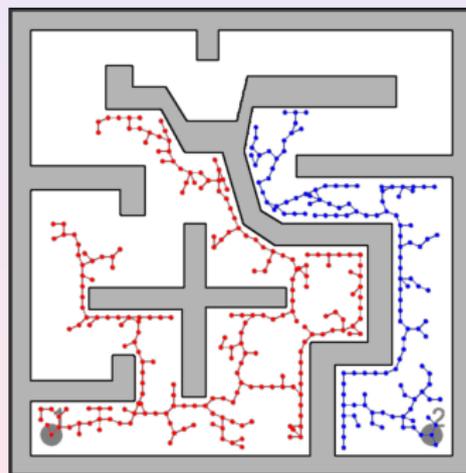
# MP in highly constrained problems

- improvement to RRT algorithm
- highly-constrained problems:  
poor performance
- proposed: RRT-Blossom
- result: big speed ups ( $>10\times$ )



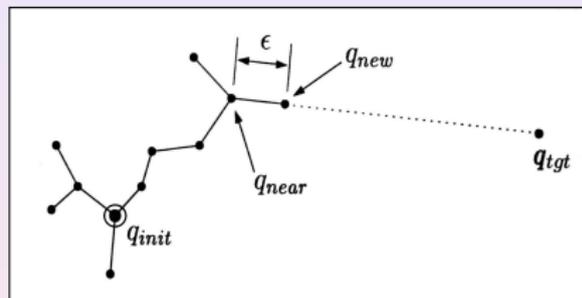
# RRT operation review

- grows two trees  
(from  $q_{init}$  and  $q_{goal}$ )
- each tree grows toward  $q_{tgt}$



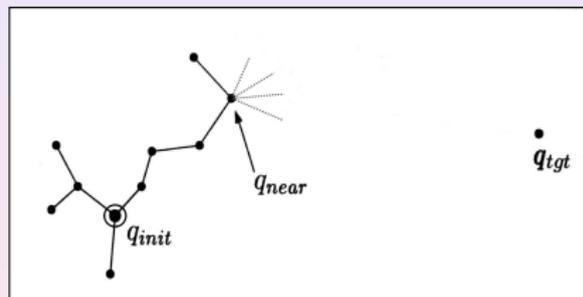
# RRT operation review

- grows two trees  
(from  $q_{init}$  and  $q_{goal}$ )
- each tree grows toward  $q_{tgt}$



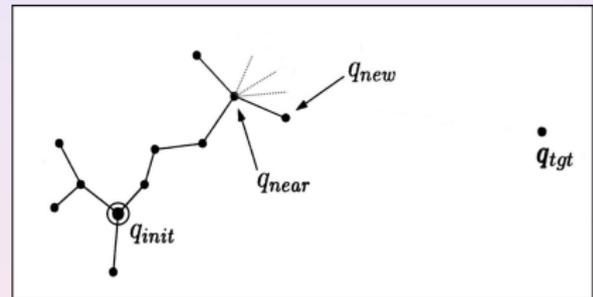
# RRT operation review

- grows two trees  
(from  $q_{init}$  and  $q_{goal}$ )
- each tree grows toward  $q_{tgt}$



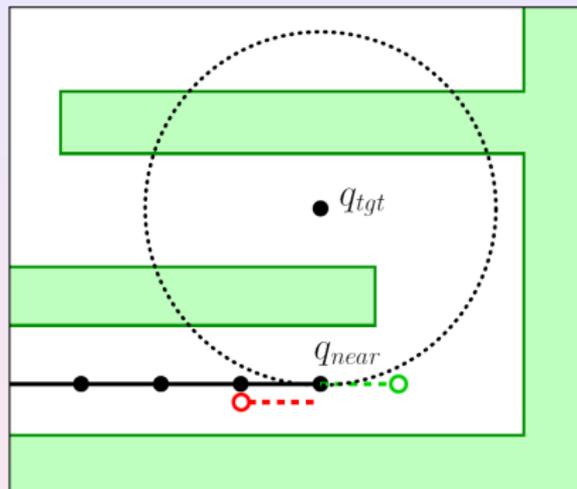
# RRT operation review

- grows two trees  
(from  $q_{init}$  and  $q_{goal}$ )
- each tree grows toward  $q_{tgt}$



