Topic 7:

3D Transformations

- Homogeneous 3D transformations
- Scene Hierarchies
- Change of basis and rotations in 3D

Representing 2D transforms as a 3x3 matrix

Translate a point $[x\ y]^T$ by $[t_x\ t_y]^T$:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Rotate a point $[x\ y]^T$ by an angle t :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Scale a point $[x\ y]^T$ by a factor $[s_x\ s_y]^T$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Representing 3D transforms as a 4x4 matrix

Translate a point $[x\ y\ z]^T$ by $[t_x\ t_y\ t_z]^T$:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

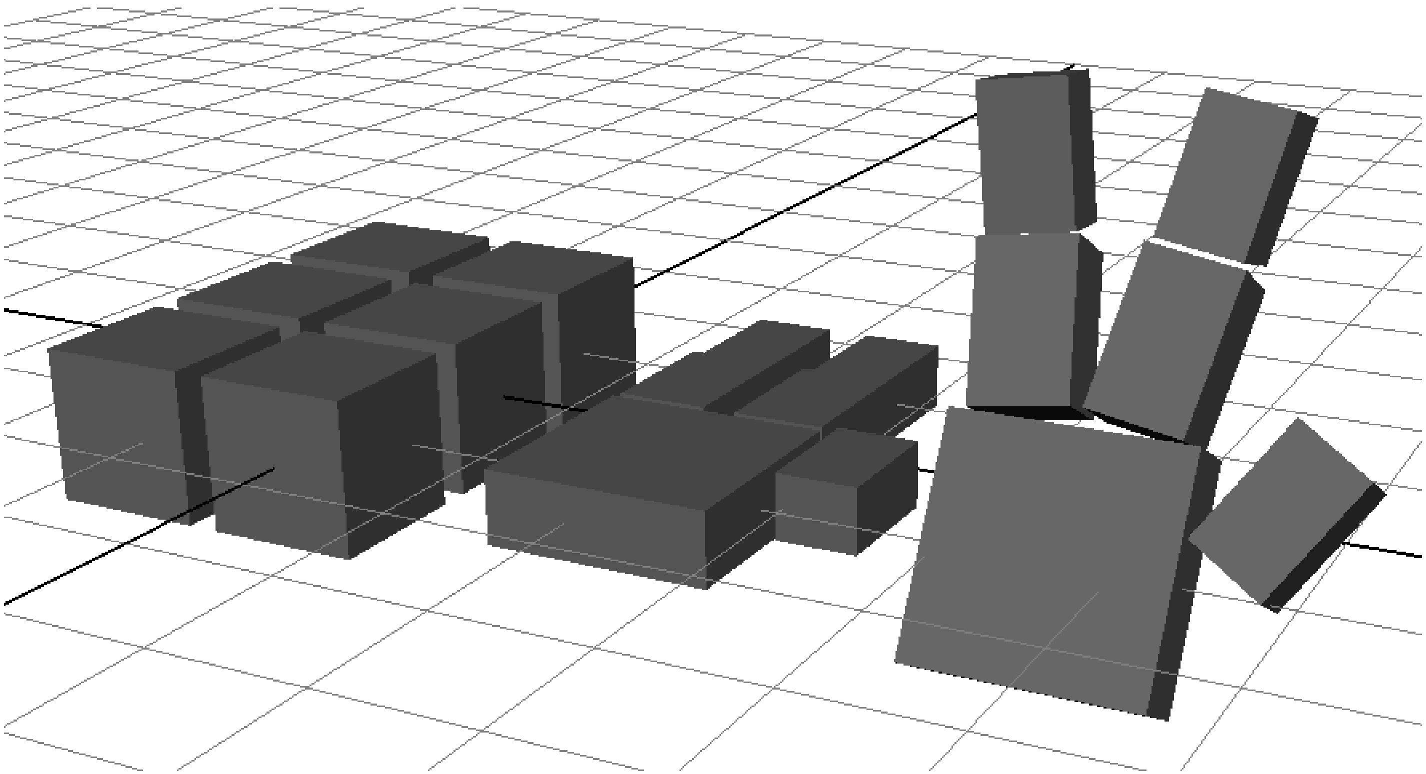
Rotate a point $[x\ y\ z]^T$ by an angle t around z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

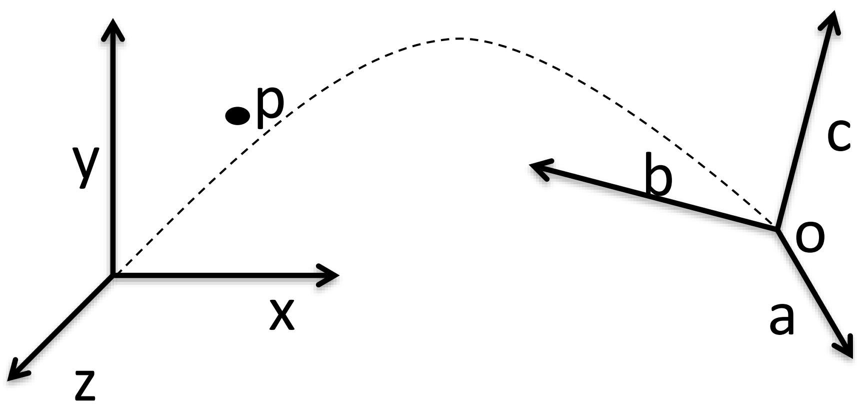
Scale a point $[x\ y\ z]^T$ by a factor $[s_x\ s_y\ s_z]^T$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scene Hierarchies



Change of reference frame/basis matrix



$$p = a p_x' + b p_y' + c p_z' + o$$

$$p = \begin{pmatrix} a & b & c & o \\ 0 & 0 & 0 & 1 \end{pmatrix} p'$$

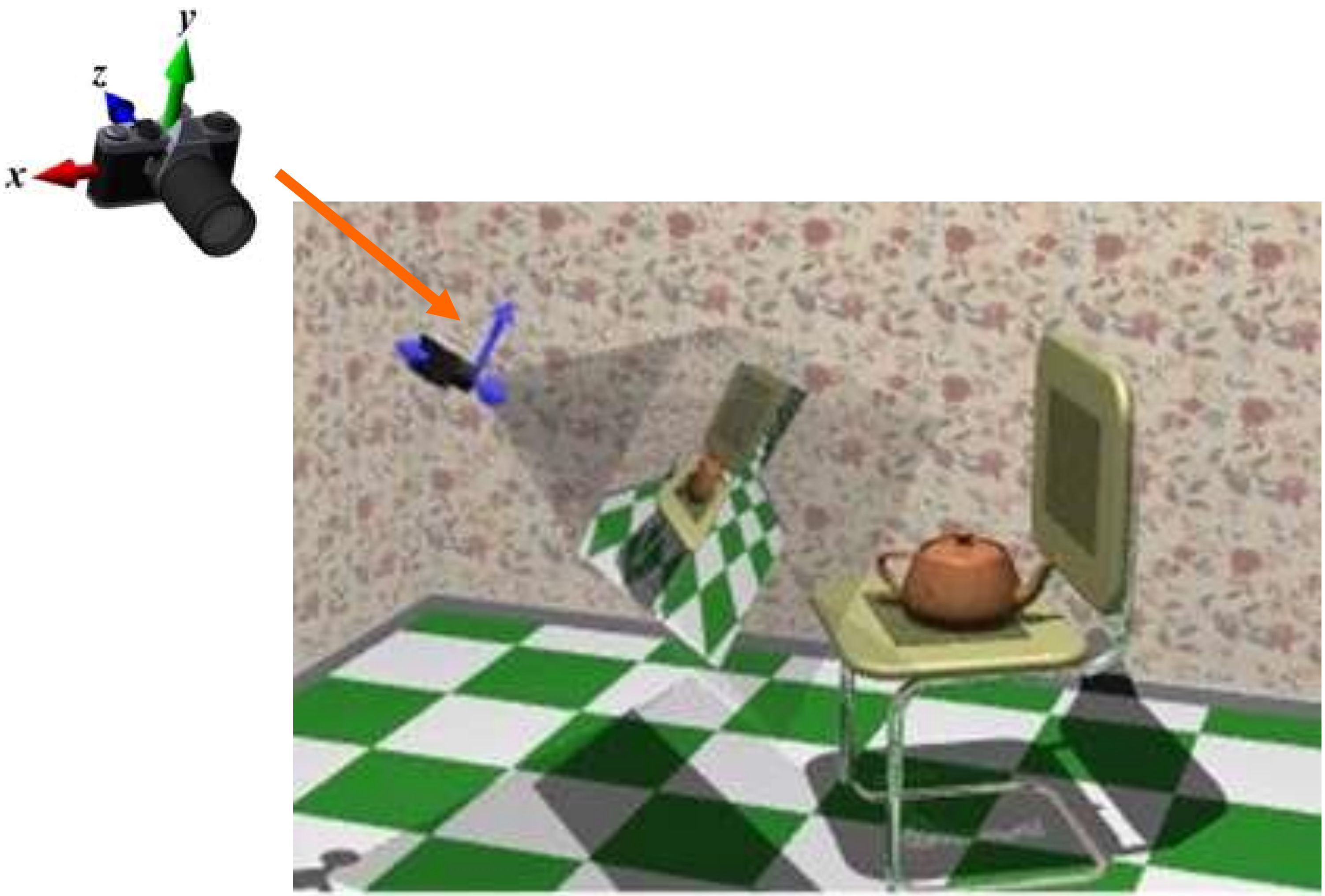
$$p' = \begin{pmatrix} a & b & c & o \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} p$$

Topic 8:

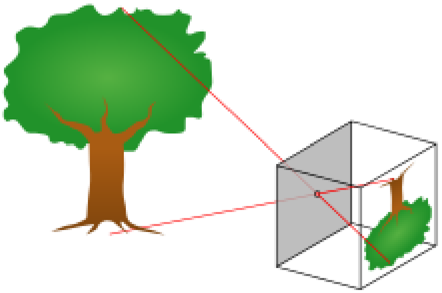
3D Viewing

- Camera Model
- Orthographic projection
- The world-to-camera transformation
- Perspective projection
- The transformation chain for 3D viewing

