

# APS 105

## Winter 2012

Jonathan Deber

jdeber -at- cs -dot- toronto -dot- edu

Lecture 29  
March 28, 2012

# Today

- Linked Lists

# Data Structures

# Data Structures

- How we represent and store data in our programs
- Individual variables
- Arrays
- Arrays of pointers
- structs

# Arrays

```
int a[] = {1,2,3};
```

- Pro:

- Simple

a: 

1	2	3
---	---	---

- Easy (and fast) access to each element

- Con:

- Only stores one type of data

- Fixed size

- Needs to be contiguous

# Arrays of structs

- structs let us group multiple variables together
- Arrays let us group multiple structs together
- Fixes the “single type” problem

```
City cities[4];
```

```
cities[0]
```

```
cities[1]
```

```
cities[2]
```

```
cities[3]
```

```
name: "Halifax"  
metroPop: 0.283
```

```
name: "Montreal"  
metroPop: 3.764
```

```
name: "Toronto"  
metroPop: 6.324
```

```
name: "Vancouver"  
metroPop: 2.254
```

# Arrays of structs

- Still need to be contiguous
- Hard to delete/move elements

```
City cities[4];
```

```
cities[0]
```

```
name: "Halifax"  
metroPop: 0.283
```

```
cities[1]
```

```
cities[2]
```

```
name: "Toronto"  
metroPop: 6.324
```

```
cities[3]
```

```
name: "Vancouver"  
metroPop: 2.254
```

# Arrays of structs

- Still need to be contiguous
- Hard to delete/move elements

```
City cities[4];    cities[0]
```

```
    name: "Halifax"  
    metroPop: 0.283
```

```
cities[1] = NULL;  cities[1]
```

Error

```
cities[2]
```

```
    name: "Toronto"  
    metroPop: 6.324
```

```
cities[3]
```

```
    name: "Vancouver"  
    metroPop: 2.254
```



# Arrays of structs

- Still need to be contiguous
- Hard to delete/move elements

```
City cities[4];
```

```
cities[0]
```

```
cities[1] = NULL;
```

```
cities[1]
```

Error

```
cities[2]
```

```
cities[3]
```

```
name: "Halifax"  
metroPop: 0.283
```

```
name: "Toronto"  
metroPop: 6.324
```

```
name: "Vancouver"  
metroPop: 2.254
```

# Arrays of structs

- Still need to be contiguous
- Hard to delete/move elements

```
City cities[4];
```

```
cities[0]
```

```
cities[1] = NULL;
```

```
cities[1]
```

Error

```
cities[2]
```

```
cities[3]
```

name: "Halifax" metroPop: 0.283
name: "Toronto" metroPop: 6.324
name: "Vancouver" metroPop: 2.254

# Arrays of structs

- Still need to be contiguous
- Hard to delete/move elements

```
City cities[4];
```

```
cities[0]
```

```
cities[1] = NULL;
```

```
cities[1]
```

Error

```
cities[2]
```

```
cities[3]
```

```
name: "Halifax"  
metroPop: 0.283
```

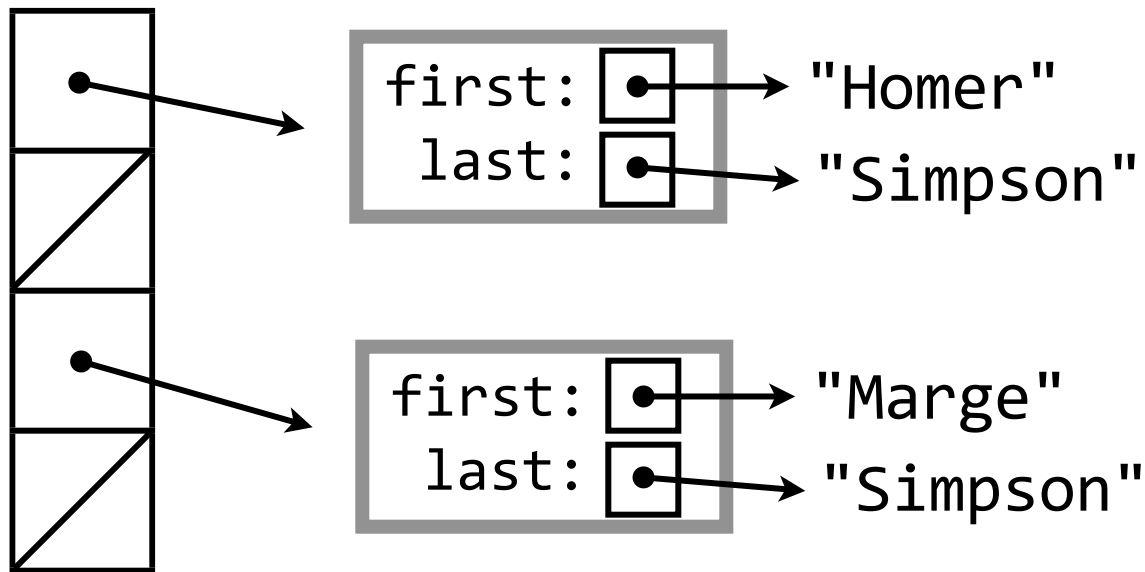
```
name: "Toronto"  
metroPop: 6.324
```

```
name: "Vancouver"  
metroPop: 2.254
```

```
name:  
metroPop:
```