# RRT-Blossom

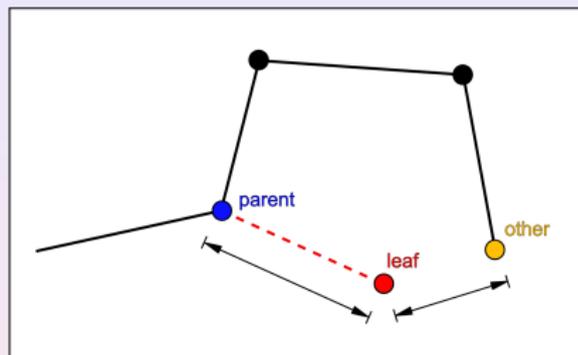
- allow **receding** edges...
- but not if **regressing**
- filter with regression test
- bottlenecks



**key idea:** “any progress” is better than “no progress”

# RRT-Blossom

- allow **receding** edges...
- but not if **regressing**
- filter with regression test
- bottlenecks



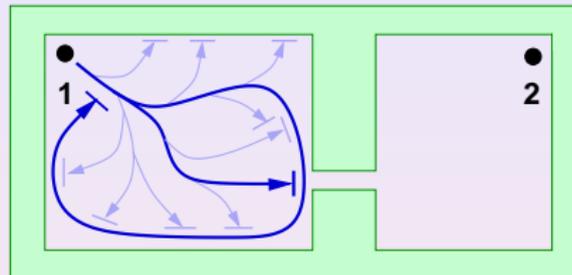
regression if:

$$\exists \textit{other} \mid \rho(\textit{parent}, \textit{leaf}) > \rho(\textit{other}, \textit{leaf})$$

**key idea:** “any progress” is better than “no progress”

# RRT-Blossom

- allow **receding** edges...
- but not if **regressing**
- filter with regression test
- bottlenecks



**key idea:** “any progress” is better than “no progress”

# Outline

- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - **MP w/viability filtering**
  - Viability-based safety enforcement
- 4 Conclusion

# MP w/viability filtering

## drawbacks of tree-based MP:

- tactile-only sensing
- search ignores prior attempts

## general idea:

- *"work smarter, not harder"*
- add "sight" + "learning" → faster MP



# MP w/viability filtering

## drawbacks of tree-based MP:

- tactile-only sensing
- search ignores prior attempts

## general idea:

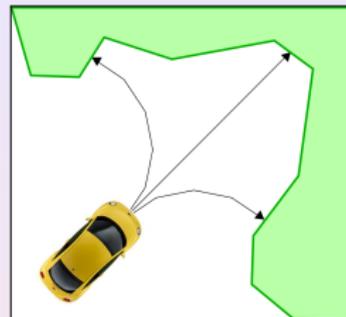
- “work smarter, not harder”
- add “sight” + “learning” → faster MP



# Key extensions

## “sight”

- virtual sensors: distance along path
- yield “locally situated” state



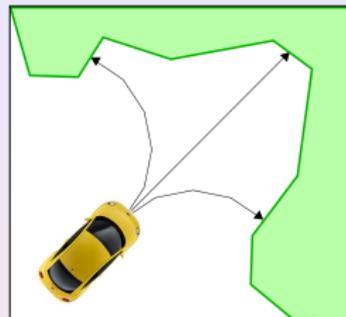
## “learning”

- prior trajectories → viability models
- models parametrized using sensors
  - local models
  - transferrable
- ideally: bootstrapping

# Key extensions

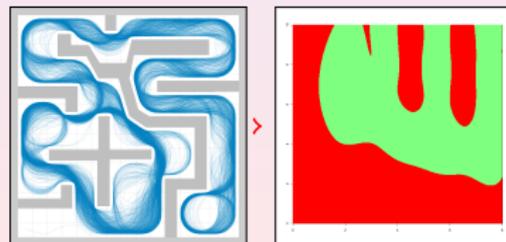
## “sight”