Camera model

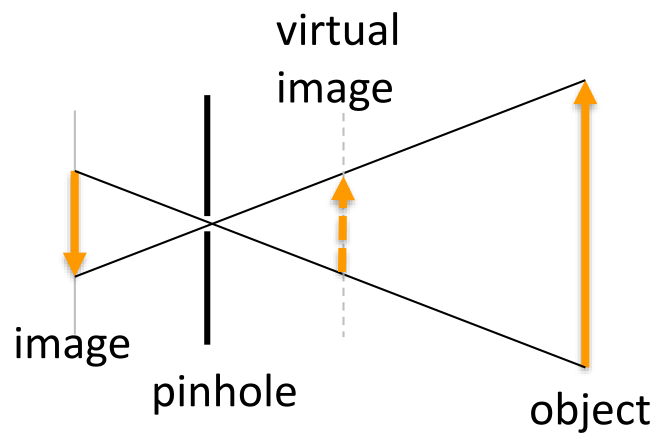


Camera model: camera obscura

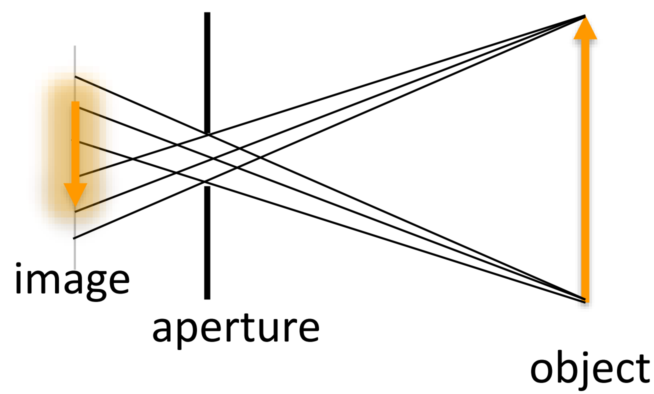


Camera model

Ideal pinhole camera

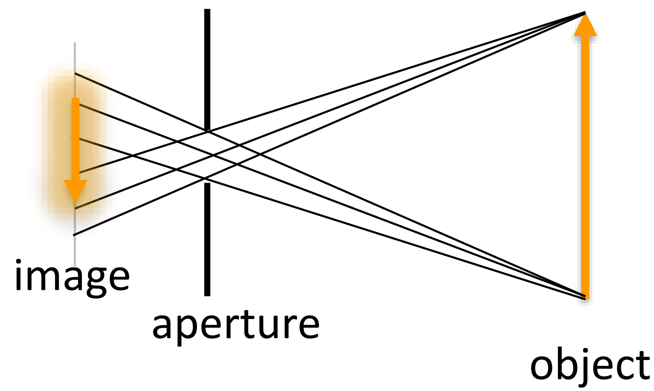


Real pinhole camera

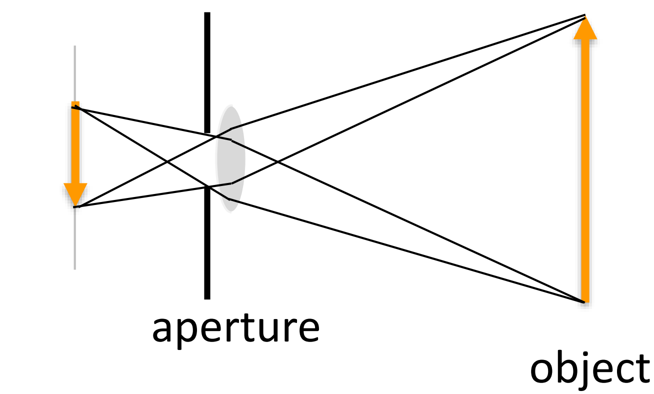


Camera model

Real pinhole camera