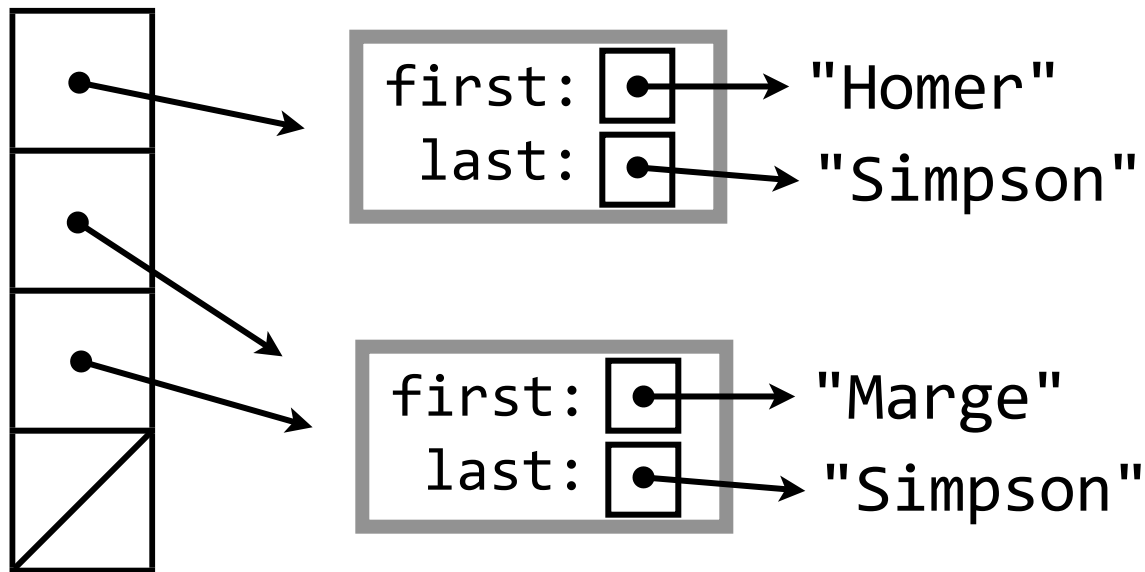
# Arrays of pointers to structs

- Array elements are much smaller
- Array elements can be NULL



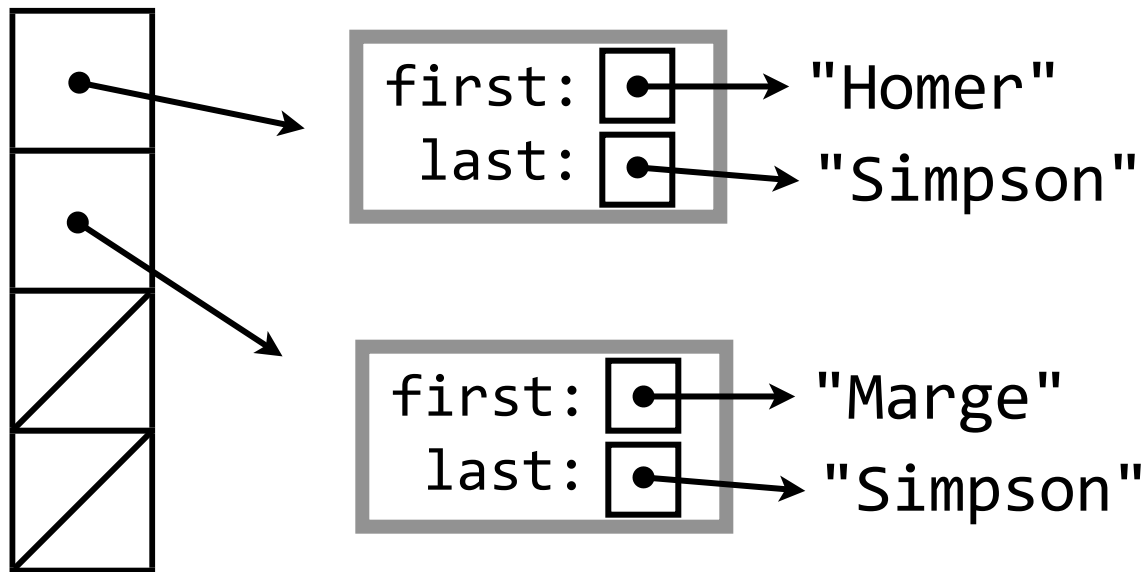
# Arrays of pointers to structs

- Array elements are much smaller
- Array elements can be NULL

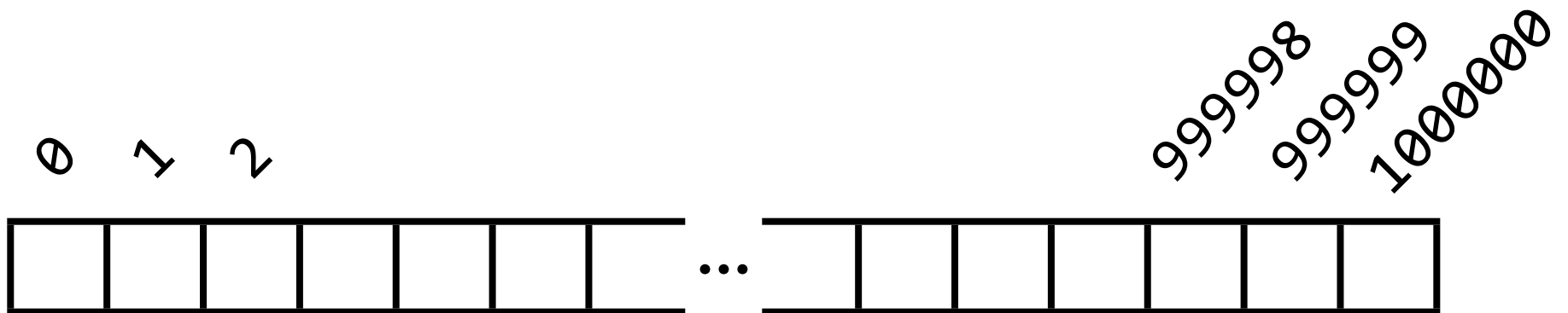


# Arrays of pointers to structs

- Array elements are much smaller
- Array elements can be NULL



# Shifting Can be Slow

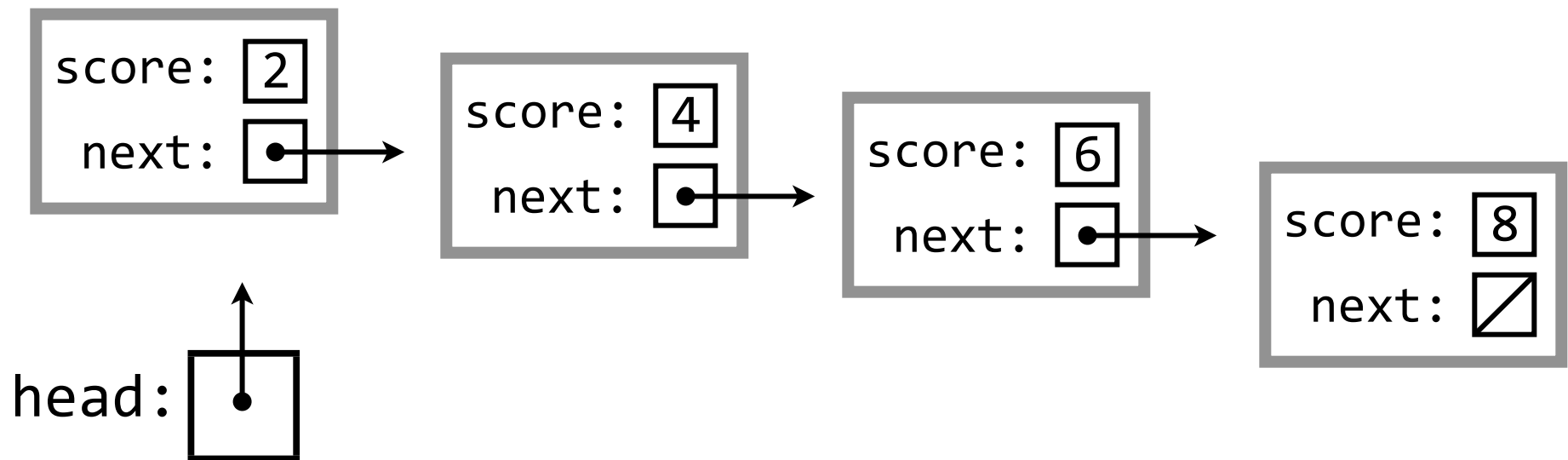


# Linked Lists

- A different way to store multiple chunks of data
- (Simplest) example of a whole category of data structures, built on indirection (i.e., using pointers)
- We have to build them ourselves

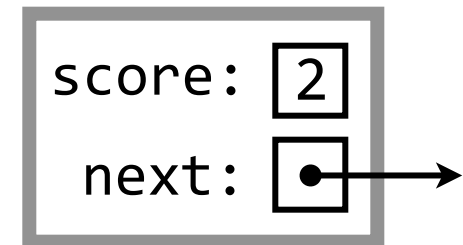


2	4	6	8
---	---	---	---

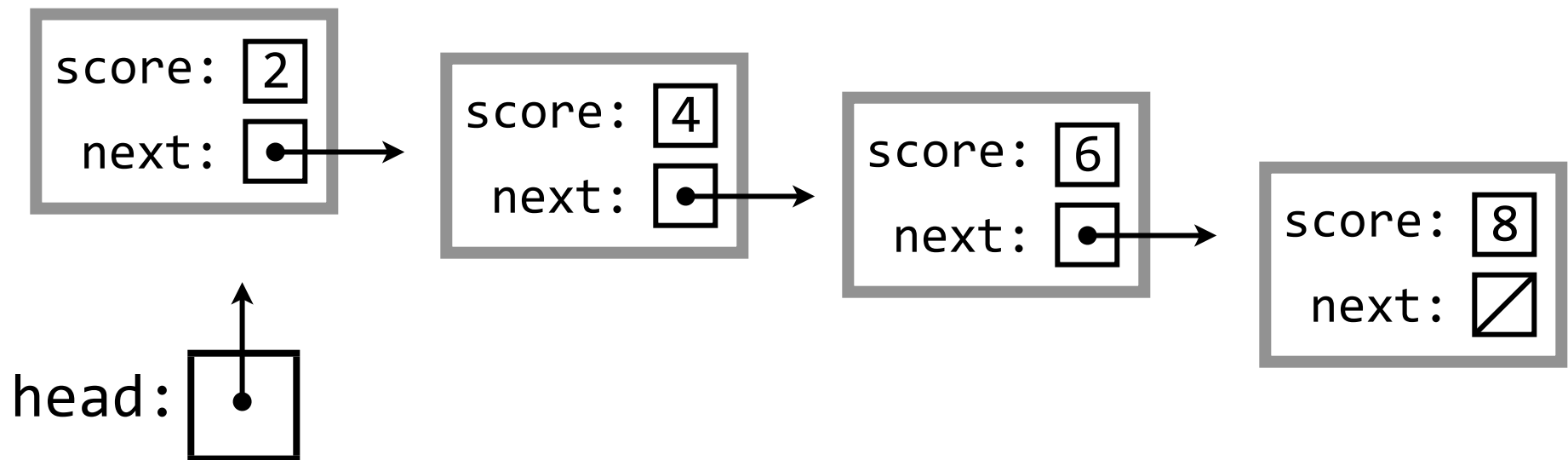


# Linked Lists

- A recursive data structure
- Made up of nodes
  - Actual data
  - A pointer to another node
- Each node points to the next node
- Last node points to NULL



2	4	6	8
---	---	---	---



# A Pointer to Another Node?

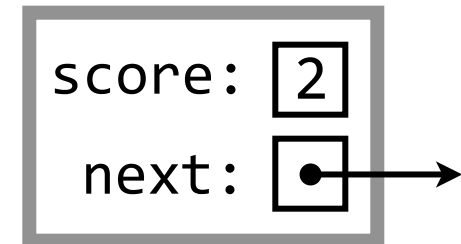
```
typedef struct node
{
    int score;
    Node next;
} Node;
```

Error

# A Pointer to Another Node?

```
typedef struct node
{
    int score;
    Node *next;
} Node;
```

Error



error: syntax error before 'Node'

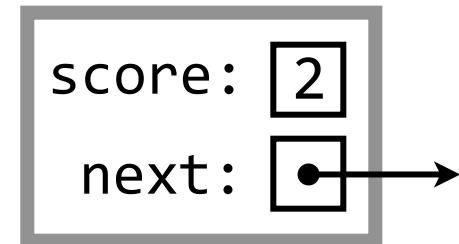
warning: no semicolon at end of struct or union

warning: type defaults to 'int' in declaration of 'Node'

warning: data definition has no type or storage class

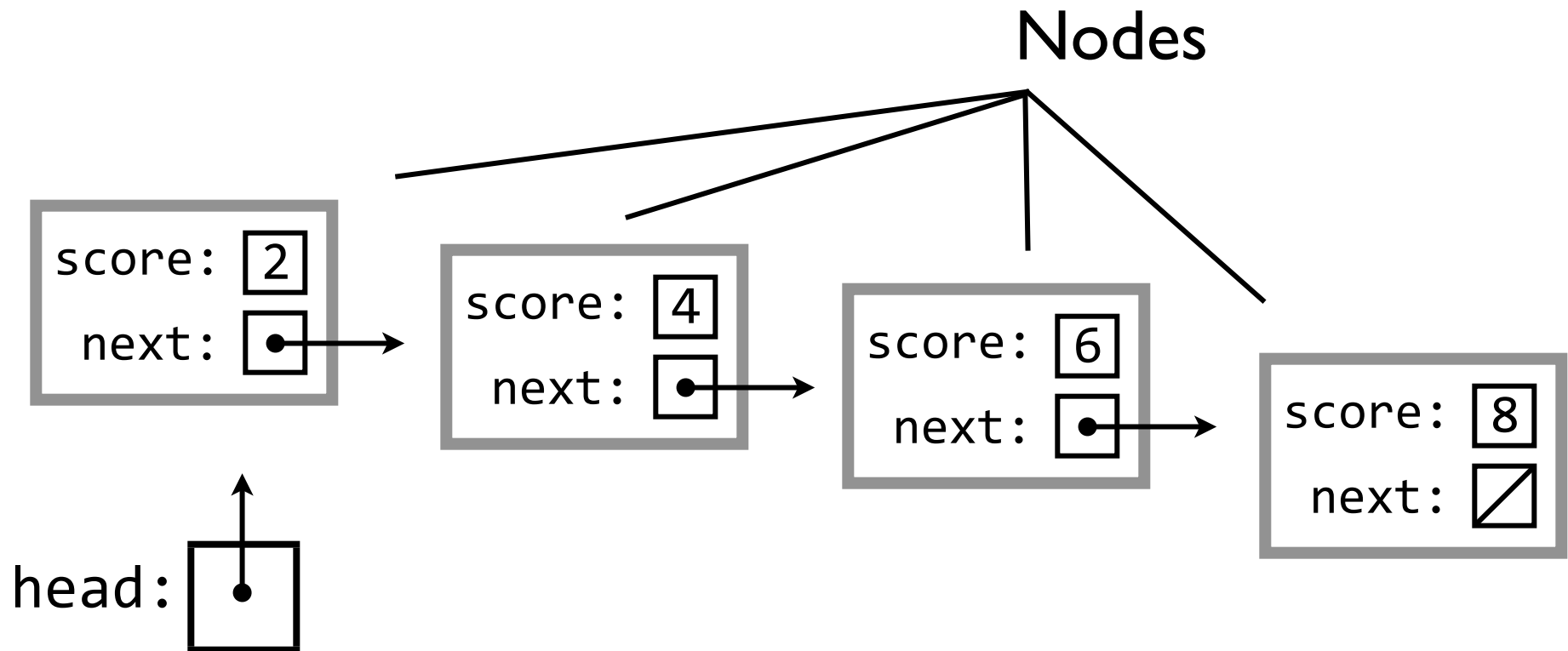
# A Pointer to Another Node?

```
typedef struct node
{
    int score;
    struct node *next;
} Node;
```



# Terminology

Whole thing is a linked list



First node is the head

Last node is the tail

(what you normally have a pointer to)

# Arrays vs. Linked Lists

- Some things get easier
- Some things get harder
- Some things are about the same

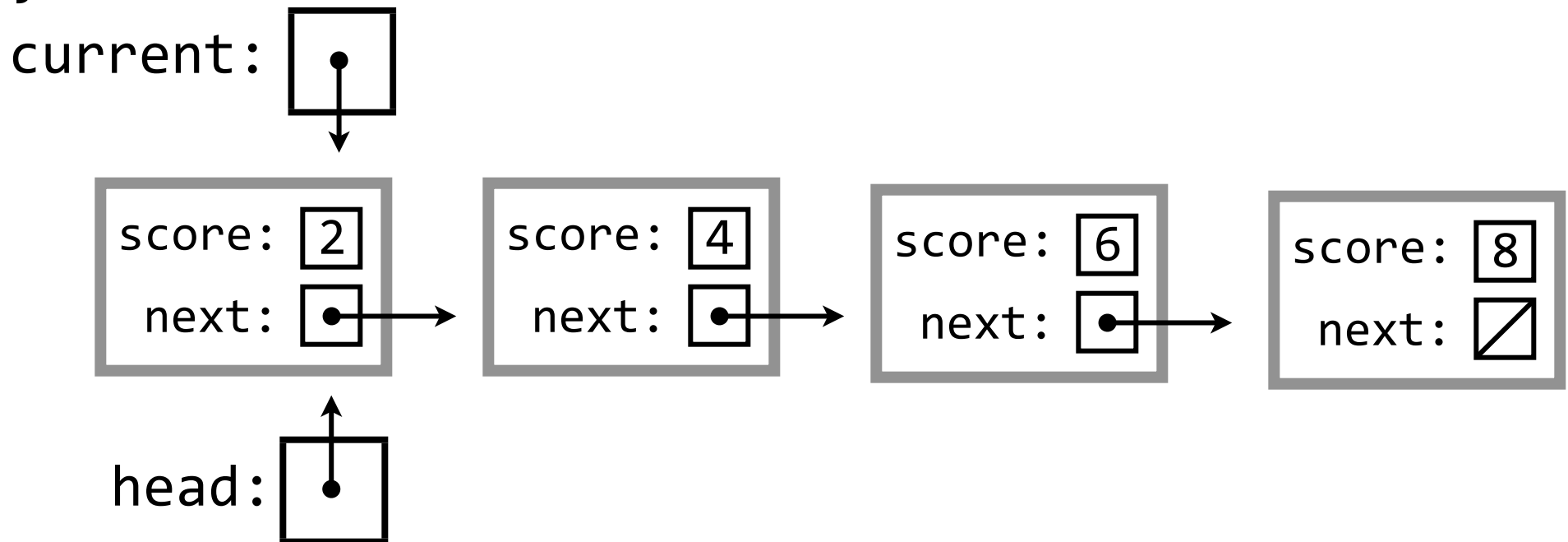


# Traversing Arrays

```
void printArray(int a[], int n)
{
    for (int i = 0; i < n; i++)
    {
        printf("%d\n", a[i]);
    }
}
```