- virtual sensors: distance along path
- yield “locally situated” state



## “learning”

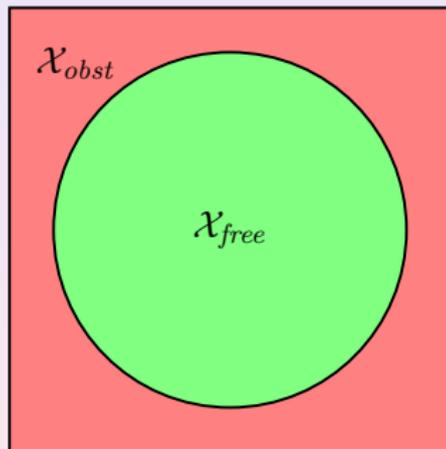
- prior trajectories  $\rightarrow$  viability models
- models parametrized using sensors
  - $\rightarrow$  local models
  - $\rightarrow$  transferrable
- ideally: bootstrapping



# Exploiting viability

## observations

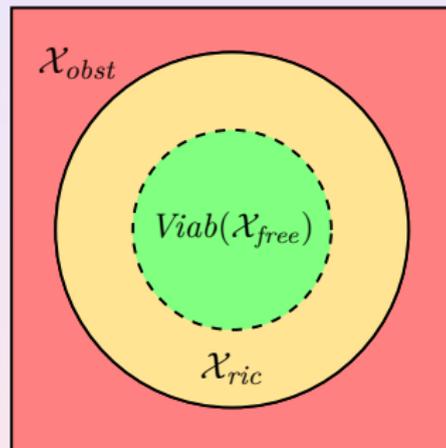
- currently: search in all of  $\mathcal{X}_{free}$
- but  $\mathcal{X}_{free}$  includes  $\mathcal{X}_{ric}$
- $x_{goal}$  usually unreachable from  $x \in \mathcal{X}_{ric}$



# Exploiting viability

## observations

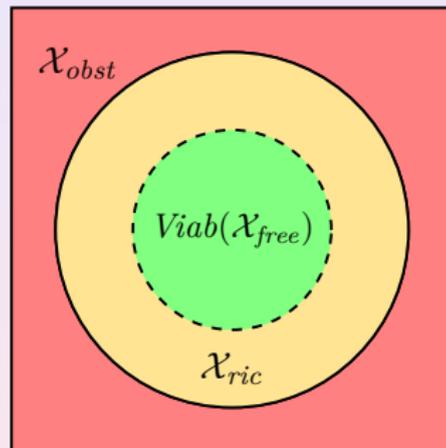
- currently: search in all of  $\mathcal{X}_{free}$
- but  $\mathcal{X}_{free}$  includes  $\mathcal{X}_{ric}$
- $x_{goal}$  usually unreachable from  $x \in \mathcal{X}_{ric}$



# Exploiting viability

## observations

- currently: search in all of  $\mathcal{X}_{free}$
- but  $\mathcal{X}_{free}$  includes  $\mathcal{X}_{ric}$
- $x_{goal}$  usually unreachable from  $x \in \mathcal{X}_{ric}$



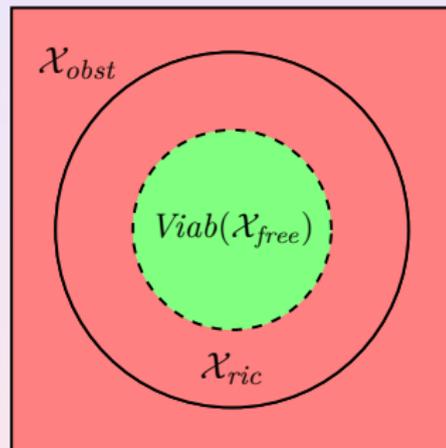
# Exploiting viability

## observations

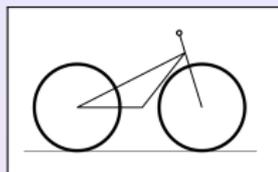
- currently: search in all of  $\mathcal{X}_{free}$
- but  $\mathcal{X}_{free}$  includes  $\mathcal{X}_{ric}$
- $x_{goal}$  usually unreachable from  $x \in \mathcal{X}_{ric}$