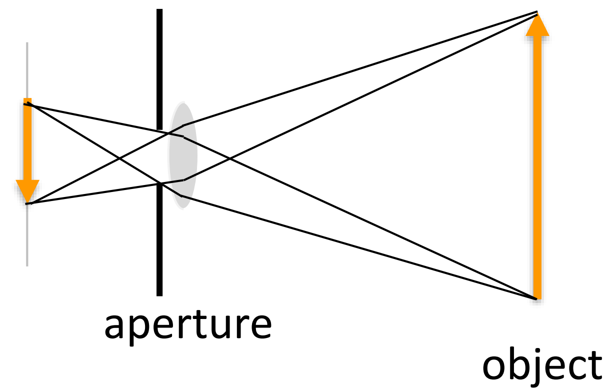


Camera with a lens

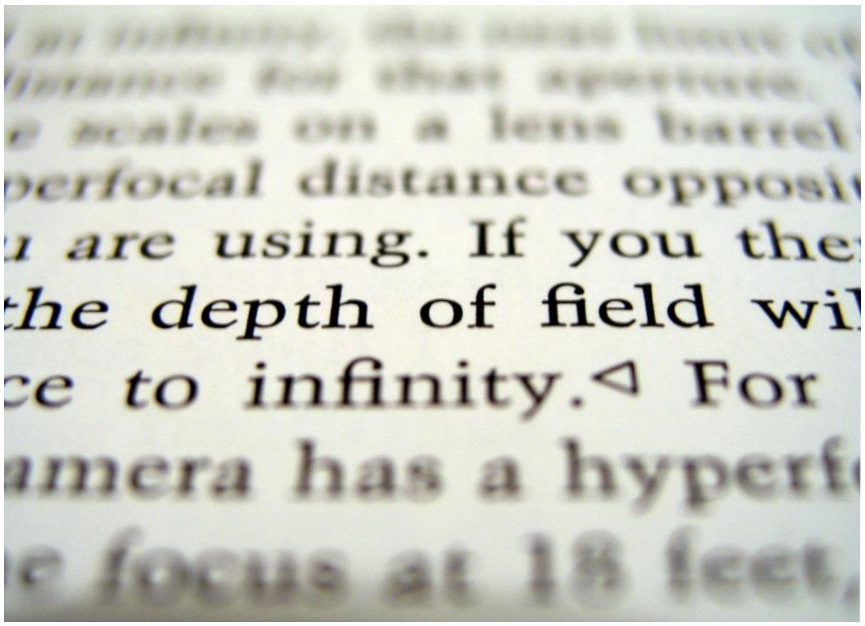


Camera model

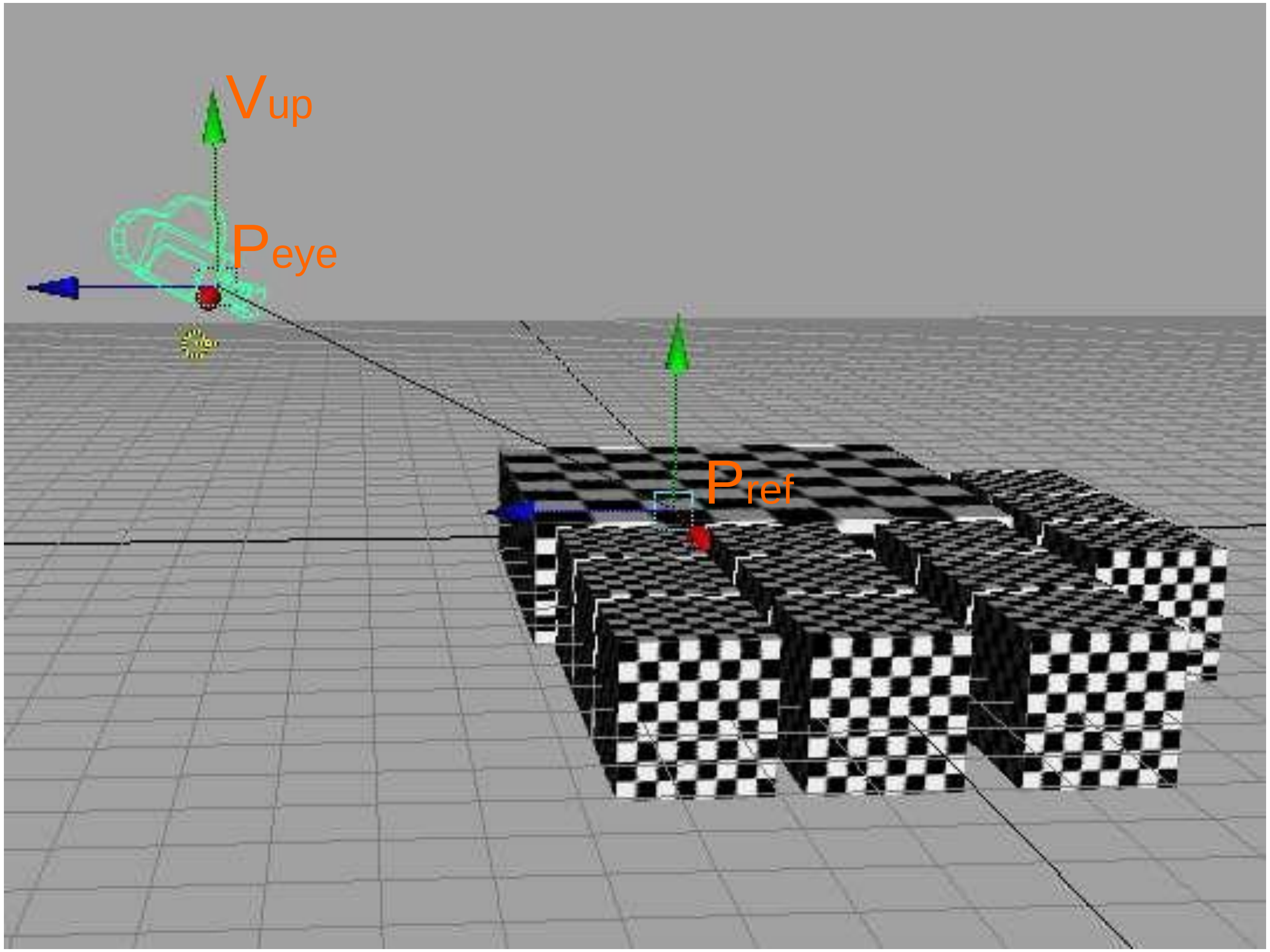
Camera with a lens



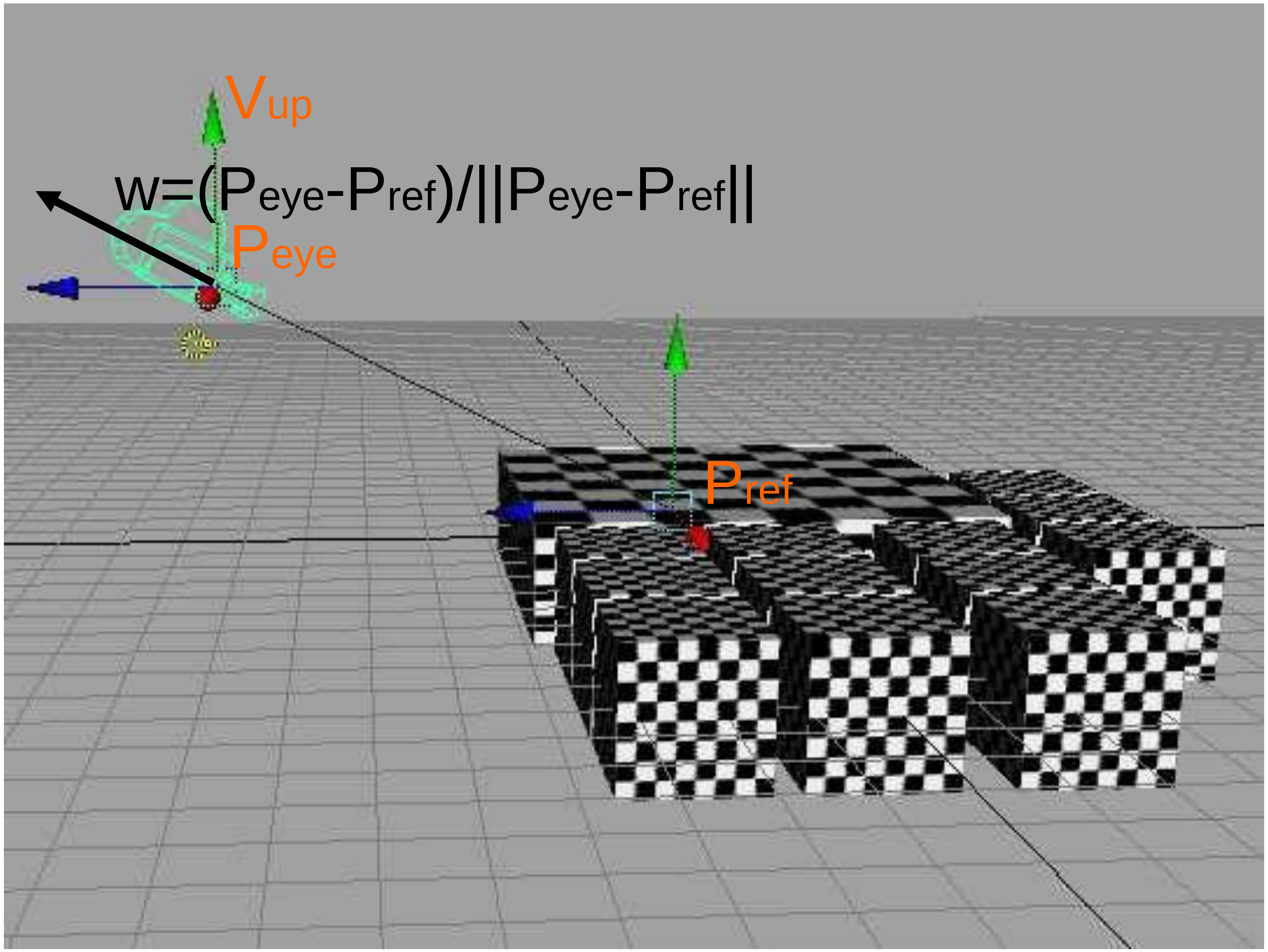
Depth of Field



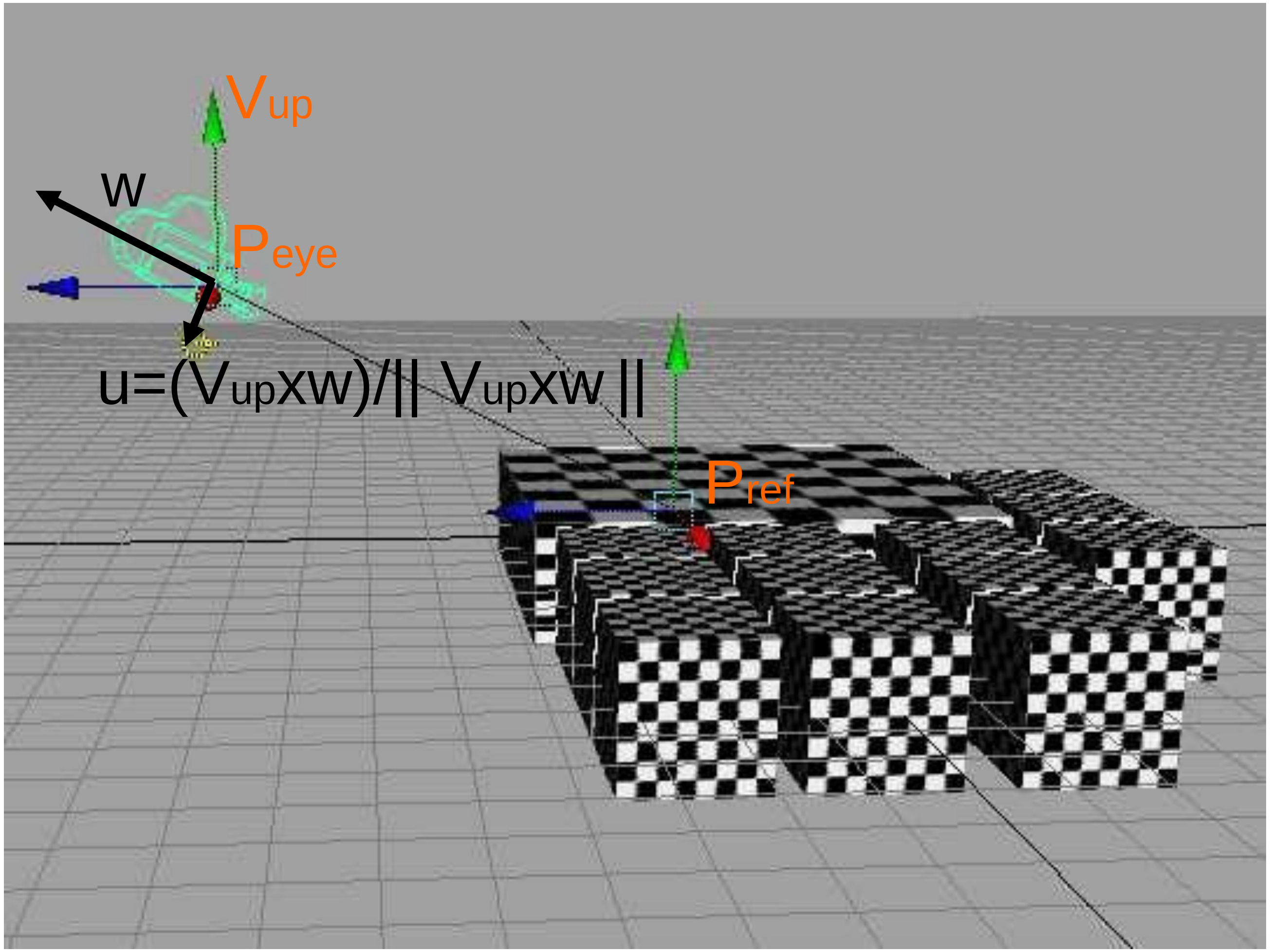
Viewing Transform



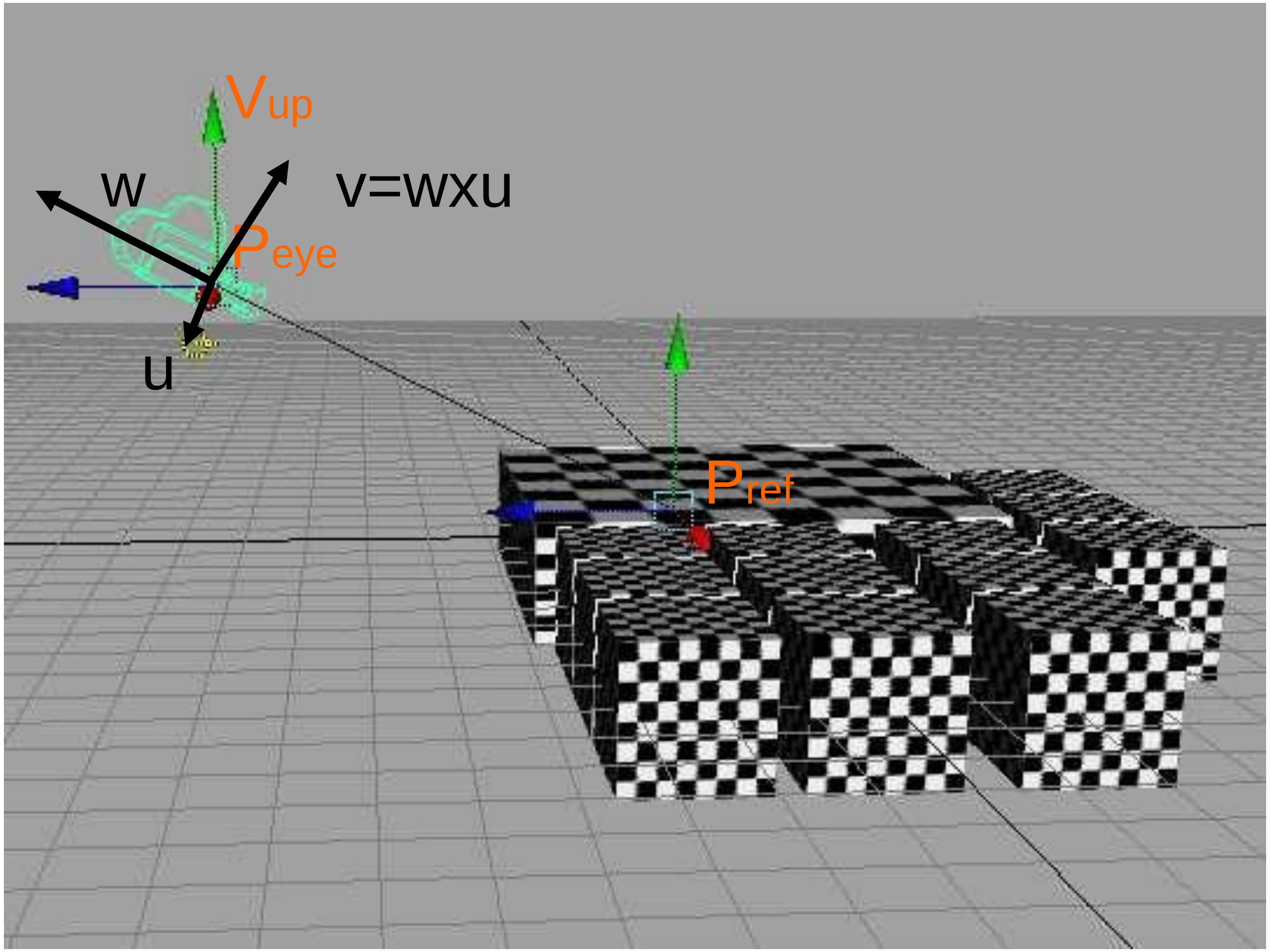