# Traversing Linked Lists

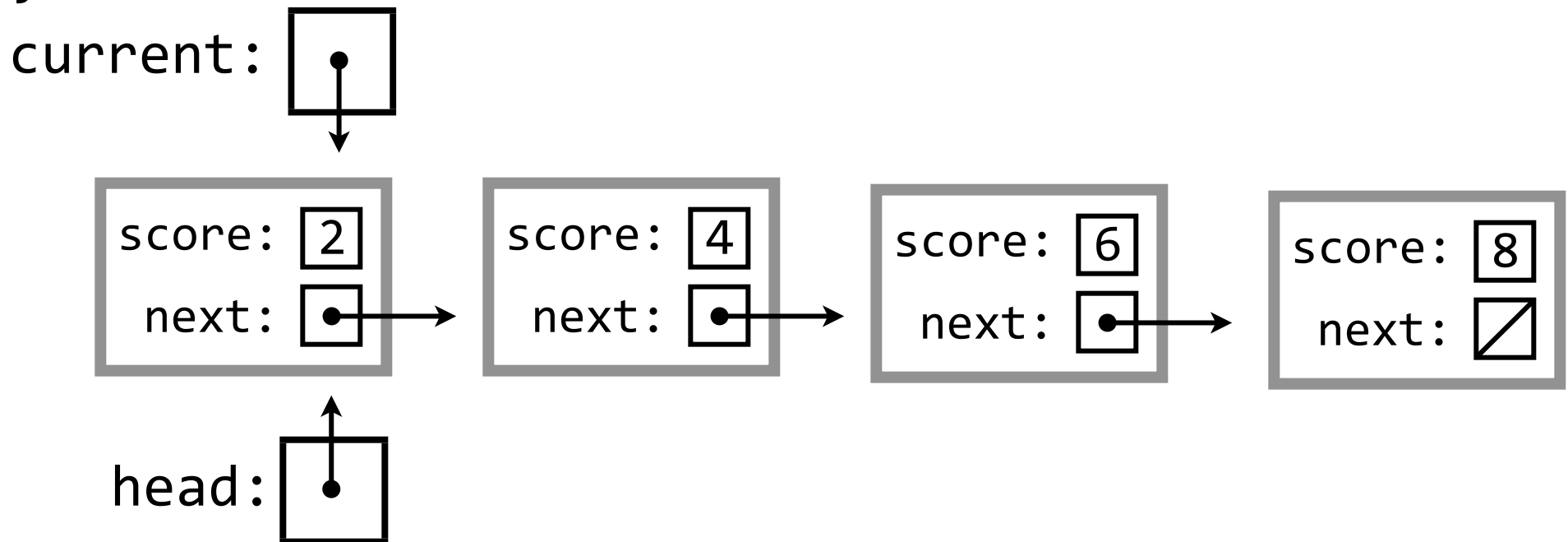
```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```



# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

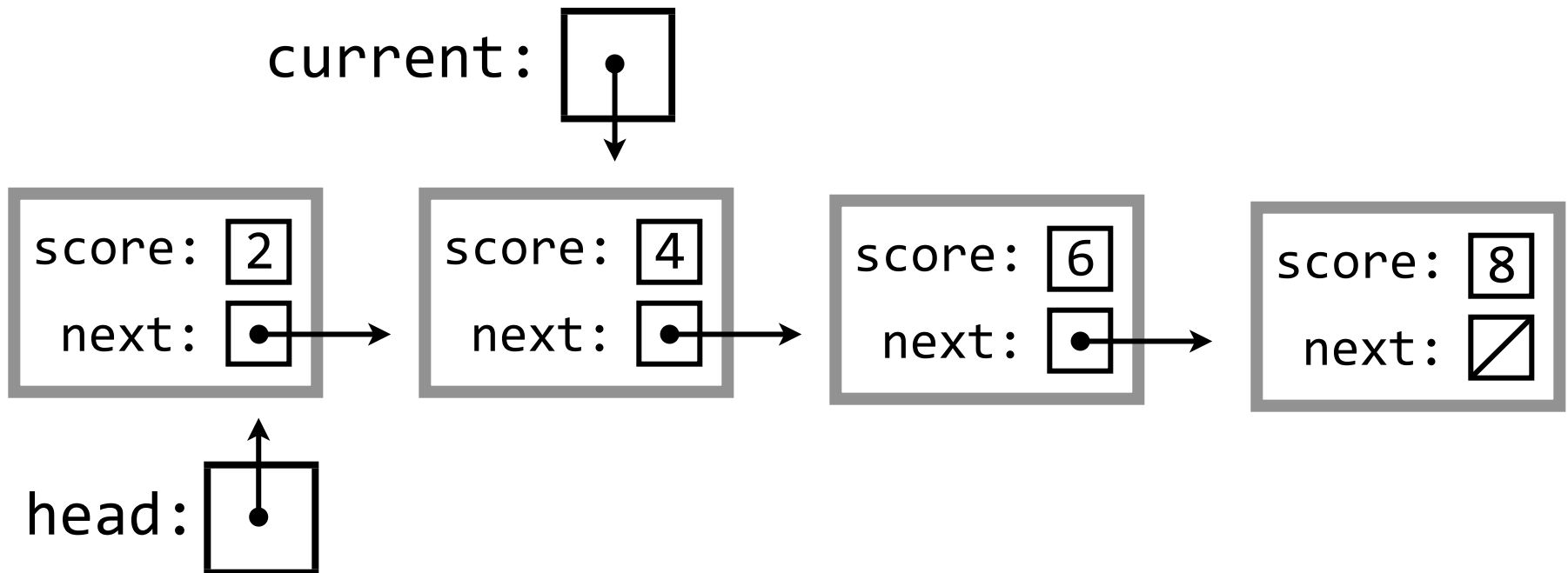
2



# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

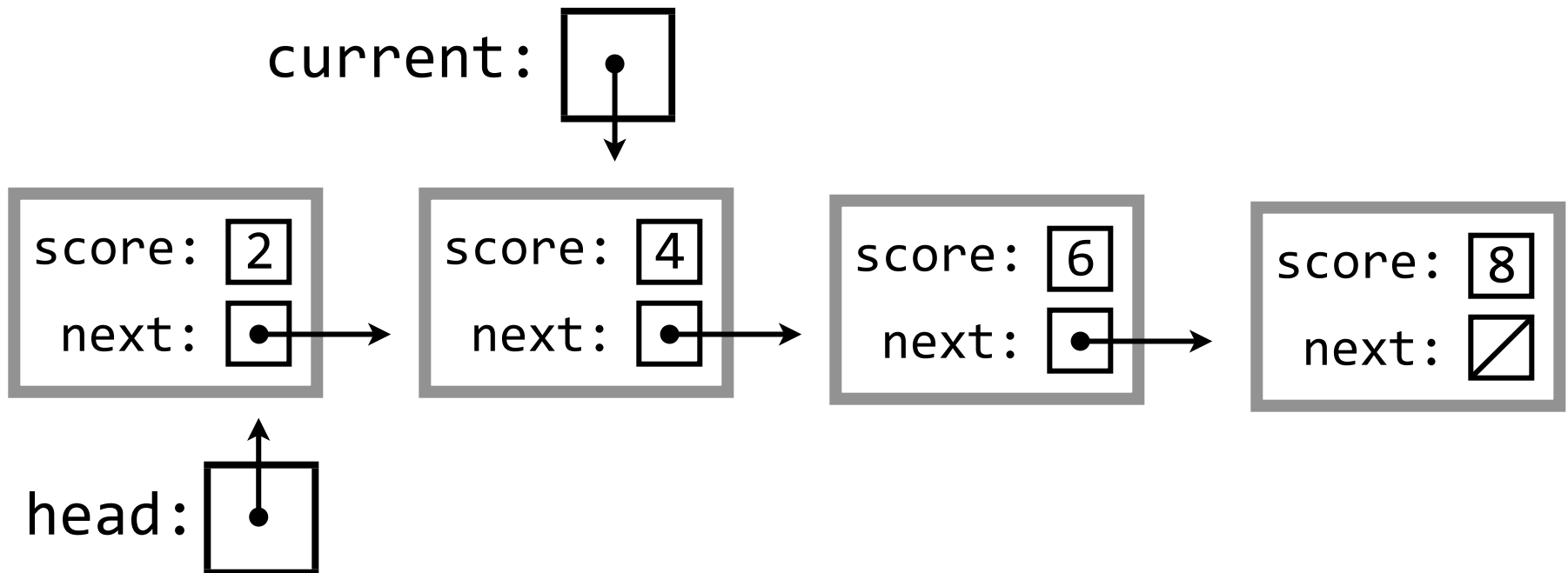
2



# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

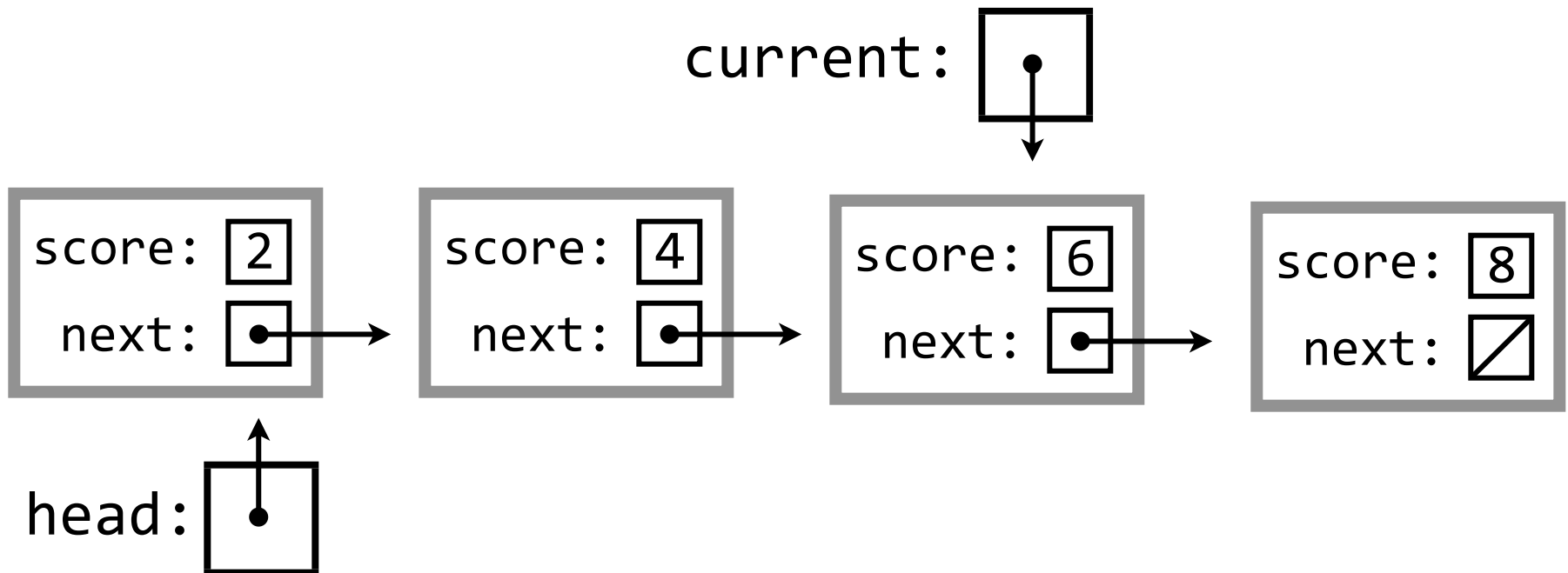
2  
4



# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