## ⇒ avoid futile searching!

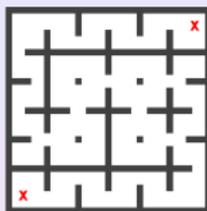
- model agent viability
- keep MP search within  $Viab(\mathcal{X}_{free})$
- observed: speed-up of up to 10x



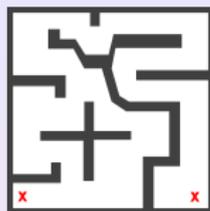
# Results: model transfer



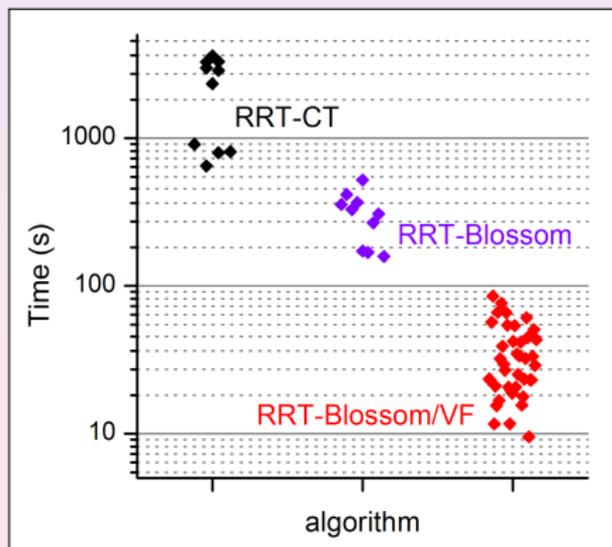
agent



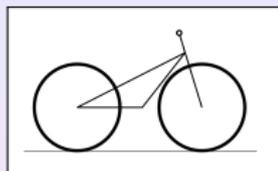
problem posed



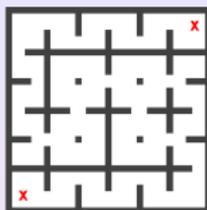
model trained on



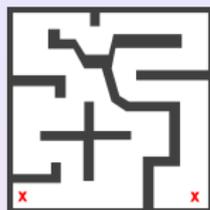
# Results: model transfer



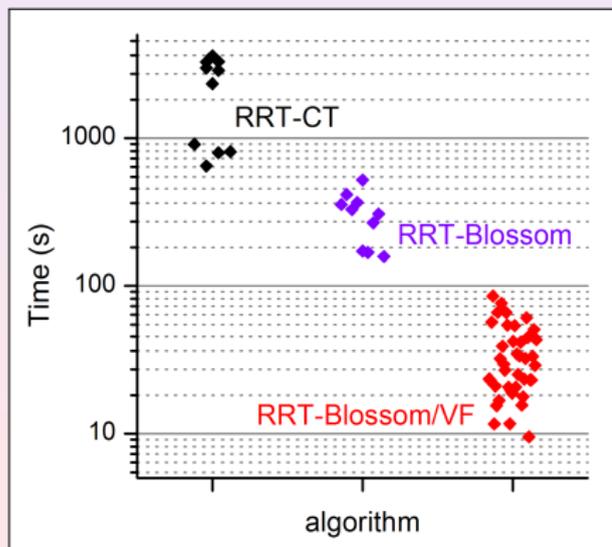
agent



problem posed



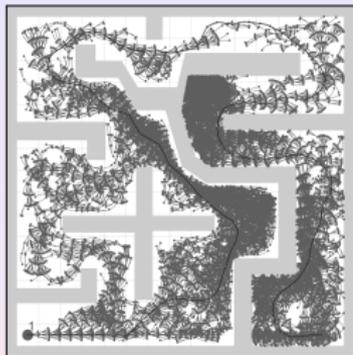
model trained on



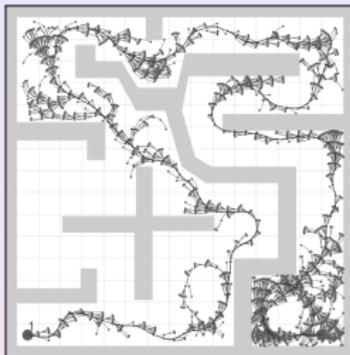
problem	algo.	runtimes
	RRT-CT	371.5s
	RRT-Blossom	21.0s
	RRT-Blossom-VF	5.6s
	RRT-CT	209.9s
	RRT-Blossom	13.5s
	RRT-Blossom-VF	3.6s
	RRT-CT	2148.6s
	RRT-Blossom	305.7s
	RRT-Blossom-VF	34.3s

# Results: tree structure

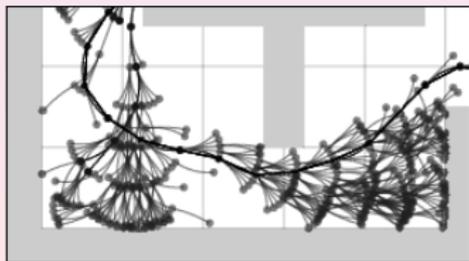
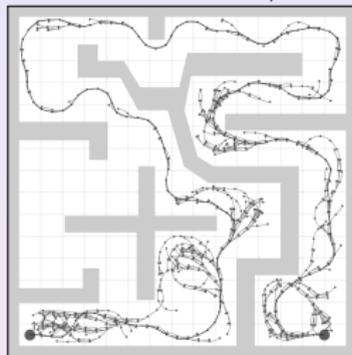
RRT-CT



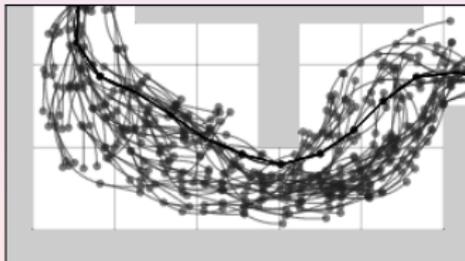
RRT-Blossom