Viewing Transform



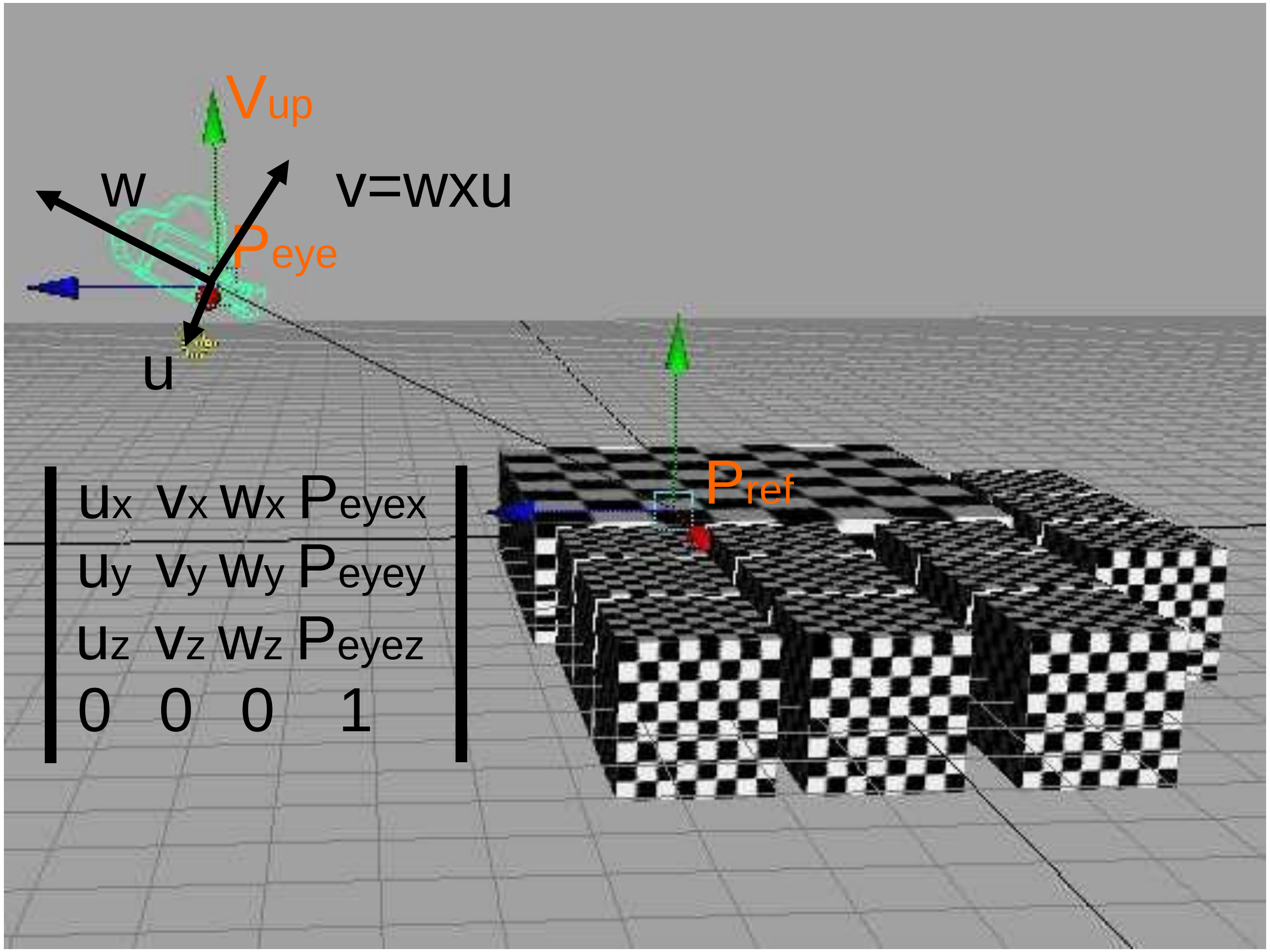
Viewing Transform



Viewing Transform

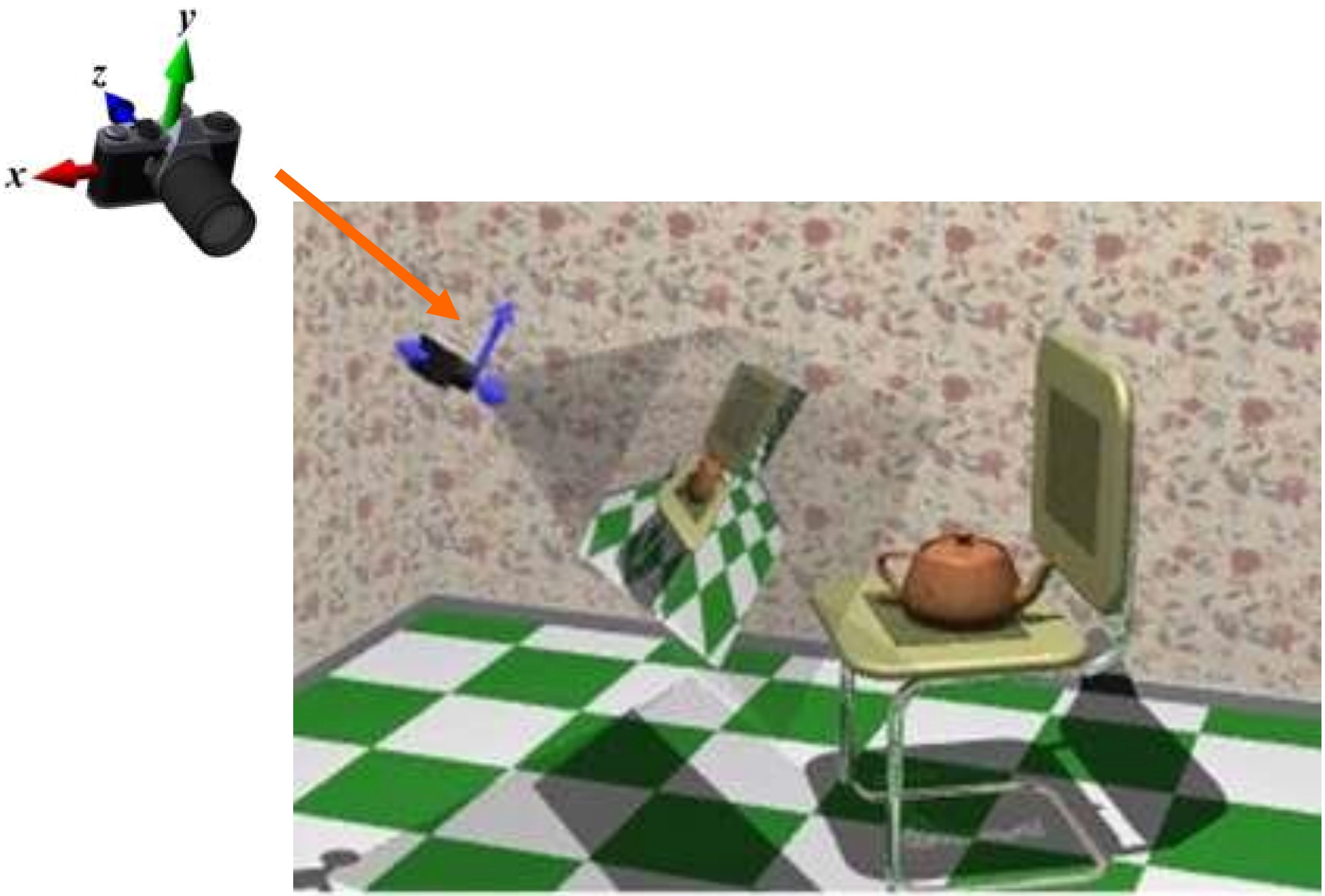


Change-of-basis Matrix



$$\begin{bmatrix} u_x & v_x & w_x & P_{eyex} \\ u_y & v_y & w_y & P_{eyey} \\ u_z & v_z & w_z & P_{eyez} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