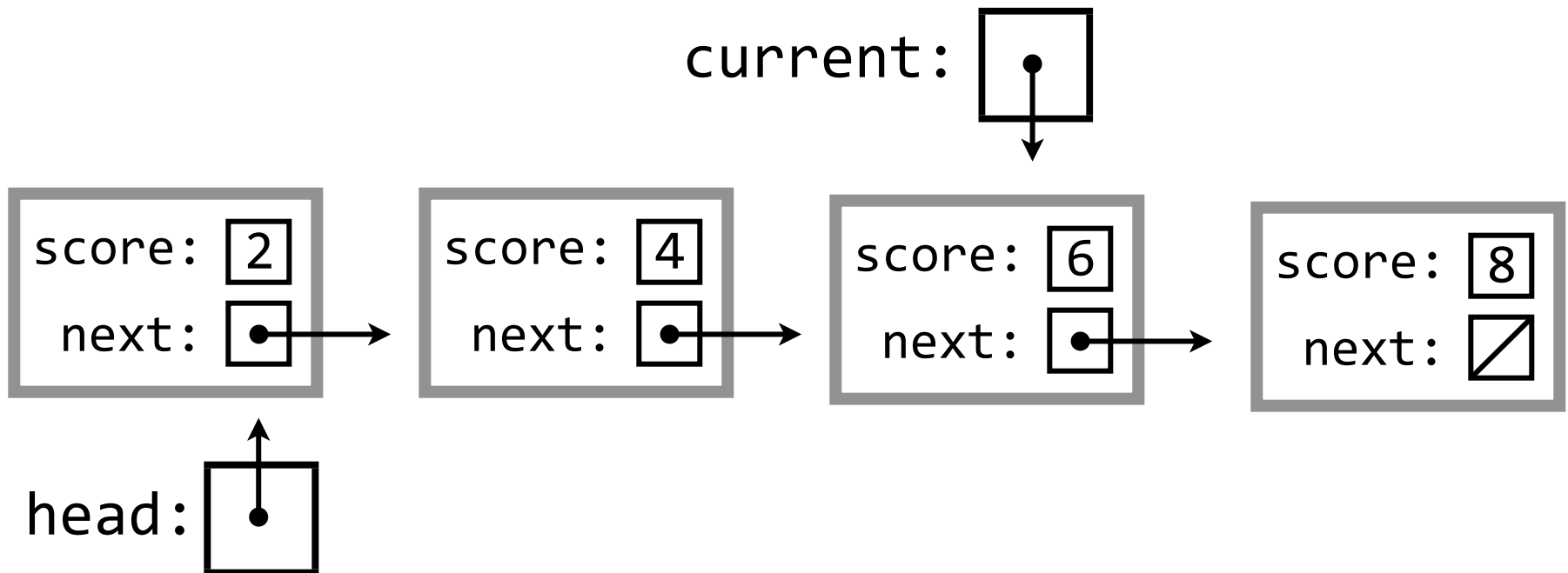
2  
4



# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

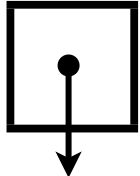
2  
4  
6

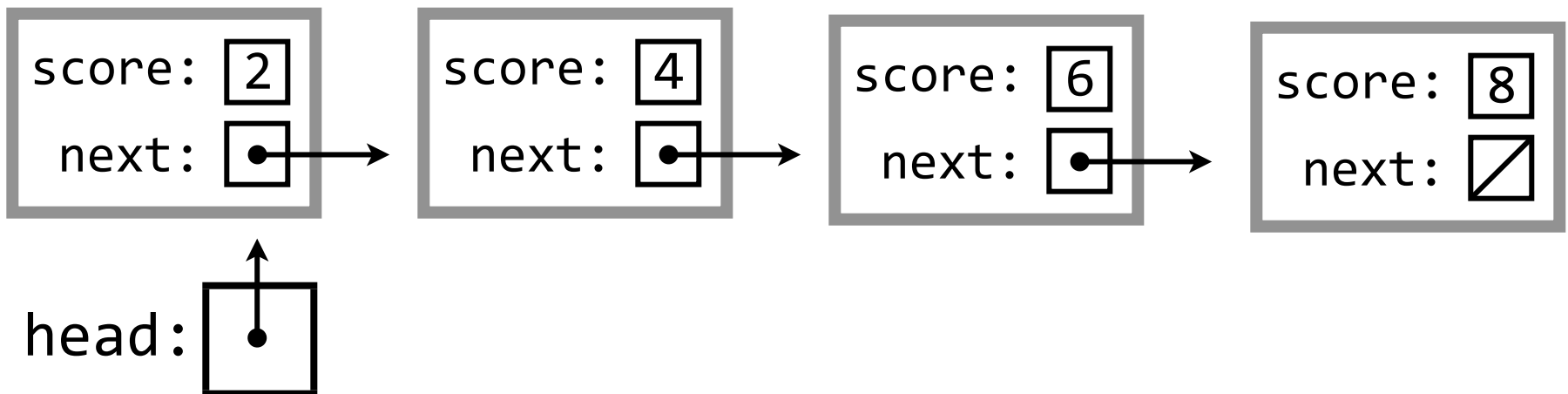


# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

2  
4  
6

current: 



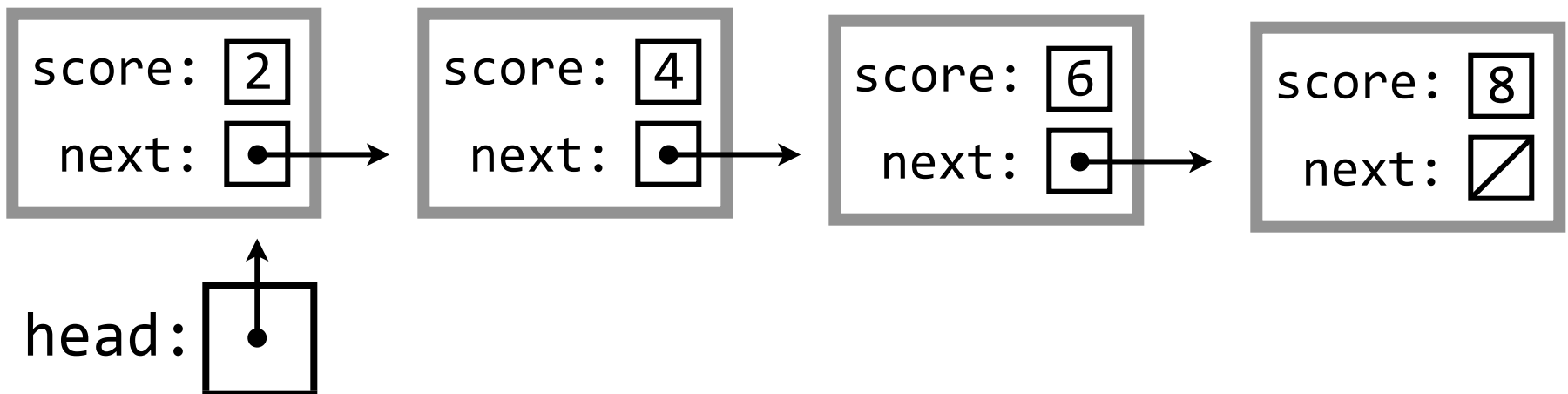


# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

2  
4  
6  
8

current: 

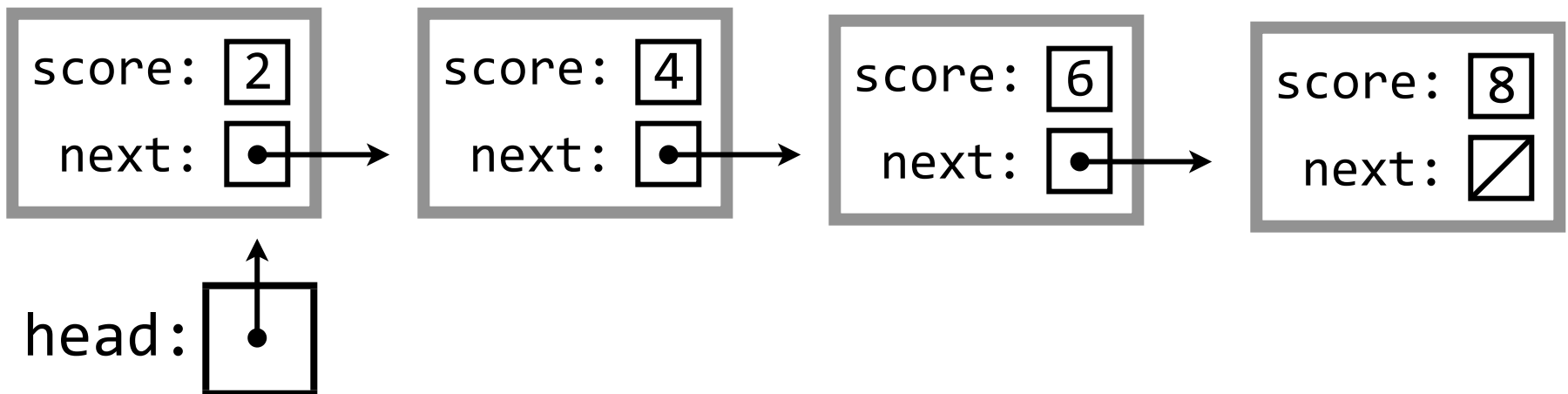


# Traversing Linked Lists

