CSC418/2504F Fall 2007: Midterm Test:
Wednesday, Oct. 24, 2005, 7:00 PM

Family Name:______________________________

First Name: ______________________________

Student ID: ______________________________

Instructions:

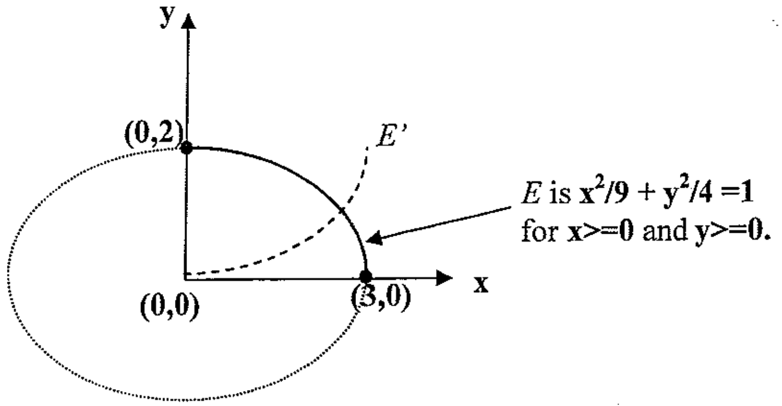
Attempt **all** questions.
There are four questions.
The total mark is 30.

You have 55 minutes to complete the test.

Aids allowed: Brain, writing implements.
Textbooks, calculators and notes are NOT allowed.

1:	/10
2:	/6
3:	/8
4:	/6
Total:	/30

1. **Ellipses - 10 marks**: We would like to develop a Bresenham-like algorithm using only integer arithmetic to draw the shape shown. The shape is an elliptical segment E defined by the equation $\mathbf{x^2/9 + y^2/4 = 1}$ with positive **x** and **y** between the points **(3,0)** and **(0,2)**.



   a. **4 marks**: Write the 2D transformation that would make the curve segment E appear like the curve segment E' with positive tangent starting at the origin. (you may leave your answer as a product of 3x3 transformation matrices).

(+4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ or other combinations that yield same result.

or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

+3 if order reversed
+2 if one matrix is wrong.

   b. **2 marks**: What is the tangent vector to the ellipse at a point **(a,b)**?

$\left(\frac{df}{dx}, \frac{df}{dy}\right) = \left(\frac{2}{9}x, \frac{2}{4}y\right) \Rightarrow$ normal vector $= \left(\frac{2}{9}a, \frac{b}{2}\right) \Rightarrow$ tangent vector $= \left(-\frac{b}{2}, \frac{2}{9}a\right)$ (+2)

+1 if normal vector is given instead

   c. **4 marks**: A Bresenham-like algorithm iterates over one coordinate (say **x**) to generate a curve segment between two points with integer coordinates. Is it possible to generate the curve segment E' (whose equation is $\mathbf{x^2/9 + (y-2)^2/4 = 1}$) in this manner iterating over **x** from 0 to 3? If not, explain why and how many curve segments E' should be broken into for the Bresenham algorithm to work. **[EXTRA CREDIT 2 marks** : Actually find the point(s) at which to break the curve E']

+0.5 if some understanding is shown

No. (+1) Because the tangent is not alway ≤ 1 (+1)
Should be broken into 2 pieces. because the tangent increases mo~~n~~otically from 0 to $+\infty$ therefore only passes 1 once. For second segment, should swap x and y in the algorithm (+2)

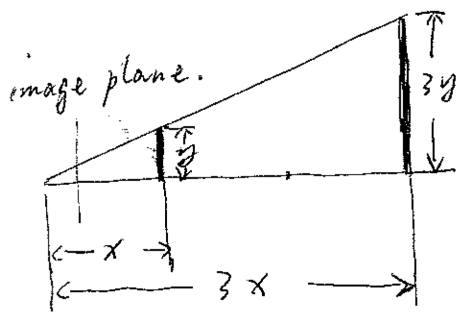
Extra: $\left.\begin{cases} x^2/9 + (y-2)^2/4 = 1 \\ \text{Tangent} = \frac{2}{9}x \Big/ \frac{2(y-2)}{4} = 1 \end{cases}\right\} \Rightarrow (x, y) = \left(\frac{9}{\sqrt{13}}, 2 - \frac{4}{\sqrt{13}}\right)$

$0 \leq x \leq 3,\ 0 \leq y \leq 2$

(+2)

2. **Projection - 6 marks**

   a. **2 marks**: Explain with an illustrated example why an object thrice as far from the viewpoint as another object a third its size appears to be of the same size as the smaller object, under perspective projection.