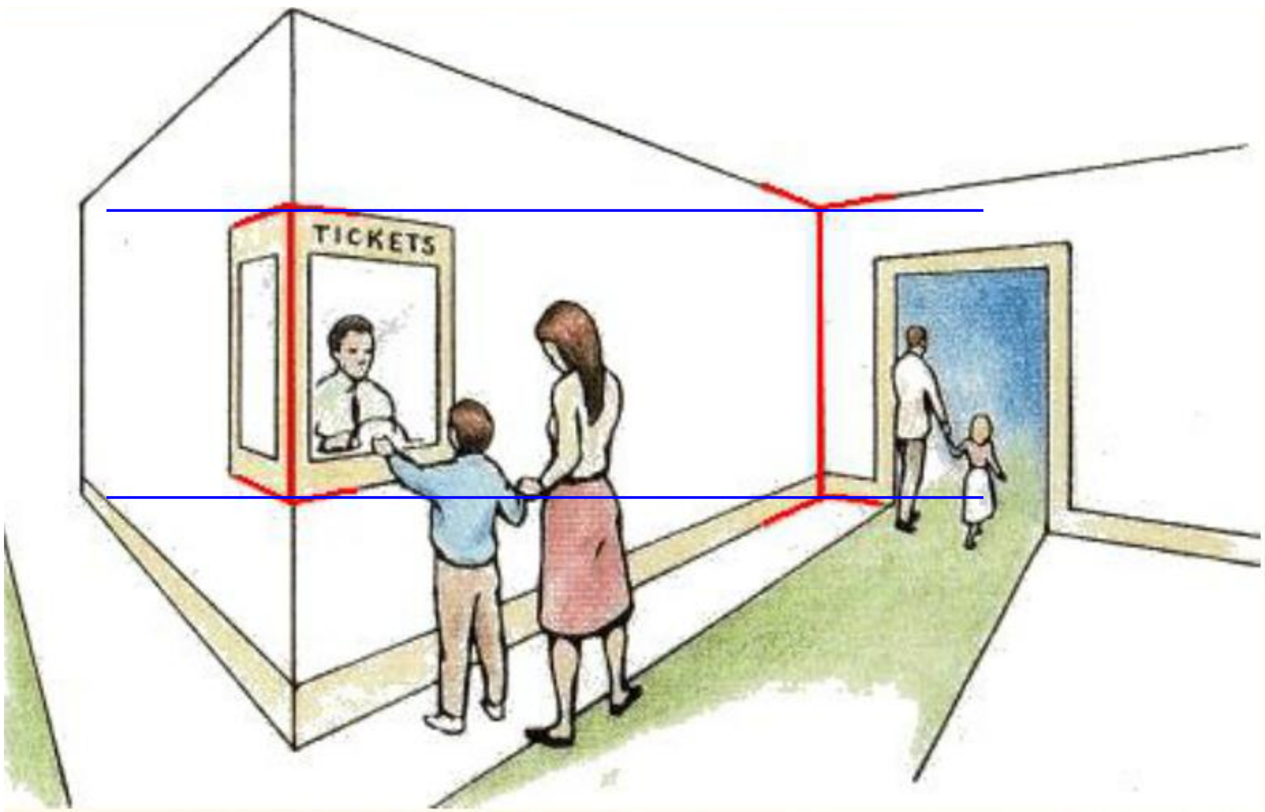
Camera model



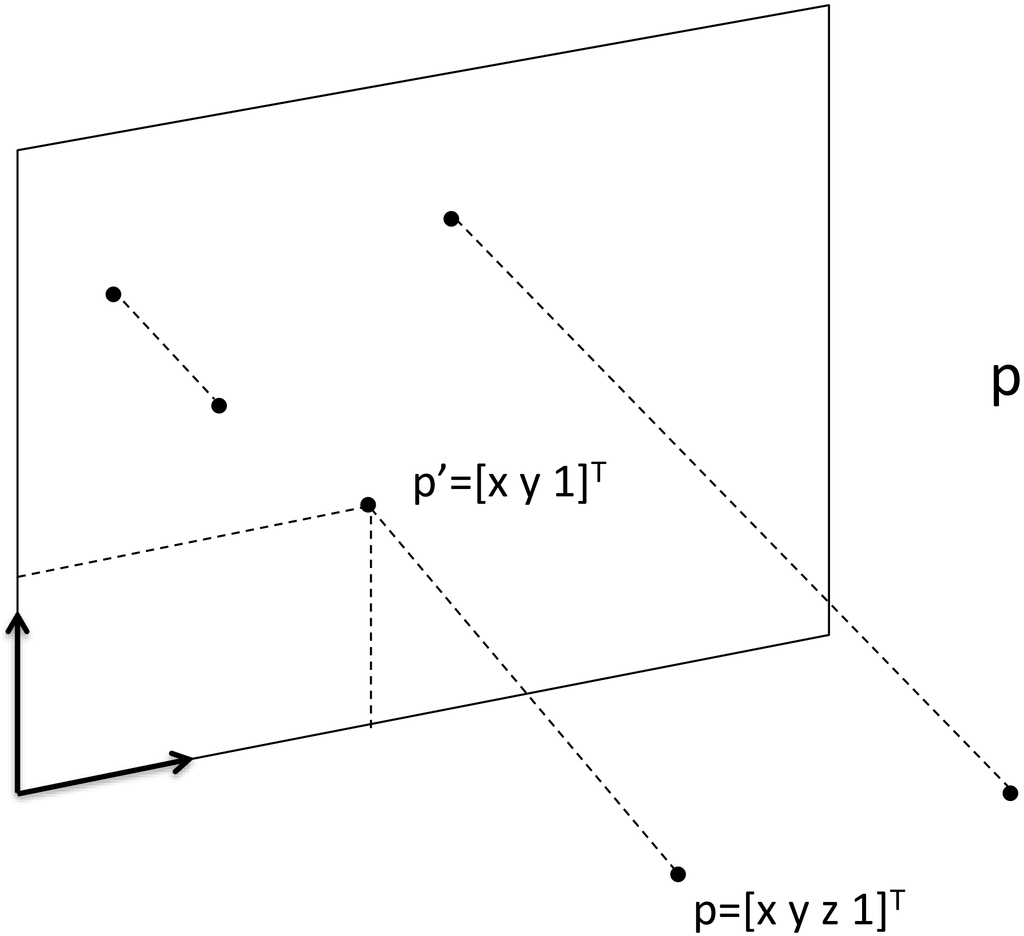
Camera model

What is the difference between these images?

Perspective: Muller-Lyer Illusion

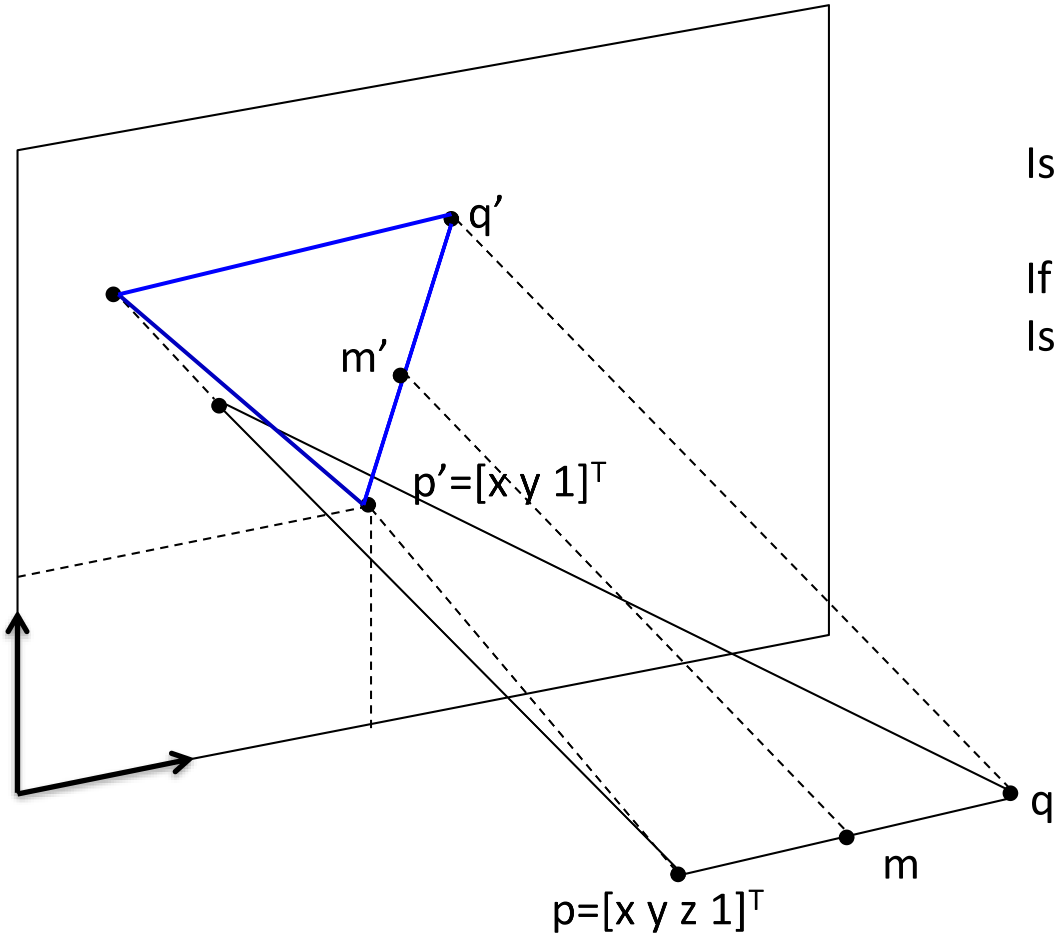


Orthographic projection



$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} p$$

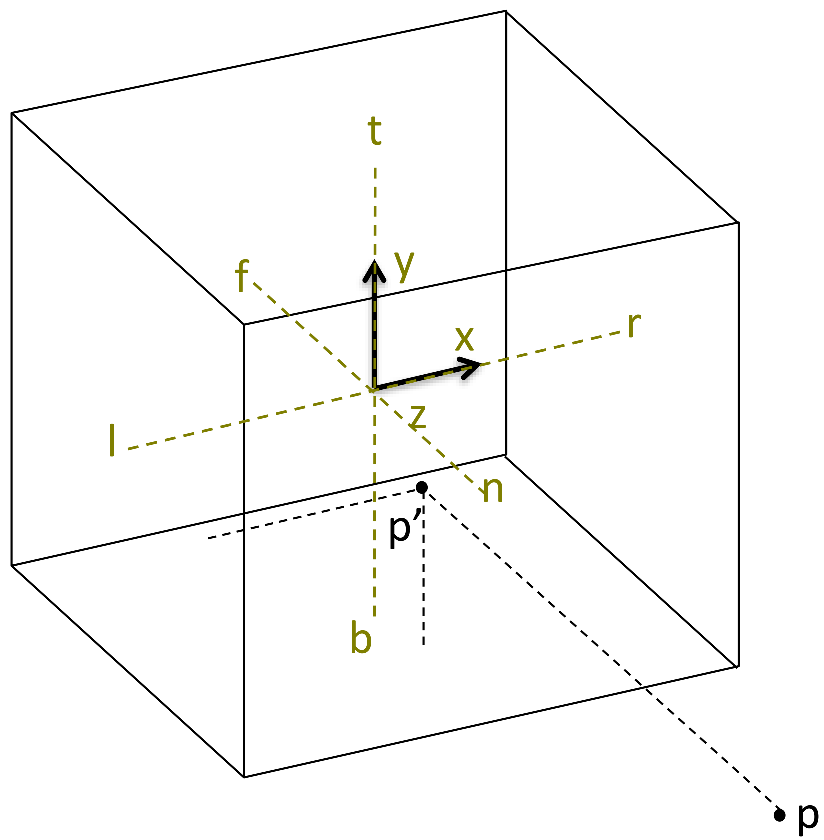
Orthographic projection



Is |p-q| = |p’-q’| ?

