Topic 6:

3D Transformations

- Homogeneous 3D transformations
- Scene Hierarchies
- Change of basis and rotations in 3D

Representing 2D transforms as a 3x3 matrix

Translate a point $[x\ y]^T$ by $[t_x\ t_y]^T$:

$$\begin{pmatrix}x'\\y'\\1\end{pmatrix}=\begin{pmatrix}1&0&t_x\\0&1&t_y\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}$$

Rotate a point $[x\ y]^T$ by an angle t :

$$\begin{pmatrix}x'\\y'\\1\end{pmatrix}=\begin{pmatrix}\cos t&-\sin t&0\\\sin t&\cos t&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}$$

Scale a point $[x\ y]^T$ by a factor $[s_x\ s_y]^T$

$$\begin{pmatrix}x'\\y'\\1\end{pmatrix}=\begin{pmatrix}s_x&0&0\\0&s_y&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}$$

Representing 3D transforms as a 4x4 matrix

Translate a point $[x\ y\ z]^T$ by $[t_x\ t_y\ t_z]^T$:

$$\begin{pmatrix}x'\\y'\\z'\\1\end{pmatrix}=\begin{pmatrix}1&0&0&t_x\\0&1&0&t_y\\0&0&1&t_z\\0&0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\\1\end{pmatrix}$$

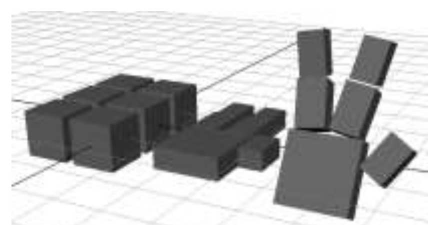
Rotate a point $[x\ y\ z]^T$ by an angle t around z axis:

$$\begin{pmatrix}x'\\y'\\z'\\1\end{pmatrix}=\begin{pmatrix}\cos t&-\sin t&0&0\\\sin t&\cos t&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\\1\end{pmatrix}$$

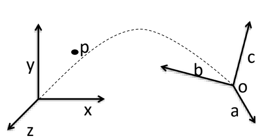
Scale a point $[x\ y\ z]^T$ by a factor $[s_x\ s_y\ s_z]^T$

$$\begin{pmatrix}x'\\y'\\z'\\1\end{pmatrix}=\begin{pmatrix}s_x&0&0&0\\0&s_y&0&0\\0&0&s_z&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\\1\end{pmatrix}$$

Scene Hierarchies



Change of reference frame/basis matrix



$$p = a p_x' + b p_y' + c p_z' + o$$

$$p = \begin{pmatrix}a&b&c&o\\0&0&0&1\end{pmatrix} p'$$

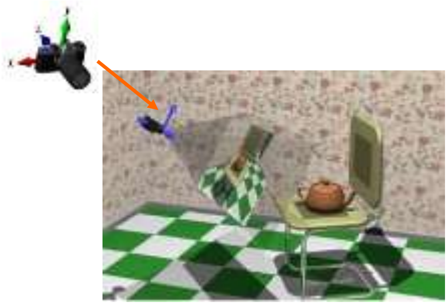
$$p' = \begin{pmatrix}a&b&c&o\\0&0&0&1\end{pmatrix}^{-1} p$$

Topic 7:

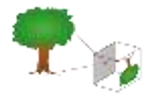
3D Viewing

- Camera Model
- Orthographic projection
- The world-to-camera transformation
- Perspective projection
- The transformation chain for 3D viewing