+2 if the illustration is correct.

+1 if the illustration is unclear but understanding is shown.

   b. **4 marks**: Parallel lines typically converge to a point in the image plane under perspective projection. Is this true for orthographic projection? Is this always true for perspective projection? If not, give an example family of parallel lines that remain parallel in the image after perspective projection along the **z** axis.

No, not true for orthographic projection. (+1)

Not always true for perspective projection (+1)

Any set of parallel lines that are perpendicular to z axis (i.e. parallel to the image plane) remain parallel (+2)

3. **Visibility and the graphics pipeline - 8 marks** (2 marks each).

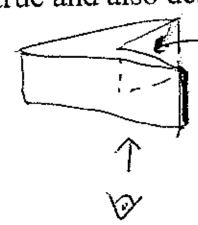
   a. Polygons can be clipped to the 3D canonical view volume after a perspective divide. (True or false with a reason).

False (+1)

Because the sign of z is lost, cannot distinguish objects in front of or behind camera. (+1)

   b. Removing back-faces completely resolves visibility for a single object in a scene (i.e. all the remaining faces are visible). Draw or describe an object for which this statement is not true and also describe the class of objects for which it holds true.

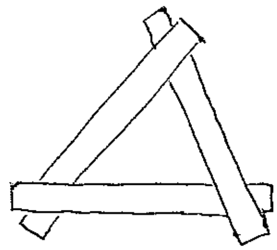
Not true :



front face but invisible (+1)

True : Convex objects (+1)

c. Draw a configuration of three polygons (that do not intersect in 3D) that can not be drawn with correct visibility using the Painter's algorithm no matter what their sorted order.

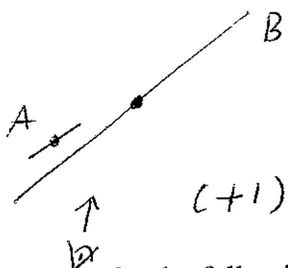


+2 if cyclic configuration is shown.

d. If the centroid of a triangle A is closer to the eye than the centroid of triangle B, visibility is resolved by rendering A after B. (True or false with reason).

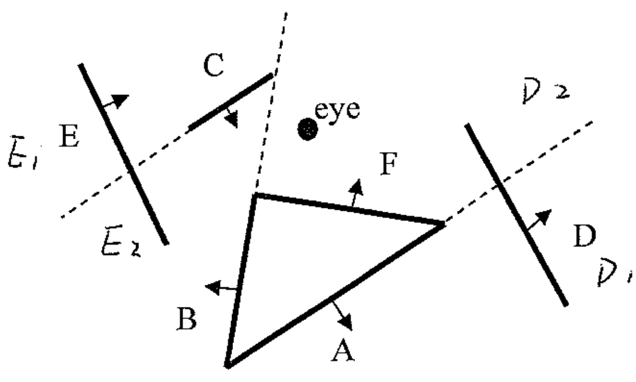
False (+1)

counter example:

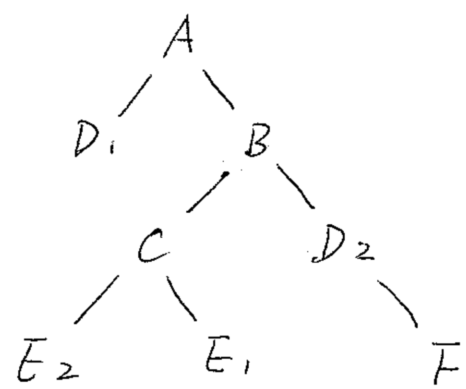


(+1) (acceptable counter example: A intersects B)

4. **BSP -6 marks** Draw the BSP tree for the following set of polygons. Break and label polygons into smaller fragments if necessary. Let the left child of a node represent the space in front of the plane (right child, the space behind). Which order should the left and right sub-trees of node **B** be drawn for correct visibility when the camera viewpoint is at **eye** as shown.



Alphabetical order:



or other orders as long as the tree is correct (+5)

Left subtree → B → Right subtree (+1)