CSC418/2504 Computer Graphics Fall 2017: Assignment 1

10% of course grade
Due 11:59PM on October 11, 2017

Part A (40 marks)

Below are different exercises covering different topics from the first weeks of class. They require thought, so you are advised to consult the relevant sections of the textbook, the online lecture notes and slides, and your notes from class well in advance of the due date. Your proofs and derivations should be clearly written, mathematically correct, and concise.

All questions require showing the steps toward the solution, and marks will be subtracted if this is not the case. Even if you cannot answer a question completely, it is very important that you show your (partial) answers and your reasoning. Otherwise your TA will not be able to award you partial marks.

Both Part A and Part B must be electronically submitted. Part A must be in PDF format, by scanning your handwritten solution or by using LaTeX/MS-Word to typeset it.

1. A 2D curve can be described parametrically as follows.

$$x(t) = t\cos t$$
$$y(t) = t\sin t$$
$$0 < t < 3\pi$$

   a) [4 marks] Draw a picture of this curve. Remember, it is a 2D graph with x and y coordinates. You can use Python, MATLAB or any other tool of your choice to draw this graph.
   b) [6 marks] Find the tangent and normal vectors to this curve as a function of t. (A normal vector is any vector perpendicular to the tangent at a curve point.)

2. Two transformations f_1 and f_2 commute when $f_1 \circ f_2 = f_2 \circ f_1$. For each pair of transformations below, specify whether or not they commute. In each case, your solution can either be a derivation that proves/disproves commutativity, or if f_1 and f_2 do not commute, a specific counter-example.
   a) [3 marks] Two different translations.
   b) [3 marks] Two different rotations.
   c) [3 marks] A rotation and a uniform scaling.
   d) [3 marks] A rotation and a non-uniform scaling.

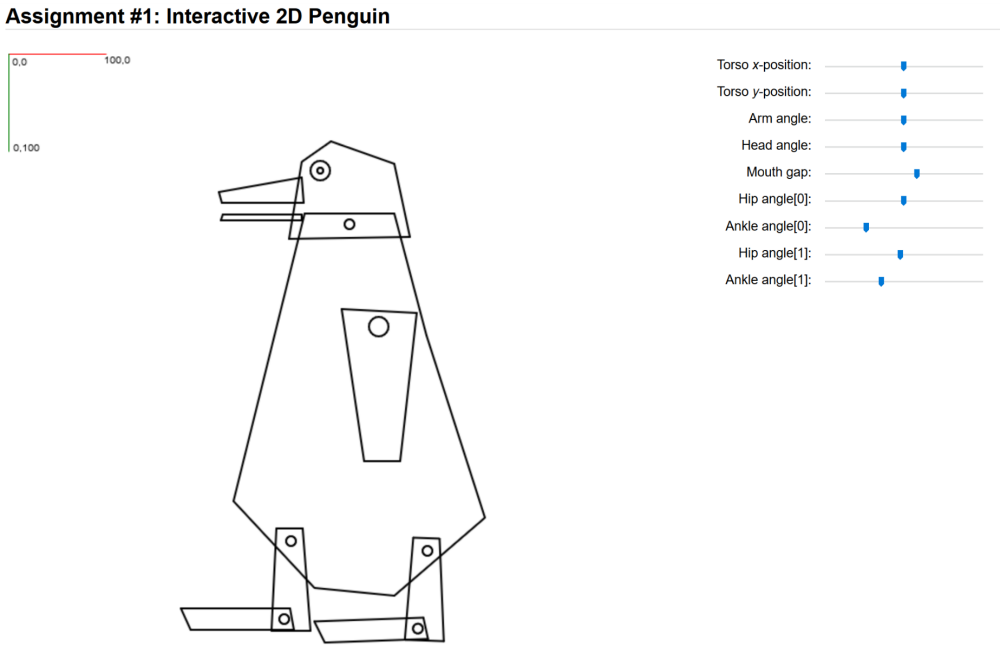
3. Points $(-1, 0), (0, 1), (1, 1), (0, 0)$ will be mapped to points $(6.3, 2.3), (6.3, 3.7), (7.7, 3.7), (7.7, 2.3)$ by an affine transformation.
   a) [8 marks] Derive the affine transformation. Simply write the steps needed to find it.
   b) [2 marks] Where does the point $(-3, 1)$ get mapped to under this transformation?

4. Decomposing transformations.

   [8 marks] Decompose the following 2D affine transformation into a translation followed by a scale followed by a rotation. Simply write the steps of how to calculate them.

$$\begin{bmatrix} 0 & 1.5 & 4.5 \\ -1.5 & 0 & -1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Part B (60 marks)



The figure above shows an articulated nine-part, seven degree-of-freedom (DoF) planar robot penguin. It has six rotational joints (depicted by circles), each with one rotational degree of freedom. The beak has a single translational degree of freedom: it can move up or down (in the local coordinate frame of the head). The whole penguin can also move up and down and left and right. Your task will be to draw such a robot penguin using Javascript and Canvas.

Hierarchical objects like this are often defined by specifying each part in a natural, part-based coordinate frame, along with transformations that specify the relative position and orientation of one part with respect to another. These transformations are often organized into a kinematic tree (e.g., with the torso as the root, and the jaw as a leaf). In addition to the kinematic tree, one must also specify the transformation from the root (e.g., the torso) to the world coordinate frame. Then, for example, to draw the torso you transform the points that define the torso from the torsos coordinate frame to the world coordinate frame. Then to draw an arm, you must transform the points that define the arm in the arm's coordinate frame to that of the torso, and then from the torsos coordinate frame to the world coordinate frame. The same process should be repeated for drawing other parts.

Your Programming Task. Your task is to draw the articulated robot penguin using Javascript and Canvas. In this task, your code should be able to control the penguin by the sliders (top-right). After program, you should be able to run it in a browser and use the sliders to control the robot penguin.

a) [20 marks] Fill up the data transformation functions. When you move the sliders, the penguin should also move.
b) [20 marks] Draw the missing parts and display a complete penguin in the browser.
c) [20 marks] Hierarchical transformation should be utilized. For example, rotating the hip joint should also move the foot.

Helper Code. A line drawing function, a point transformation function, a transformation composition function, and the controlling functions (sliders, callback functions) are given. You need to fill up the remaining drawing and transformation functions, and utilize them to draw the missing parts of the penguin.

Turning in your Solution to Parts A and B. Your pdf file for part A and your submission for part B should go inside the same folder named `a1` in a `tgz` file. All your code should remain in the directory `a1/penguin`. Note that this file should not be thought of as a substitute for putting detailed comments in your code. Your code should be well commented if you want to receive full (or even partial) credit for it.

To pack and submit your solution, execute the following commands from the directory containing your code (i.e., `a1/penguin`).

```
cd ../../
tar cvfz a1-solution.tgz a1
```

Submit your assignment using the submission script on the course website.