```
void printList(Node *head)
{
    Node *current = head;
    while (current != NULL)
    {
        printf("%d\n", current->score);
        current = current->next;
    }
}
```

2  
4  
6  
8

current: 



# Verdict?

- Basically the same

# Length of Arrays

- No general technique
  - `sizeof()` under some circumstances
  - “Sentinel value” (e.g.,  $\backslash 0$ ) under some circumstances
  - Keep another variable around

# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
{
```

```
    int length = 0;
```

length: 0

```
    Node *current = head;
```

```
    while (current != NULL)
    {
```

```
        length++;
```

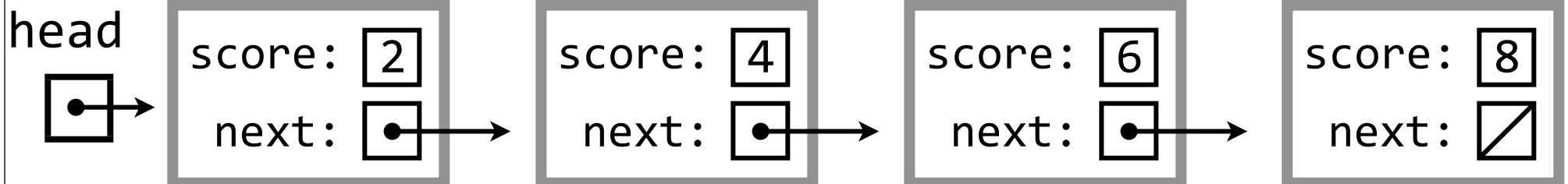
```
        current = current->next;
```

```
    }
```

```
    return length;
```

```
}
```

current: ●



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

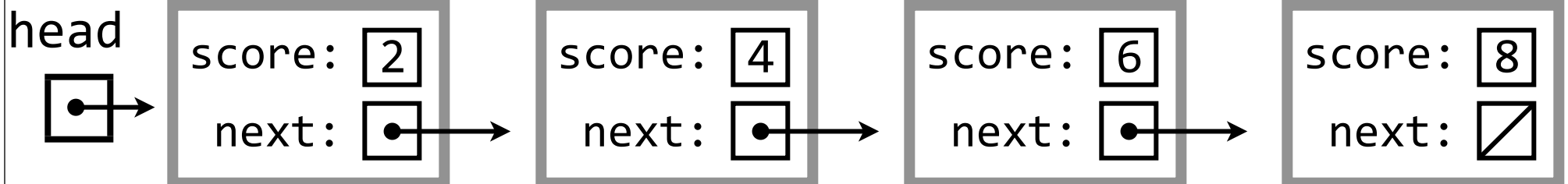
```
    }
```

```
    return length;
```

```
}
```

length: 1

current: ●



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

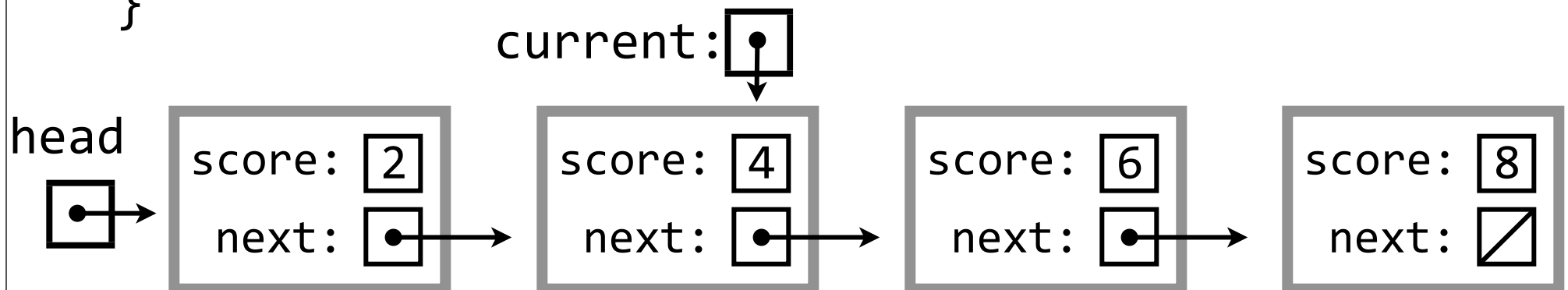
```
    }
```

```
    return length;
```

```
}
```

length:

1



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

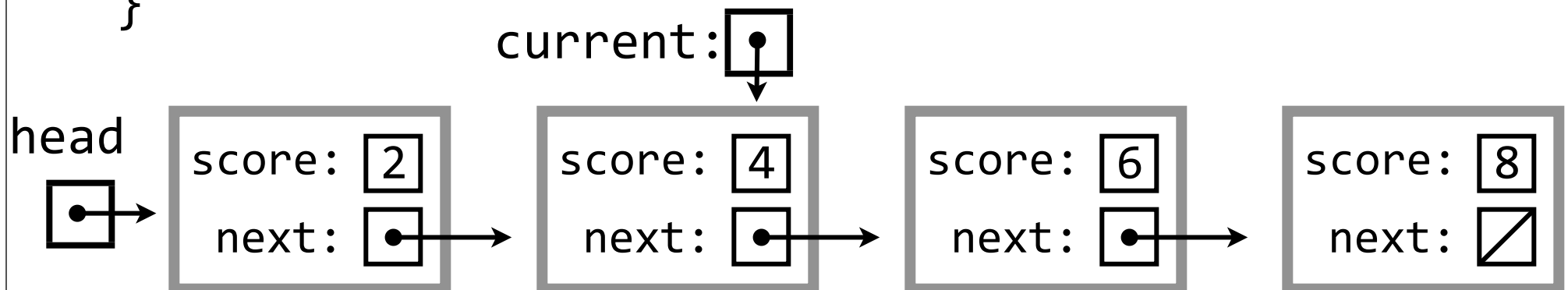
```
    }
```

```
    return length;
```

```
}
```

length:

2





# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

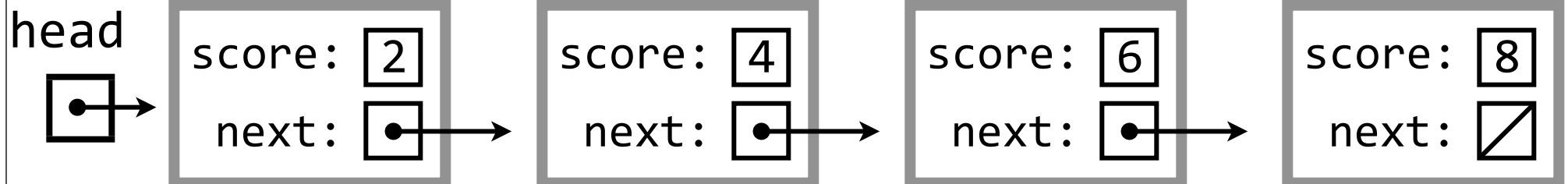
```
    }
```

```
    return length;
```

```
}
```

length:

2



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

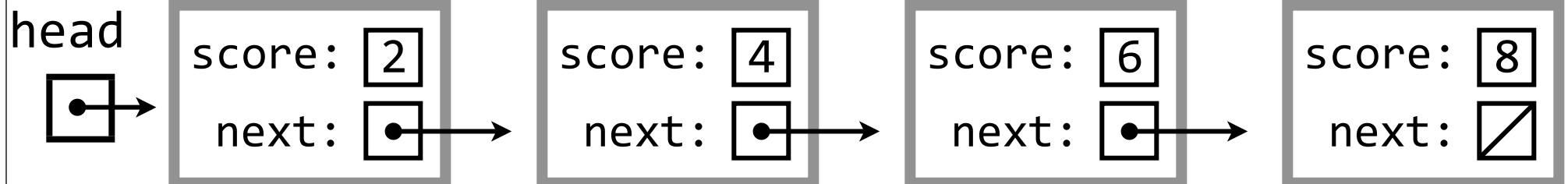
```
    }
```

```
    return length;
```

```
}
```

length:

3



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

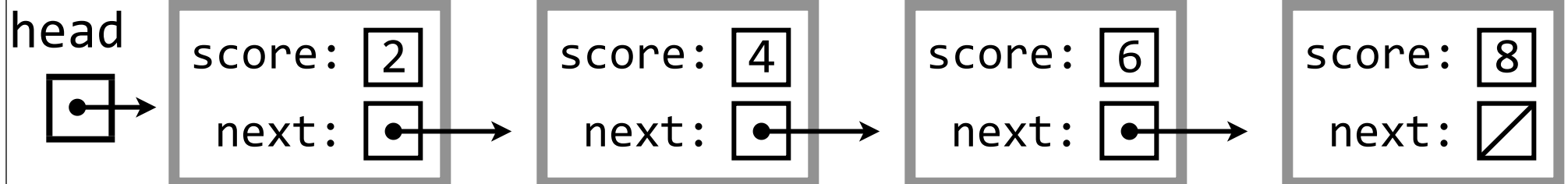
```
    }
```

```
    return length;
```

```
}
```

length:

3



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

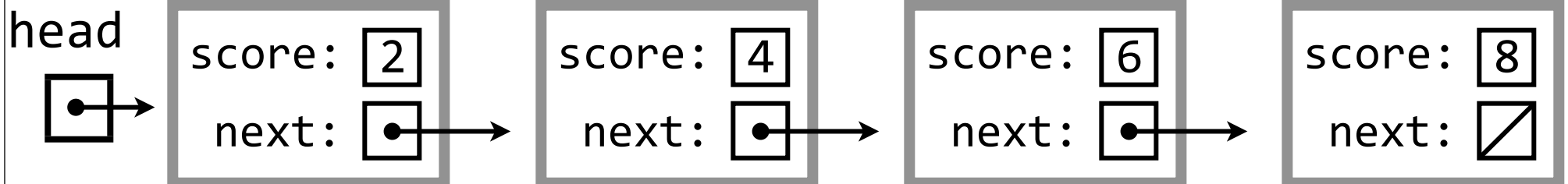
```
    }
```

```
    return length;
```

```
}
```

length:

4



# Length of Linked Lists

- Need to traverse it, but that will always work

```
int length(Node *head)
```

```
{
```

```
    int length = 0;
```

```
    Node *current = head;
```

```
    while (current != NULL)
```

```
    {
```

```
        length++;
```

```
        current = current->next;
```

```
    }
```

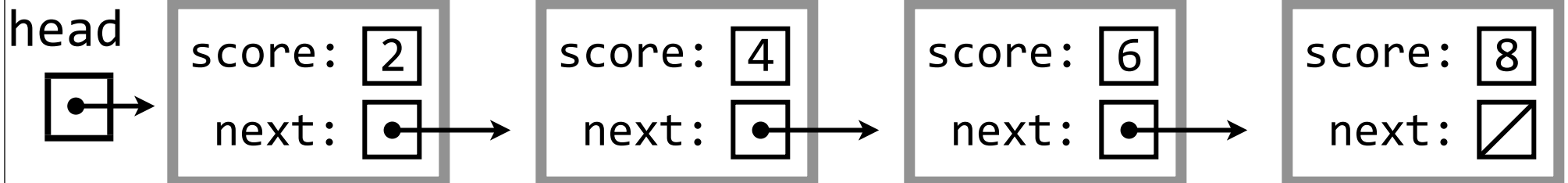
```
    return length;
```

```
}
```

length:

4

current: 



# Verdict?

- You can always figure out the length of a linked list, but it might take you a while
- Different

# $i^{\text{th}}$ Element of Array

- Really easy
- Really fast (basic math to find address)
  - Same speed for any  $i$
- Potential for buffer overrun if not careful

City toronto = cities[2];

City vancouver = cities[29348];

Wrong?

# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 1

```
Node *getElement(Node *head, int index)
```

```
{
```

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

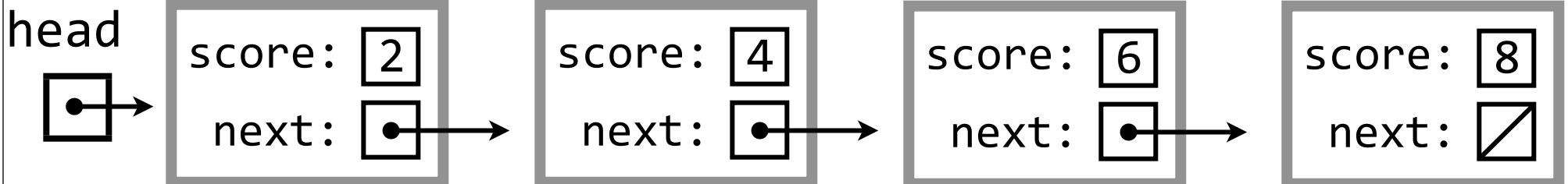
```
        current = current->next;
```

```
    }
```

```
    return current;
```

```
}
```

current: ●  
↓





# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 1

```
Node *getElement(Node *head, int index)
```

```
{
```

i: 1

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

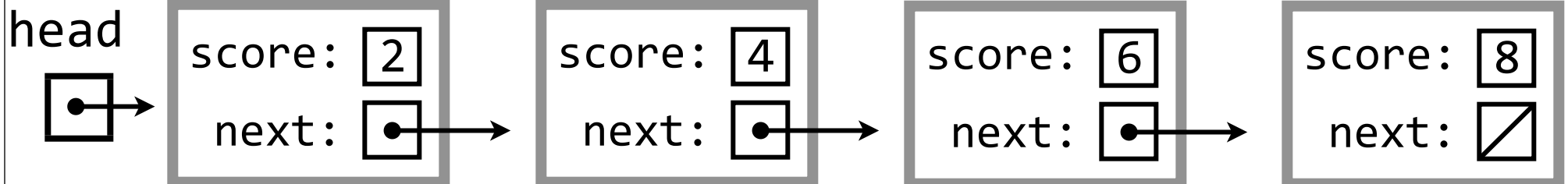
```
        current = current->next;
```

```
    }
```

```
    return current;
```

```
}
```

current: ●  
↓



# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 10

```
Node *getElement(Node *head, int index)
```

i: 1

```
{
```

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

```
        current = current->next;
```

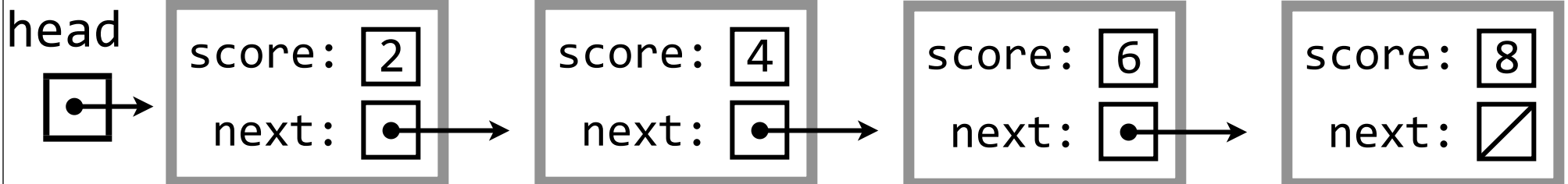
```
    }
```

```
    return current;
```

```
}
```

What if index is larger than the number of elements in the list?

current: •



# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 10

```
Node *getElement(Node *head, int index)
```

i: 2

```
{
```

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

```
        current = current->next;
```

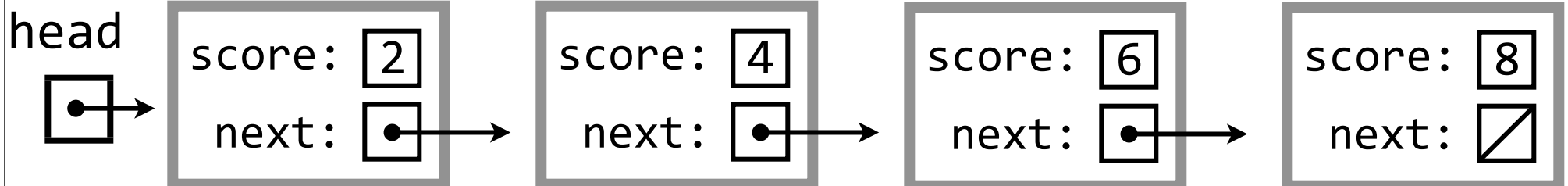
```
    }
```

```
    return current;
```

```
}
```

What if index is larger than the number of elements in the list?

current: •



# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 10

```
Node *getElement(Node *head, int index)
```

i: 3

```
{
```

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

```
        current = current->next;
```

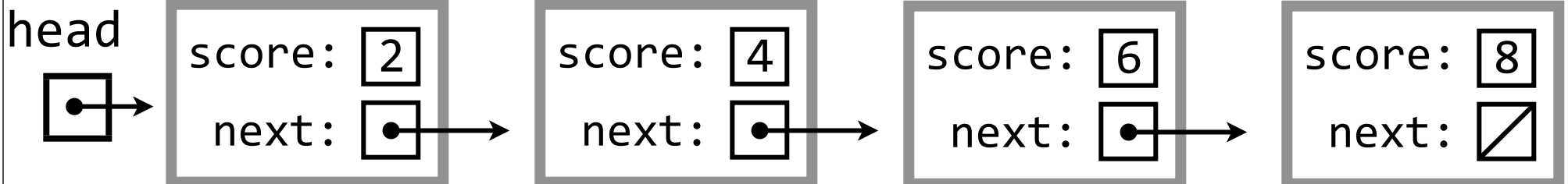
```
    }
```

```
    return current;
```

```
}
```

What if index is larger than the number of elements in the list?

current: •  
↓



# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 10

```
Node *getElement(Node *head, int index)
```

i: 4

```
{
```

```
    Node *current = head;
```

```
    for (int i = 0; i < index; i++)
```

```
    {
```

```
        current = current->next;
```

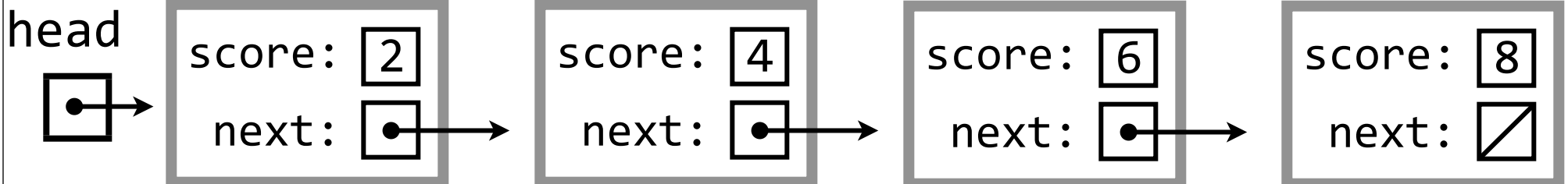
```
    }
```

```
    return current;
```

```
}
```

What if index is larger than the number of elements in the list?

current: /



# $i^{\text{th}}$ Element of Linked List

- Need to traverse until we find it

index: 10

```
Node *getElement(Node *head, int index)
```

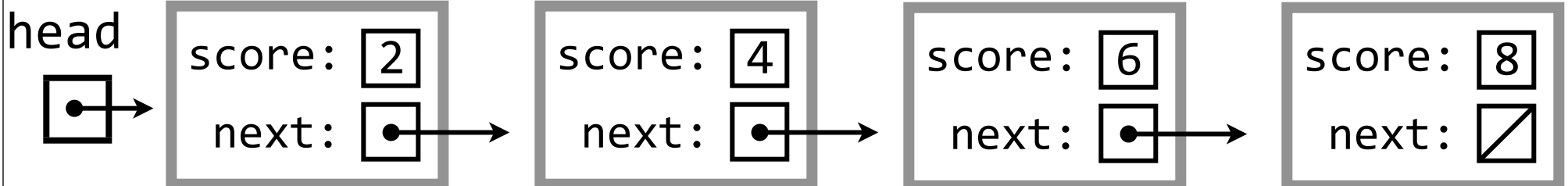
i: 4

```
{  
    Node *current = head;  
    for (int i = 0; i < index; i++)  
    {  
        current = current->next;  
    }  
    return current;  
}
```

Wrong

What if index is larger than the number of elements in the list?

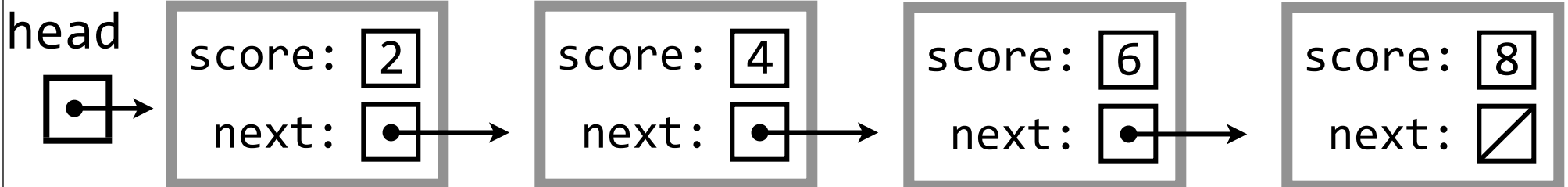
current: /



# $i^{\text{th}}$ Element of Linked List

```
Node *getElement(Node *head, int index)
{
    Node *current = head;
    for (int i = 0; i < index && current != NULL; i++)
    {
        current = current->next;
    }
    return current;
}
```

current: 



# Verdict?

- Arrays are much easier
- Arrays are much faster, and have constant run time



# Insert at End of Array

```
// If the array has space  
list[numElements] = newItem;  
numElements++;
```

```
// If the array is full but dynamically allocated  
list = realloc(list, (numElements+1) * sizeof(list[0]));  
list[numElements] = newItem;  
numElements++;
```

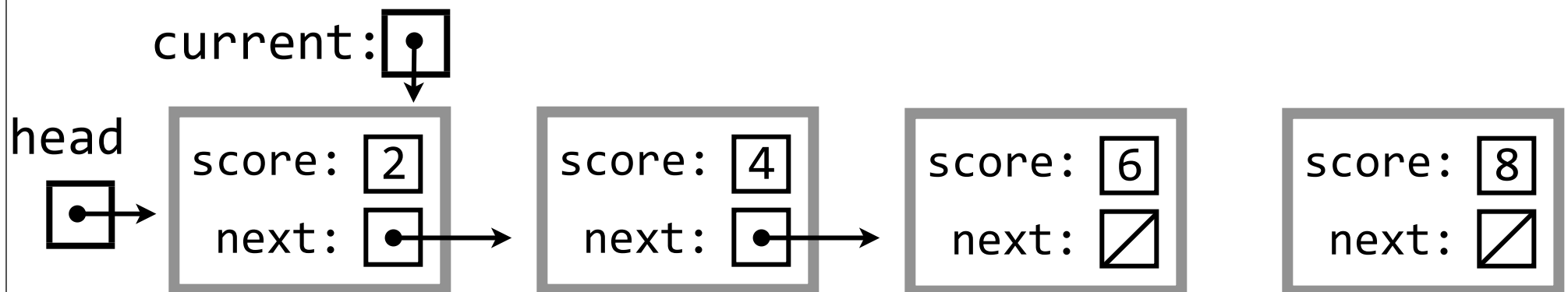
```
// If the array is full but automatically allocated  
// Out of luck!
```