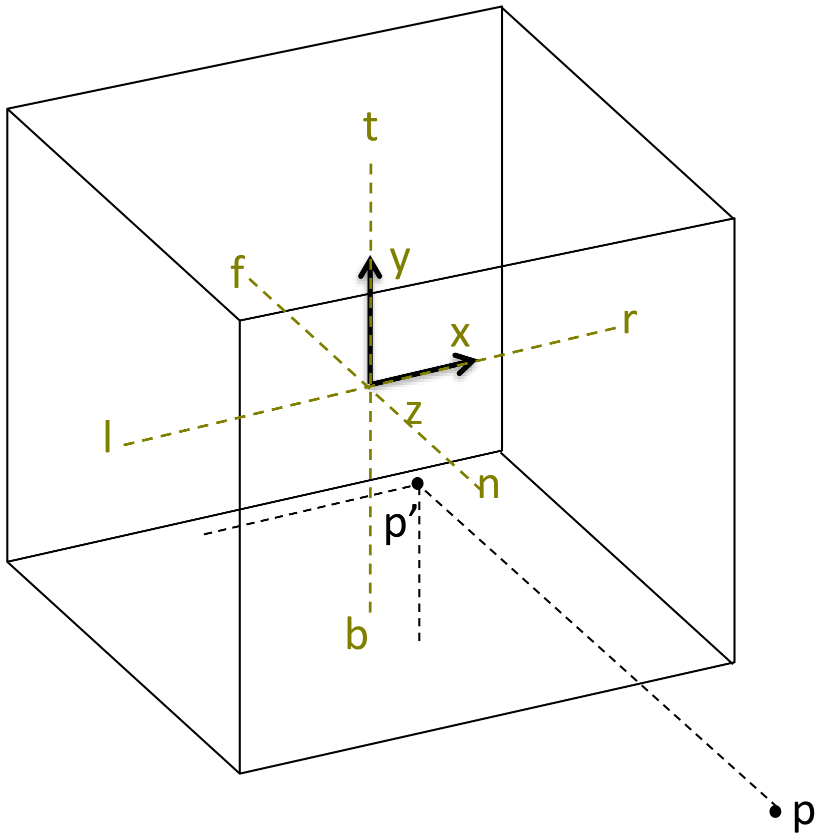
If m= (p+q)/2,
Is m’ = (p’+q’)/2?

Cannonical view volume



Map 3D to a cube centered at the origin of side length 2!

Cannonical view volume

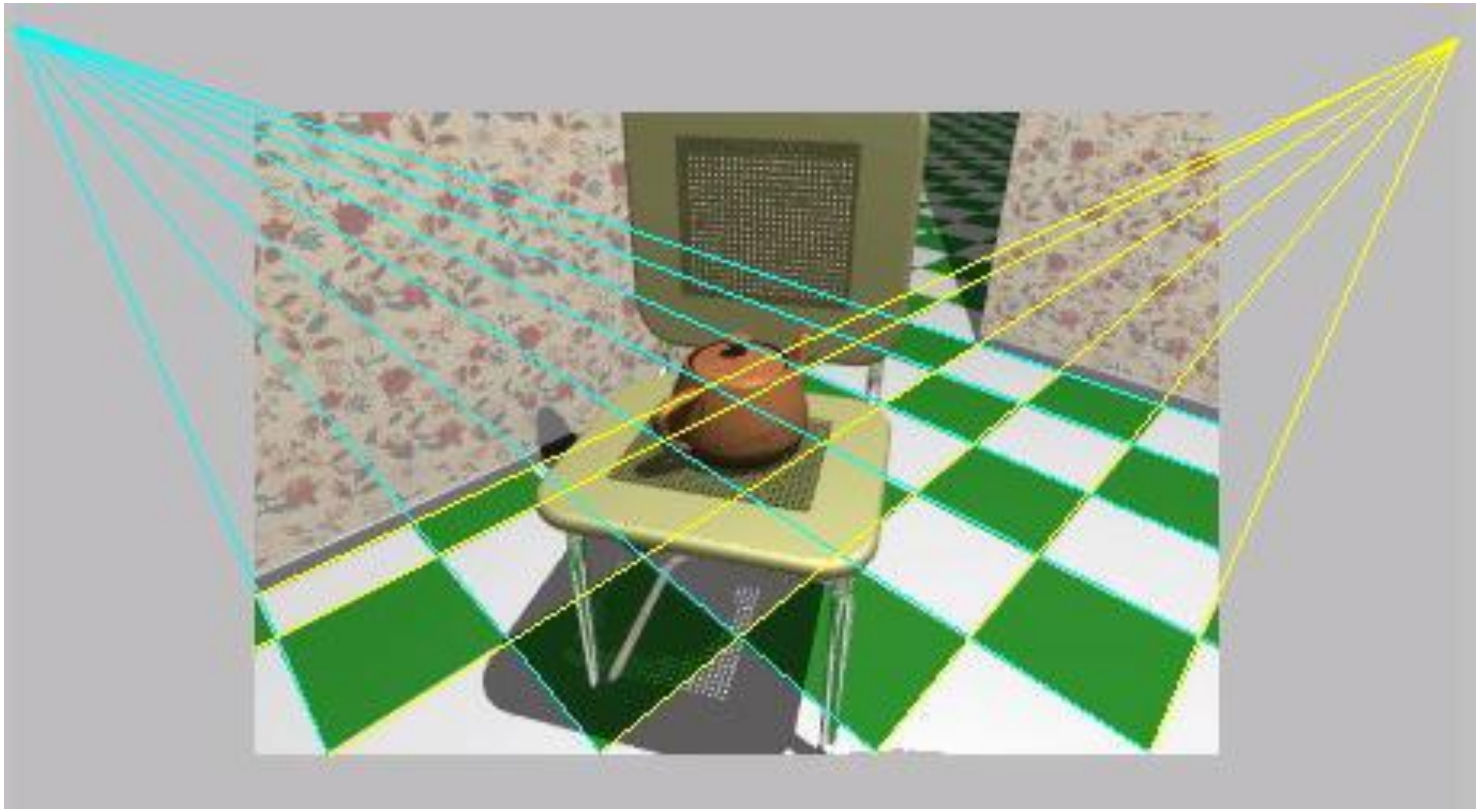


Map 3D to a cube centered at the origin of side length 2!

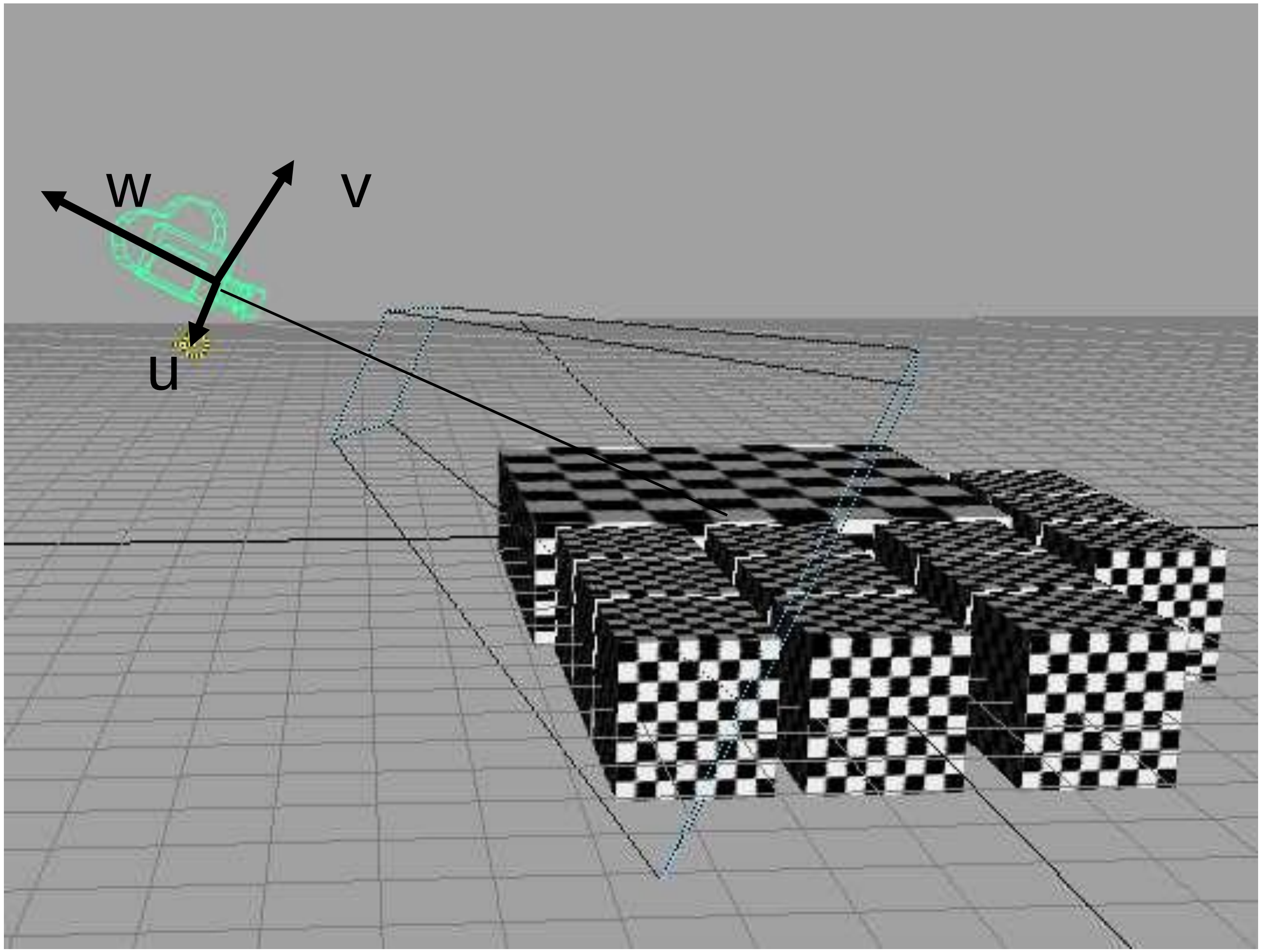
Translate(-(l+r)/2,-(t+b)/2,-(n+f)/2))
Scale(2/(r-l), 2/(t-b), 2/(f-n))