RRT-Blossom w/**VF**



no filtering



viability filtering

# Outline

- 1 Motion Planning (MP)
  - What is MP?
  - Types of MP problems
  - MP is hard
- 2 Viability
- 3 Contributions
  - MP in highly constrained problems
  - MP w/viability filtering
  - Viability-based safety enforcement
- 4 Conclusion

# Viability-based safety enforcement

- $\Rightarrow$  assisted control:
  - inherently useful
  - facilitates obtaining user-demonstrated training data
  - helpful in user-assisted MP (future work)
  
- **key idea:** viability more reliable for detecting imminent danger

# Viability-based safety enforcement

- ⇒ assisted control:
  - inherently useful
  - facilitates obtaining user-demonstrated training data
  - helpful in user-assisted MP (future work)
  
- **key idea:** viability more reliable for detecting imminent danger

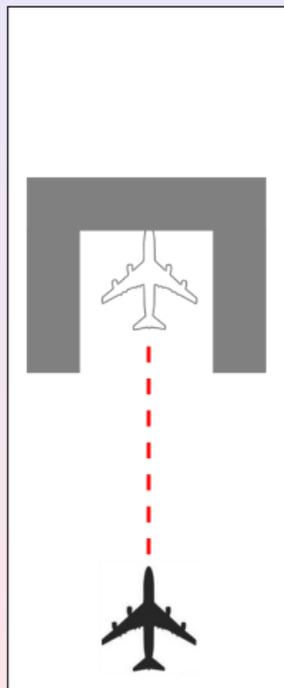
# Collision avoidance

## typical (collision-based)

- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety  $\uparrow$  as  $T_h \rightarrow \infty$