Camera model

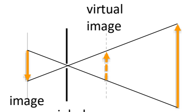


Camera model: camera obscura

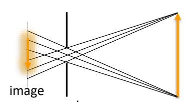


Camera model

Ideal pinhole camera

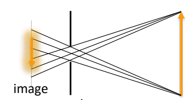


Real pinhole camera

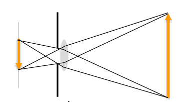


Camera model

Real pinhole camera

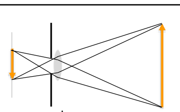


Camera with a lens

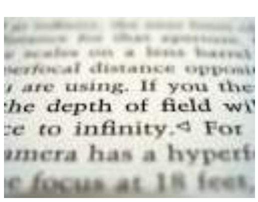


Camera model

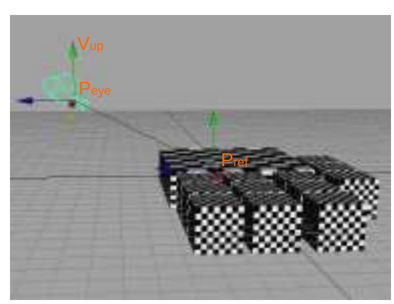
Camera with a lens



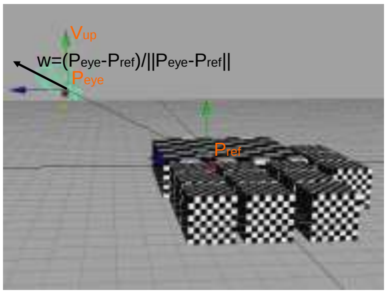
Depth of Field



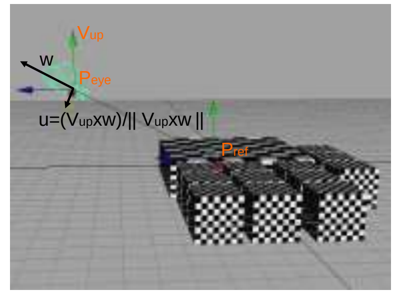
Viewing Transform



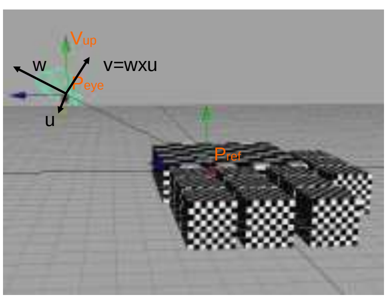
Viewing Transform



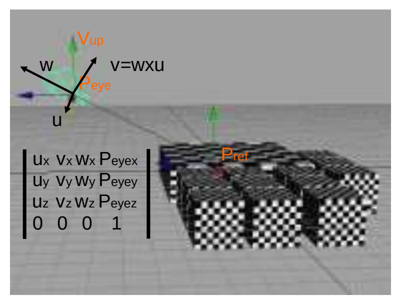
Viewing Transform



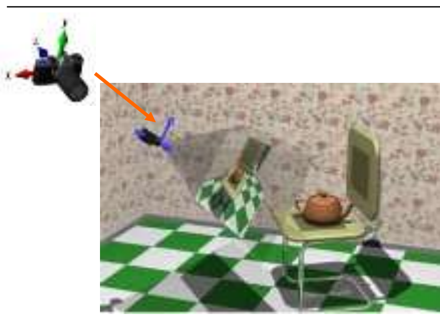
Viewing Transform



Change-of-basis Matrix

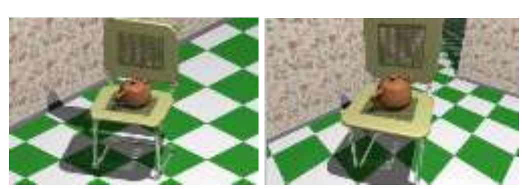


Camera model



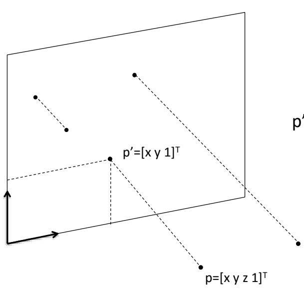
Camera model

What is the difference between these images?



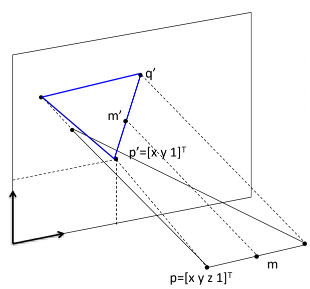
Perspective: Muller-Lyer Illusion

Orthographic projection



p'= [1 0 0 0] p
[0 1 0 0]
[0 0 0 1]

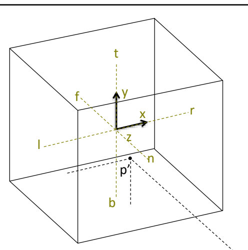
Orthographic projection



Is |p-q| = |p'-q'| ?