Camera model

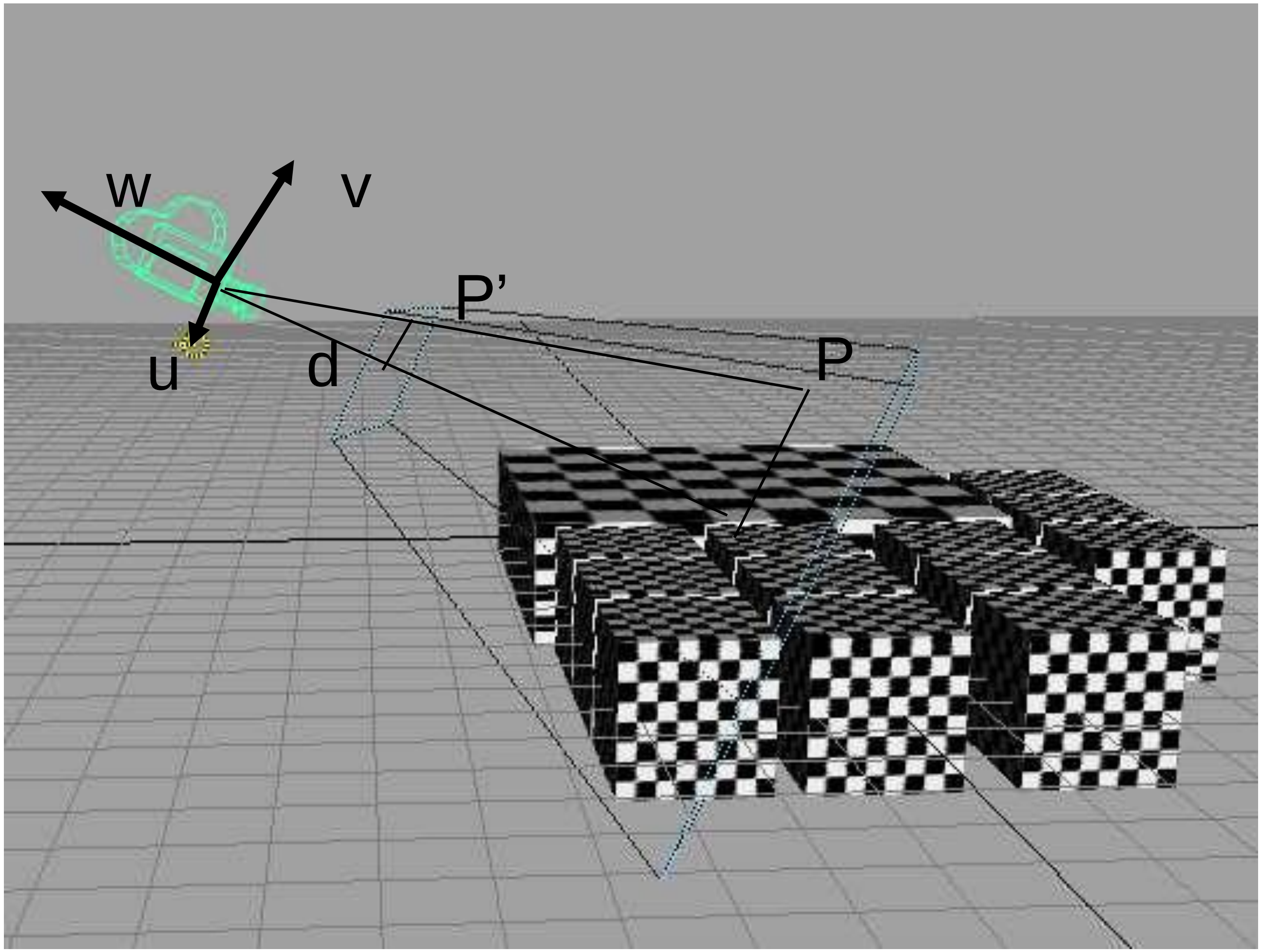
Perspective Projection



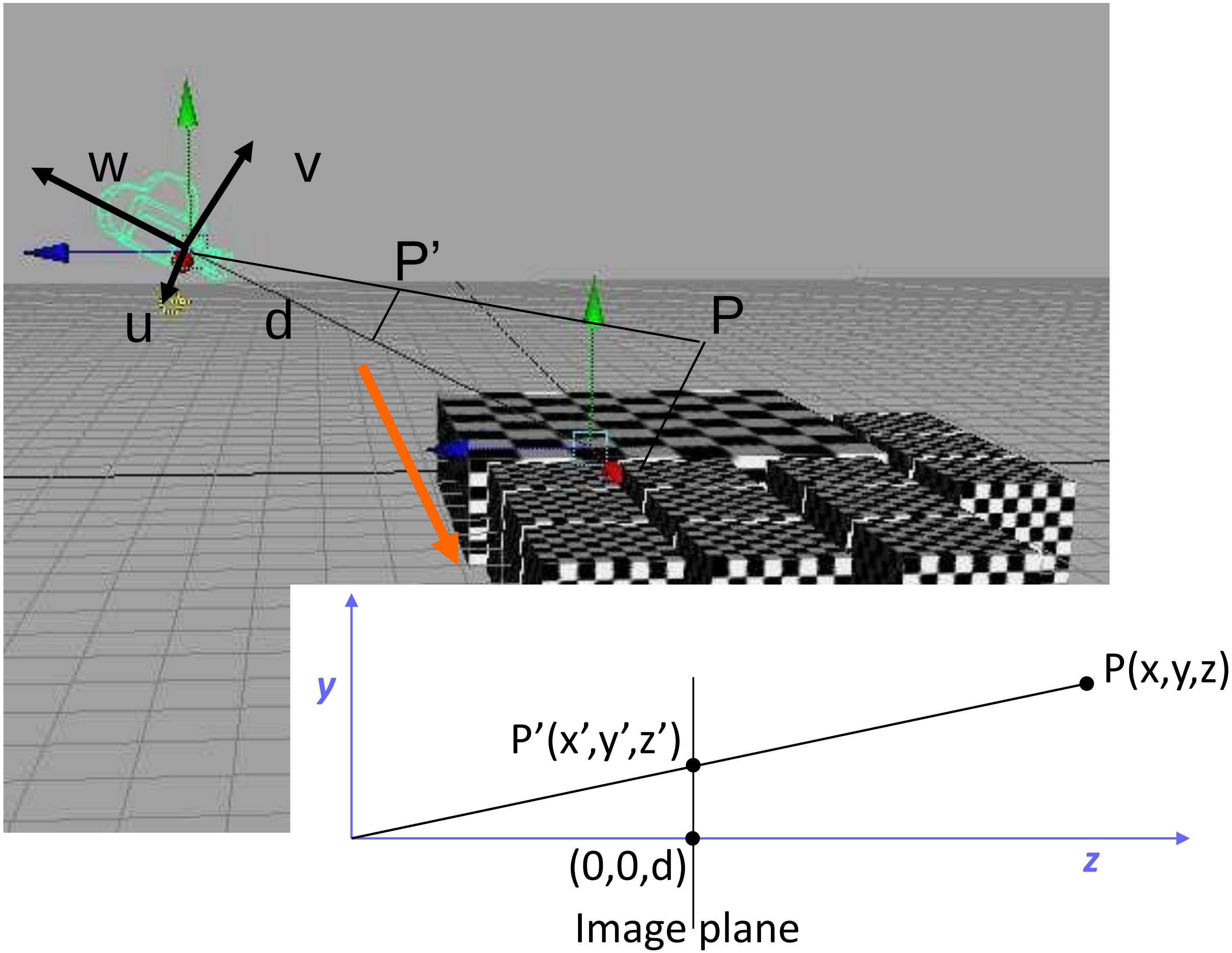
Perspective projection



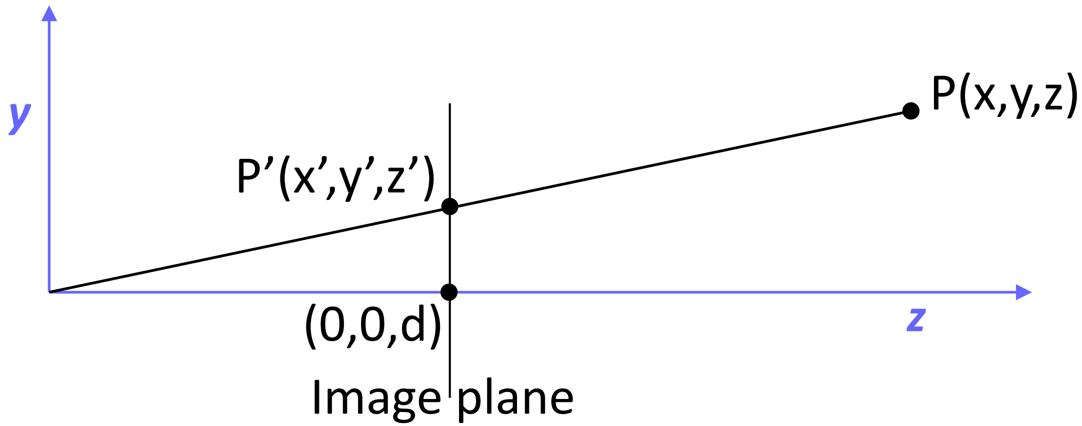
Perspective projection



Simple Perspective



Simple Perspective

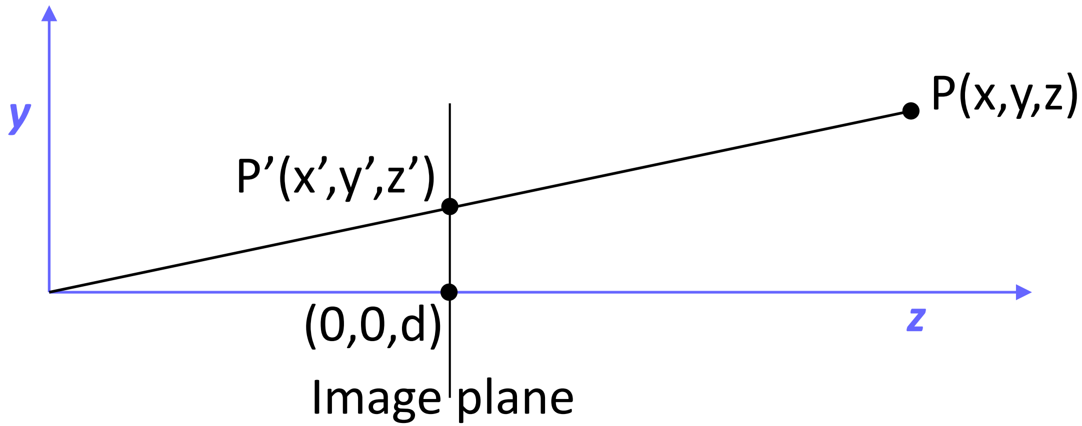


y'= yd/z

x'= xd/z

z'=d

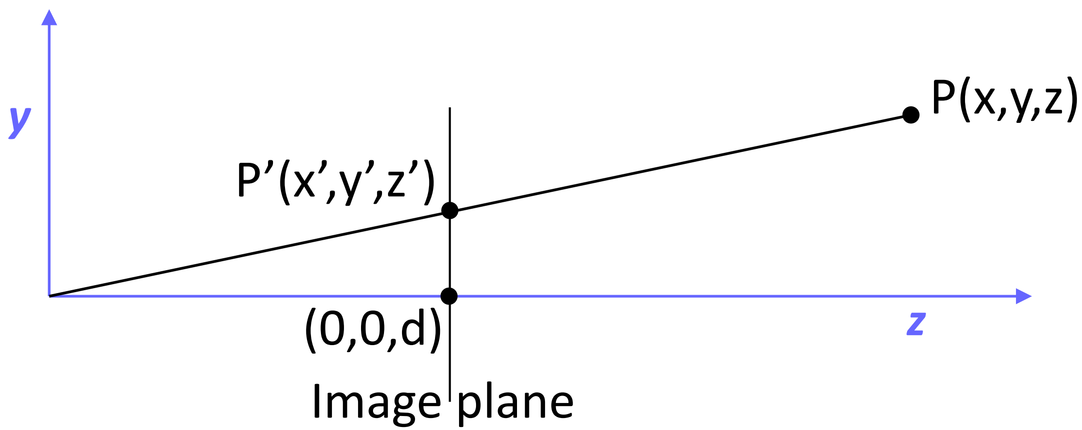
Simple Perspective



$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$w' = z/d$

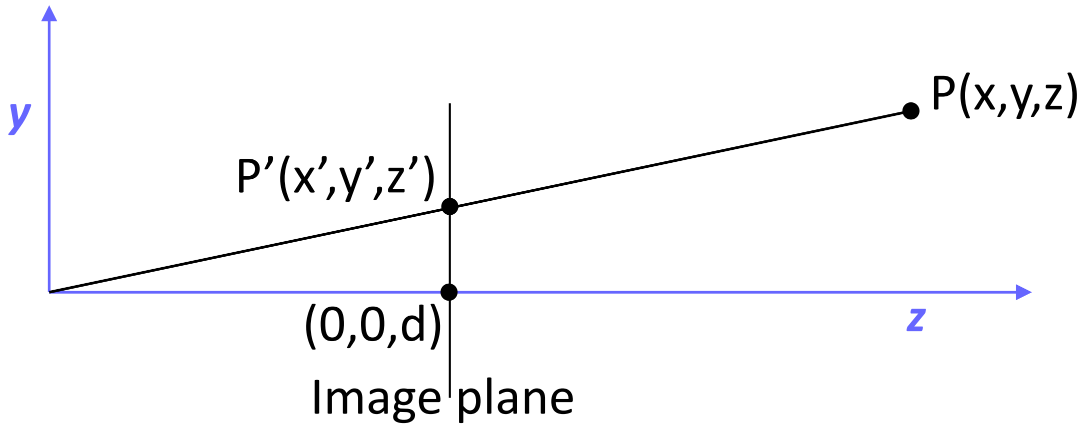
Simple Perspective