# Insert at End of Linked List

Find the end of the list

Update that node's pointer to point at the new node

Make sure the new node's pointer is NULL

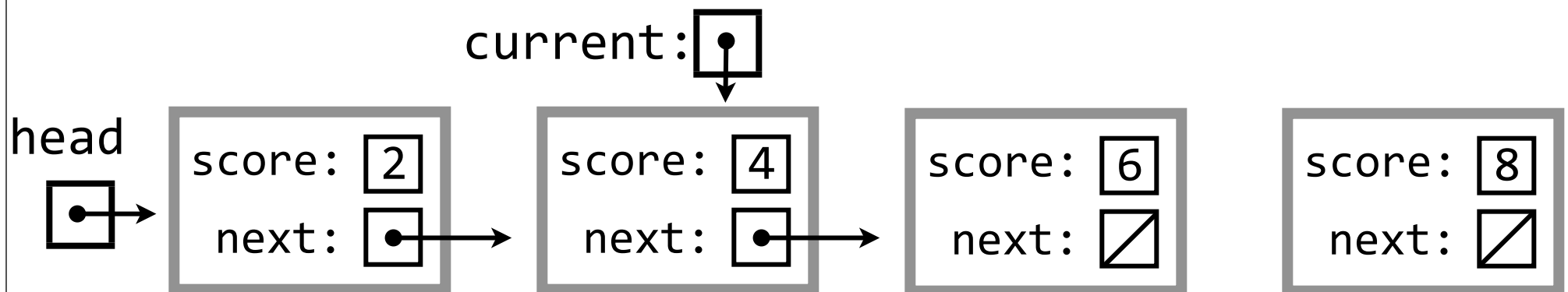


# Insert at End of Linked List

Find the end of the list

Update that node's pointer to point at the new node

Make sure the new node's pointer is NULL

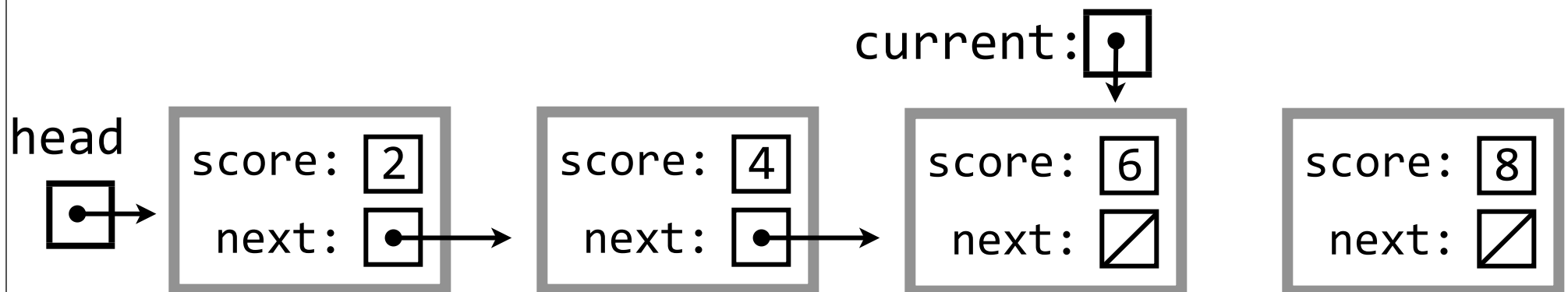


# Insert at End of Linked List

Find the end of the list

Update that node's pointer to point at the new node

Make sure the new node's pointer is NULL

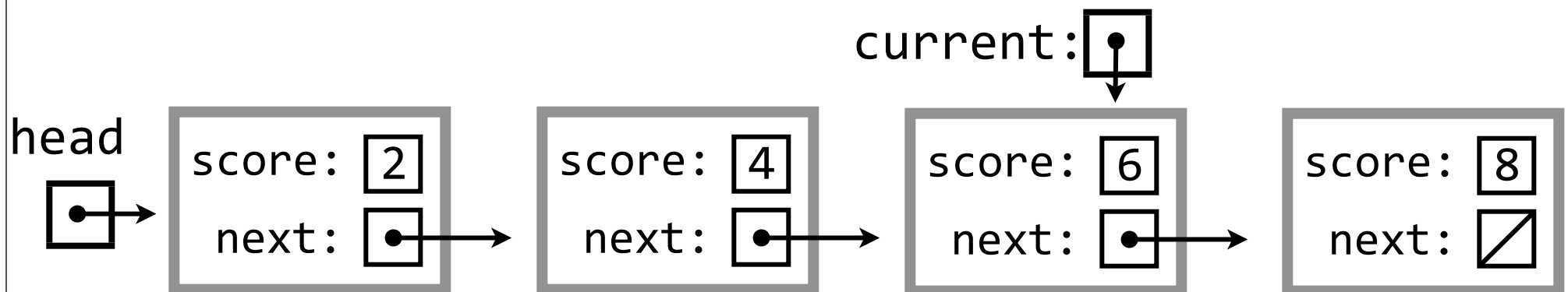


# Insert at End of Linked List

Find the end of the list

Update that node's pointer to point at the new node

Make sure the new node's pointer is NULL



# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current != NULL)
    {
        current = current->next;
    }

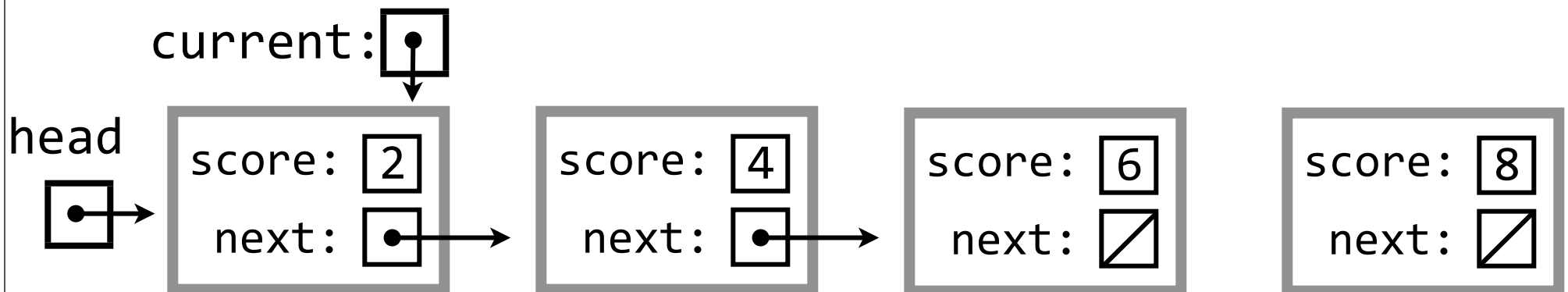
    current->next = newNode;
    newNode->next = NULL;
}
```

Wrong

# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current != NULL)
    {
        current = current->next;
    }

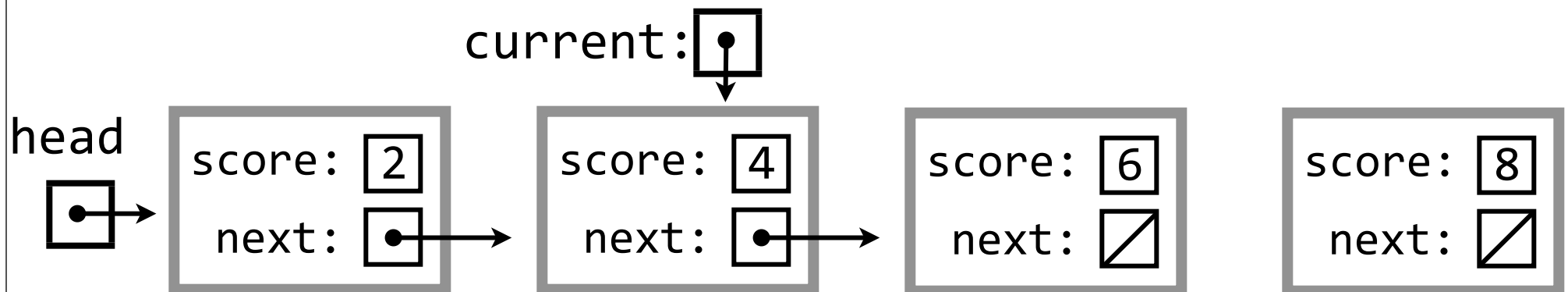
    current->next = newNode;
    newNode->next = NULL;
}
```



# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current != NULL)
    {
        current = current->next;
    }

    current->next = newNode;
    newNode->next = NULL;
}
```

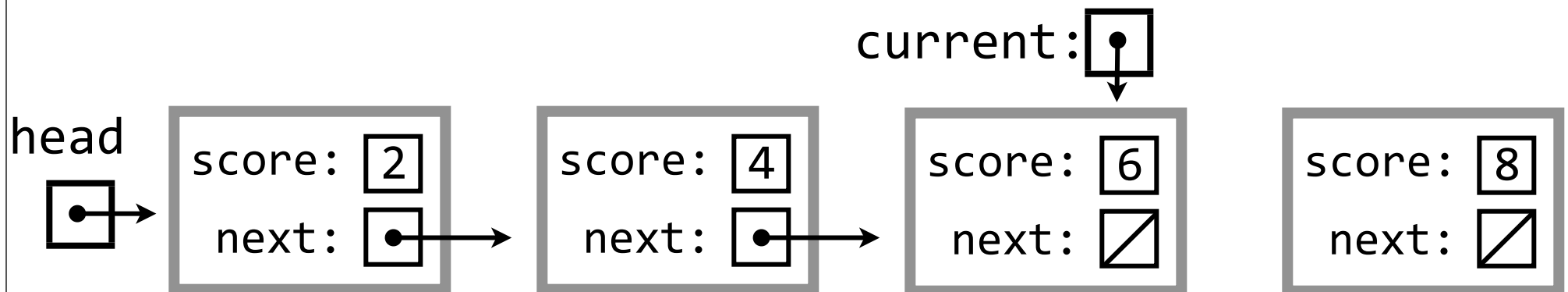




# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current != NULL)
    {
        current = current->next;
    }

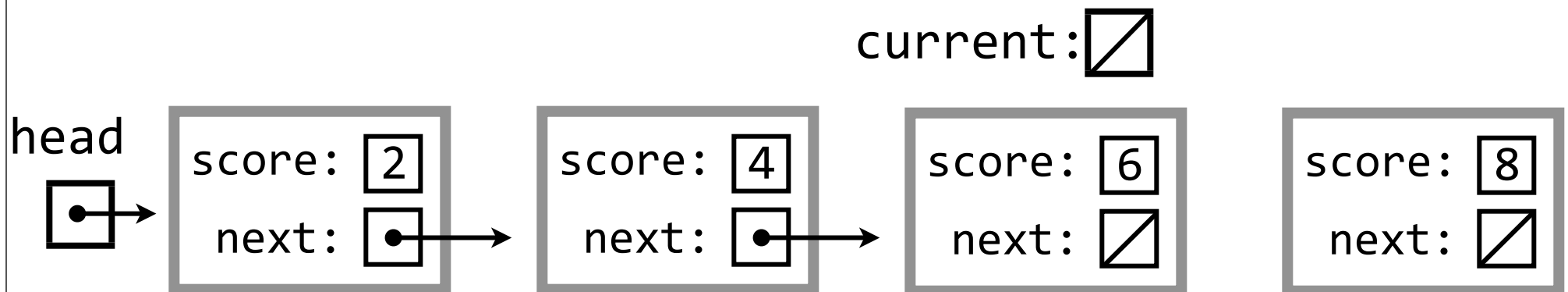
    current->next = newNode;
    newNode->next = NULL;
}
```



# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current != NULL)
    {
        current = current->next;
    }