## better: viability-based safety enforcement

- only a minimal lookahead needed
- longer lookaheads: milder corrections



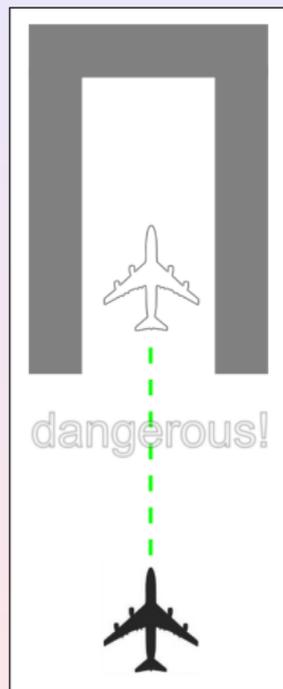
# Collision avoidance

## typical (collision-based)

- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety $\uparrow$  as  $T_h \rightarrow \infty$

better: viability-based safety enforcement

- only a minimal lookahead needed
- longer lookaheads: milder corrections



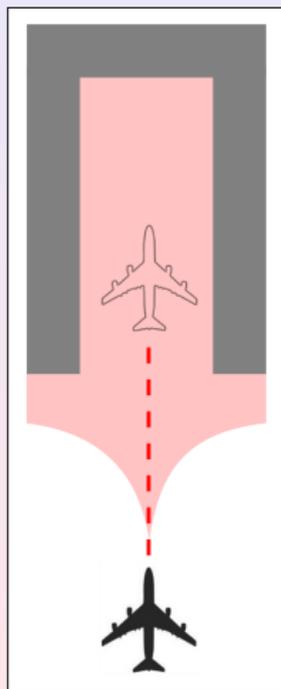
# Collision avoidance

## typical (collision-based)

- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety  $\uparrow$  as  $T_h \rightarrow \infty$

## better: viability-based safety enforcement

- only a minimal lookahead needed
- longer lookaheads: milder corrections



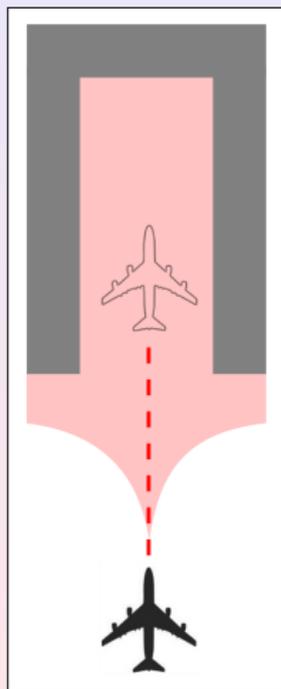
# Collision avoidance

## typical (collision-based)

- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety $\uparrow$  as  $T_h \rightarrow \infty$

## better: viability-based safety enforcement

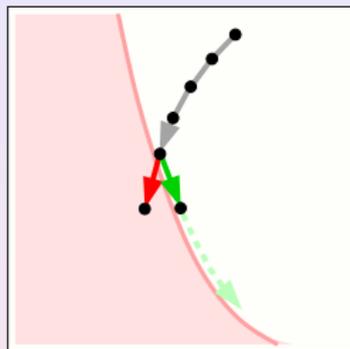
- only a minimal lookahead needed
- longer lookaheads: milder corrections



# Collision avoidance

## typical (collision-based)

- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety  $\uparrow$  as  $T_h \rightarrow \infty$



## better: viability-based safety enforcement

- only a minimal lookahead needed
- longer lookaheads: milder corrections

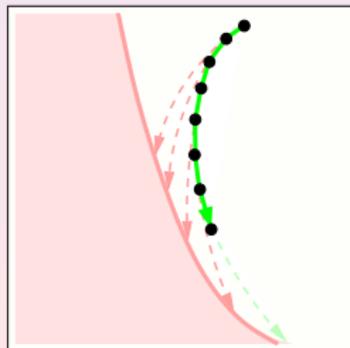
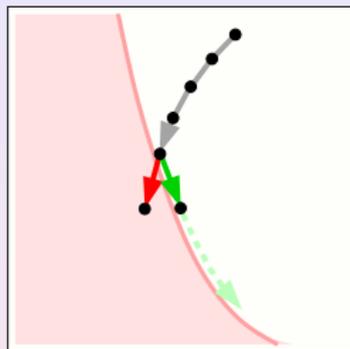
# Collision avoidance