$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Find **a** and **b** such that z'=-1 when z=d and z'=1 when z=D, where d and D are near and far clip planes.

Simple Perspective

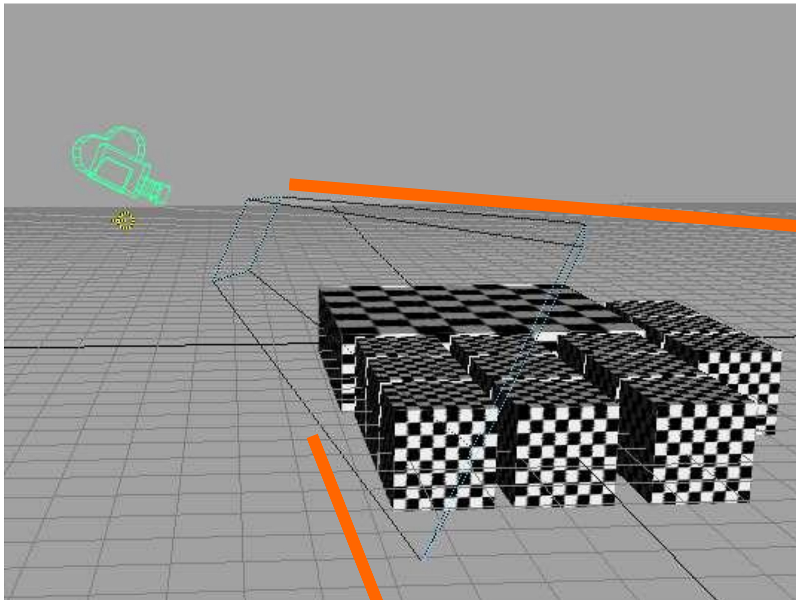


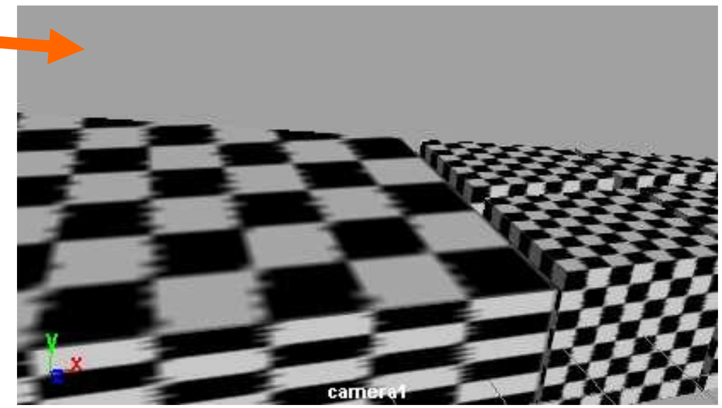
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathbf{a} & \mathbf{b} \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$z' = d(\mathbf{a}z+\mathbf{b})/z \Rightarrow -1 = \mathbf{a}d+\mathbf{b}$ and $1 = d(\mathbf{a}D+\mathbf{b})/D$

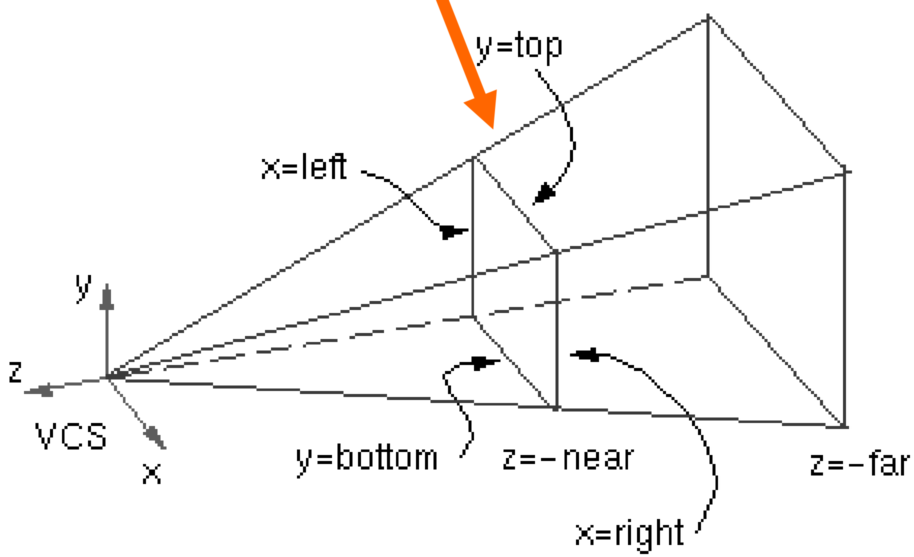
$\Rightarrow$ $\mathbf{b} = 2D/(d-D)$ and $\mathbf{a} = (D+d)/(d(D-d))$

Viewing volumes





Projected image



Viewing Pipeline