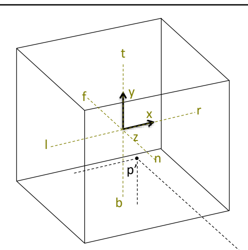
If m= (p+q)/2,
Is m' = (p'+q')/2?

Cannonical view volume



Map 3D to a cube centered at the origin of side length 2!

Cannonical view volume

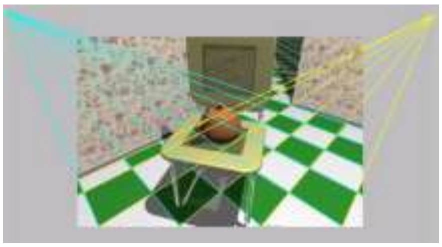


Map 3D to a cube centered at the origin of side length 2!

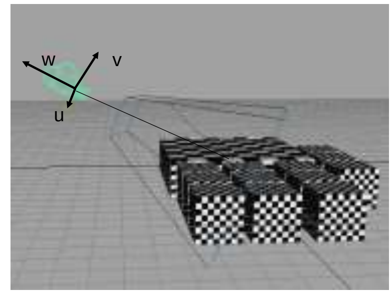
Translate(-(l+r)/2,-(t+b)/2,-(n+f)/2))
Scale(2/(r-l), 2/(t-b), 2/(f-n))

Camera model

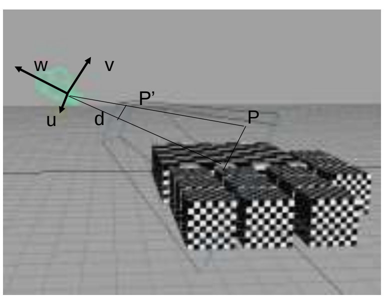
Perspective Projection



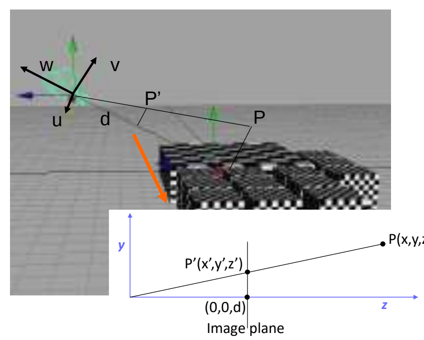
Perspective projection



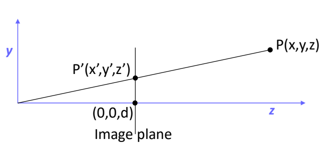
Perspective projection



Simple Perspective

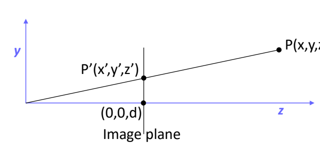


Simple Perspective



y'= yd/z
x'= xd/z
z'=d

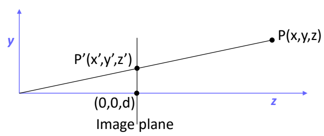
Simple Perspective



$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

w'= z/d

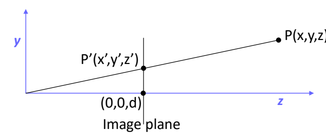
Simple Perspective



$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Find **a** and **b** such that z'=-1 when z=d and z'=1 when z=D, where d and D are near and far clip planes.

Simple Perspective

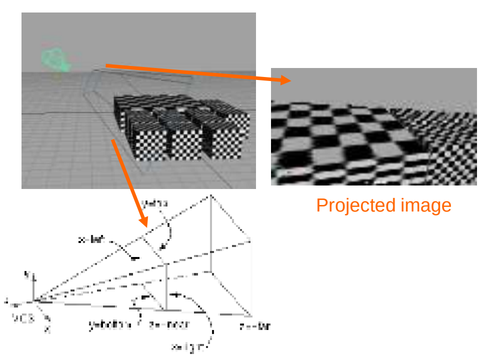


$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

z'=d(**az**+**b**)/z => -1=**a**d+**b** and 1=d(**a**D+**b**)/D

=> **b**=2D/(d-D) and **a**=(D+d)/(d(D-d))

Viewing volumes



Viewing Pipeline