## typical (collision-based)

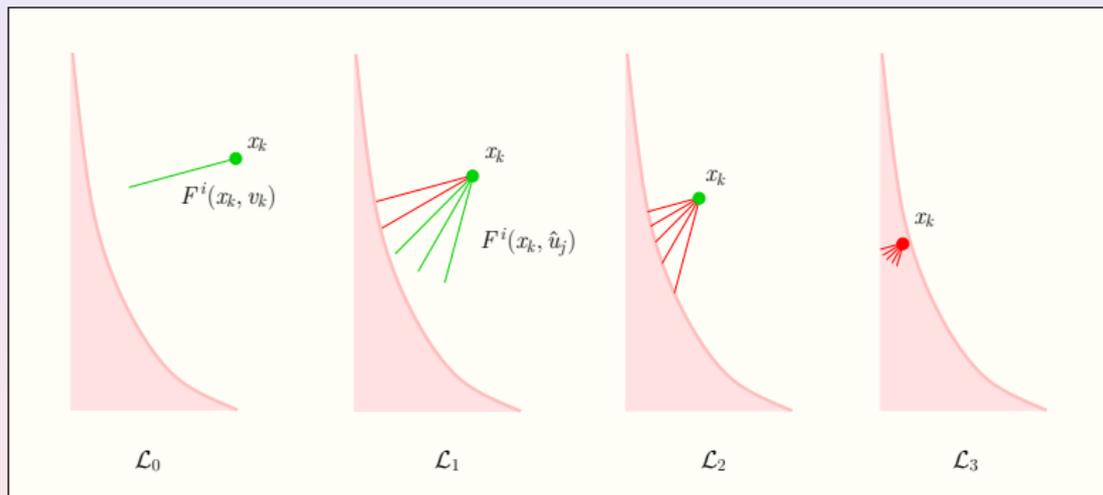
- based on predictive lookahead ( $T_h$  seconds)
- weakness:  $T_h$  is finite
  - $T_h$  may be too small
  - safety  $\uparrow$  as  $T_h \rightarrow \infty$

## better: viability-based safety enforcement

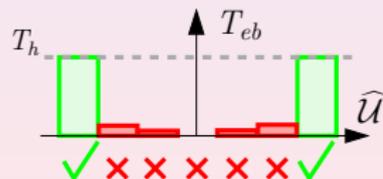
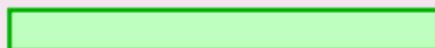
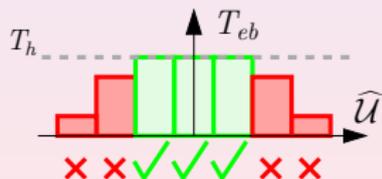
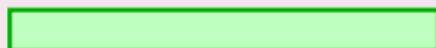
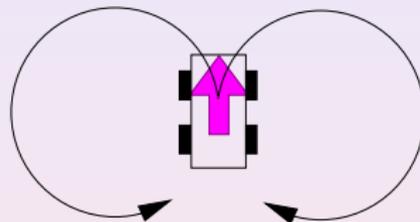
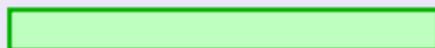
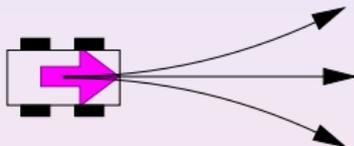
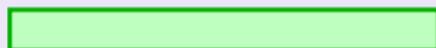
- only a minimal lookahead needed
- longer lookaheads: milder corrections



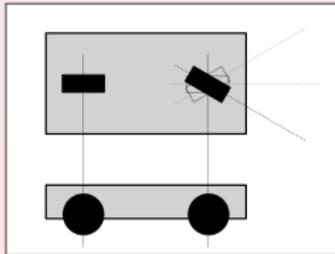
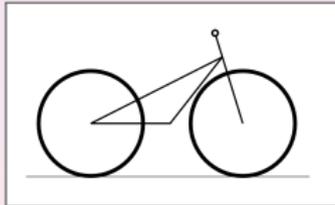
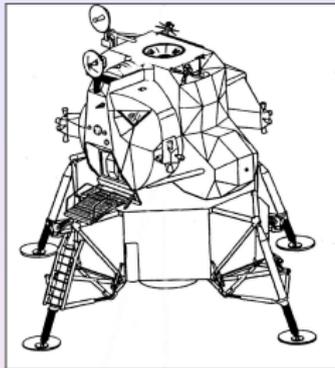
# Operation



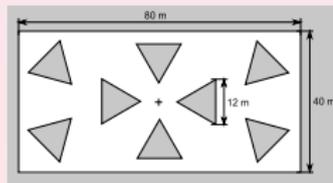
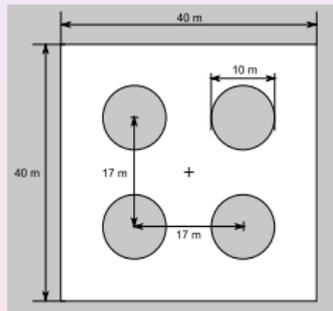
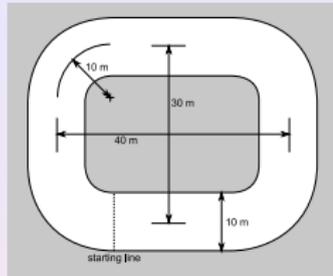
# Viability of control actions



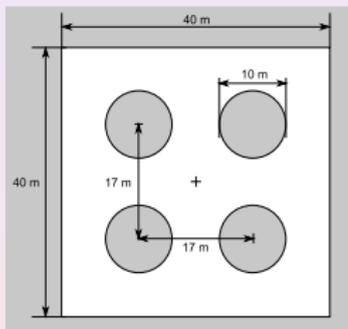
agents



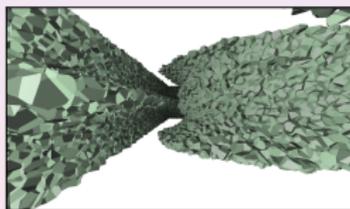
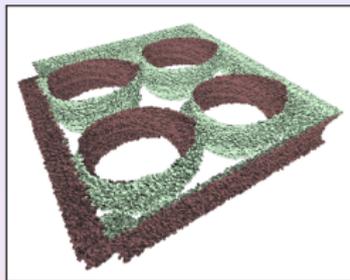
environments



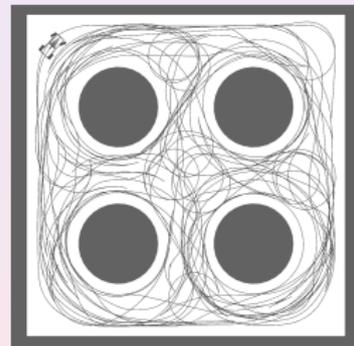
environment



$Viab(\mathcal{X}_{free})$  model



enforcement



# Conclusion

- **contributions**

- better handling of constrained environments in RRT
- more efficient MP by narrowing search to  $Viab(\mathcal{X}_{free})$
- more robust threat avoidance in computer-assisted control

- **future work:**

- learning appropriate *actions* from motion data
- MP w/motion “macro primitives”
- evaluate viability filtering with other MPs
- *local* viability models for safety enforcement
- (near-)optimal solutions for MP w/DC
- human-derived motion data (e.g., style content)
- human-guided MP: selection of style or topology

# Conclusion

- **contributions**

- better handling of constrained environments in RRT
- more efficient MP by narrowing search to  $Viab(\mathcal{X}_{free})$
- more robust threat avoidance in computer-assisted control

- **future work:**

- learning appropriate *actions* from motion data
- MP w/motion “macro primitives”
- evaluate viability filtering with other MPs
- *local* viability models for safety enforcement
- (near-)optimal solutions for MP w/DC
- human-derived motion data (e.g., style content)
- human-guided MP: selection of style or topology