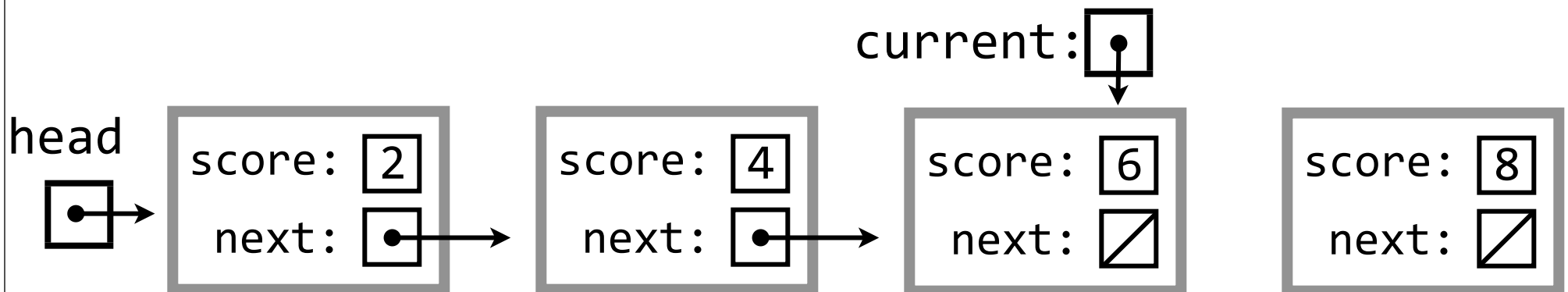
    current->next = newNode;
    newNode->next = NULL;
}
```



# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current->next != NULL)
    {
        current = current->next;
    }

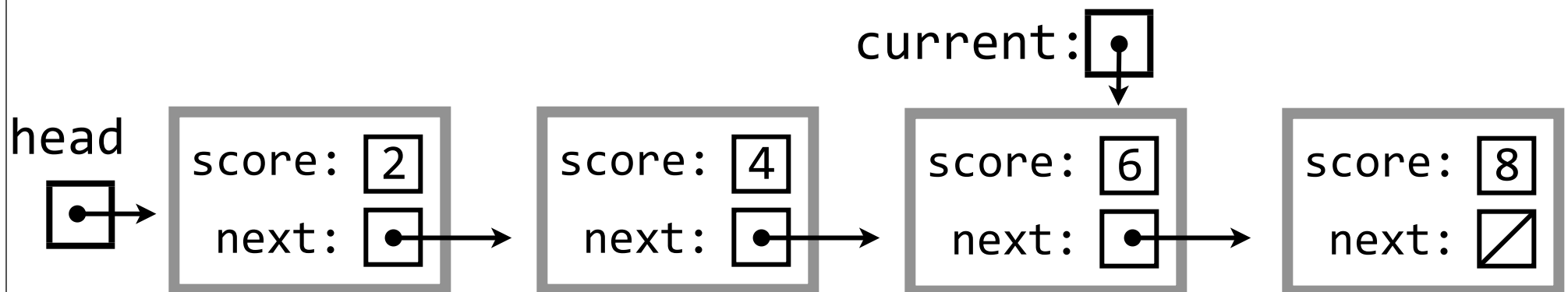
    current->next = newNode;
    newNode->next = NULL;
}
```



# Insert at End of Linked List

```
void addToEnd(Node *head, Node *newNode)
{
    Node *current = head;
    while (current->next != NULL)
    {
        current = current->next;
    }

    current->next = newNode;
    newNode->next = NULL;
}
```



# Insert at End of Linked List

```
Node *addToEnd(Node *head, Node *newNode)
{
    if (head == NULL)
    {
        newNode->next = NULL;
        return newNode;
    }

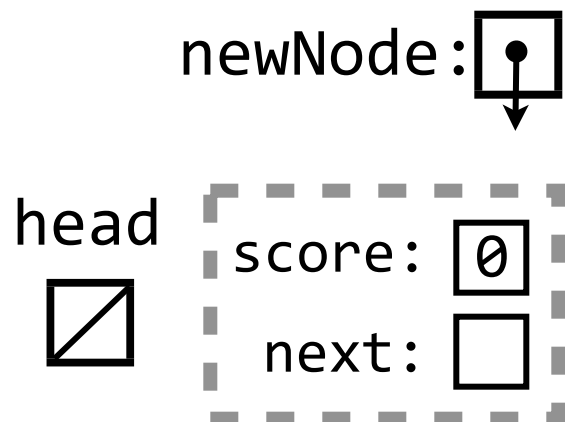
    Node *current = head;
    while (current->next != NULL)
    {
        current = current->next;
    }

    current->next = newNode;
    newNode->next = NULL;

    return head;
}
```

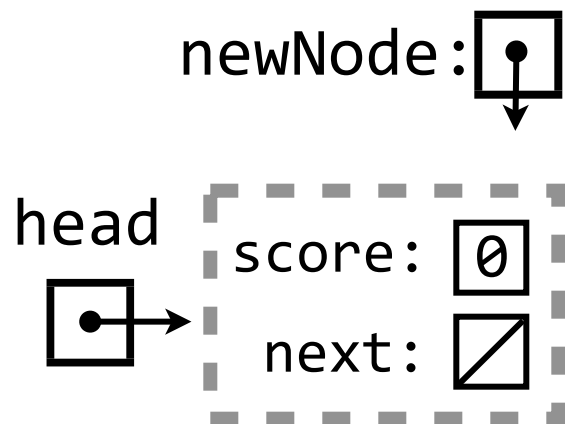
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



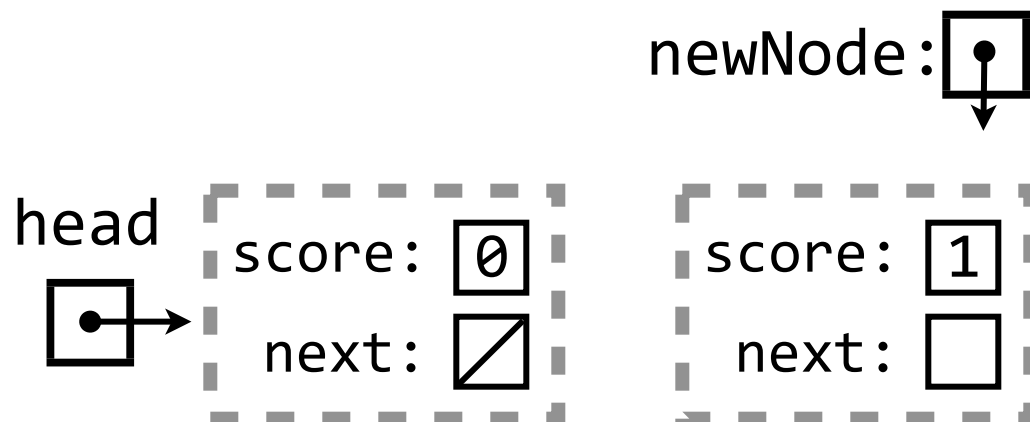
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



# Building a List

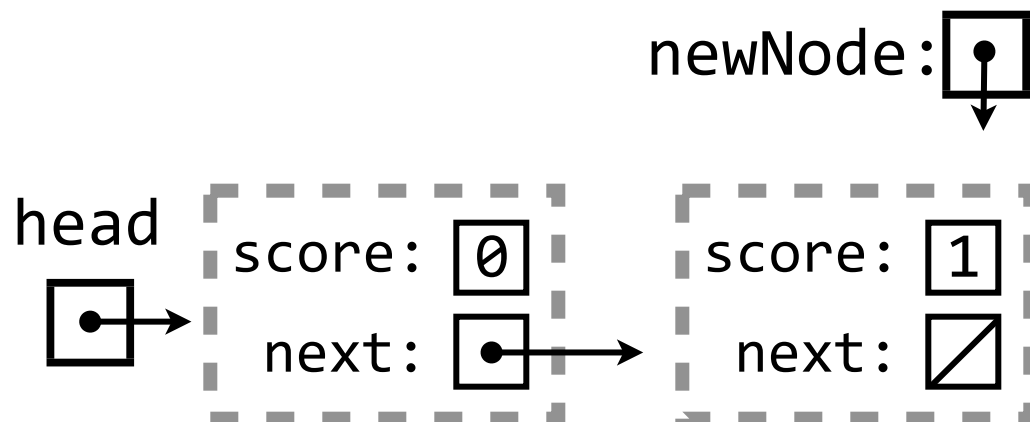
```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```





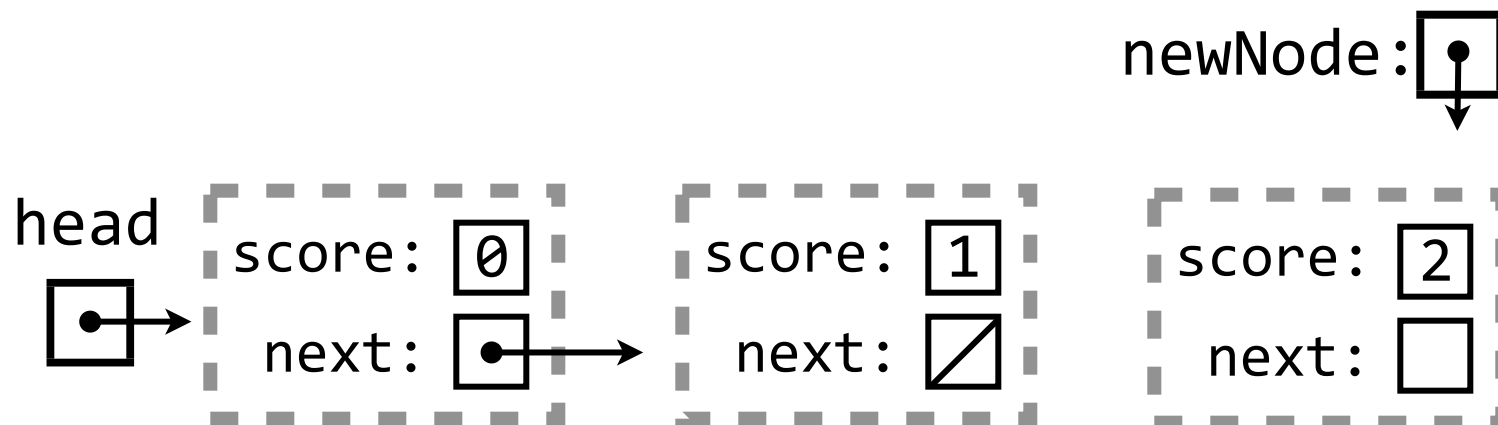
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



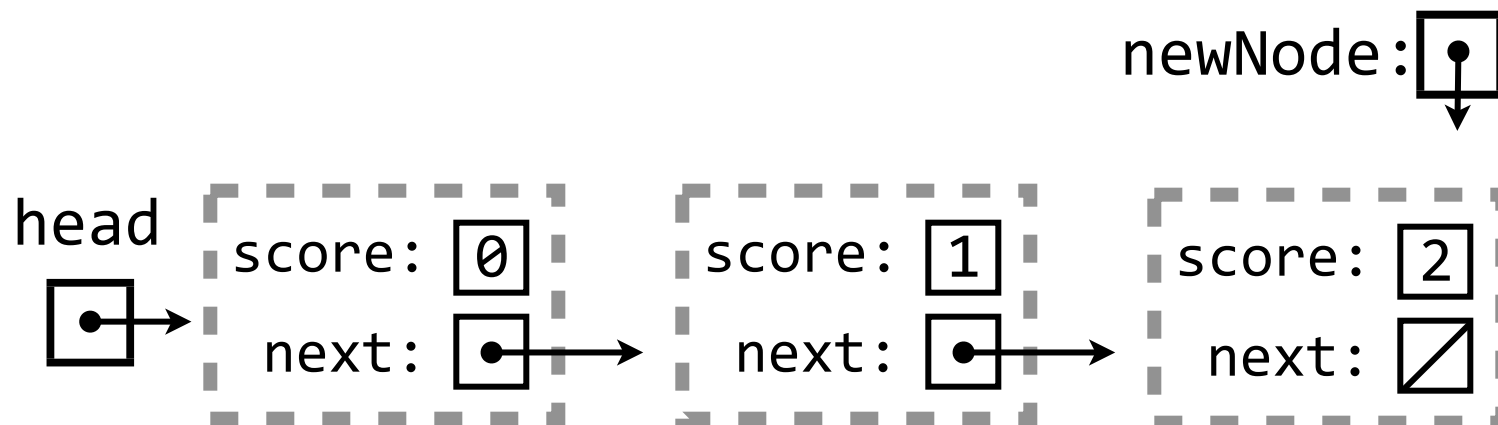
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToEnd(head, newNode);
}
```



# Verdict?

- Array easier, if it has space
- Array (much?) harder, if it doesn't
- Linked list never runs out of space
- Linked list needs to traverse entire list

# Insert at Beginning of Array

2	4	8	
---	---	---	--

# Insert at Beginning of Array

2	4		8
---	---	--	---

# Insert at Beginning of Array

2		4	8
---	--	---	---



# Insert at Beginning of Array

	2	4	8
--	---	---	---

# Insert at Beginning of Array

1	2	4	8
---	---	---	---

# Insert at Beginning of Array

```
// Assuming the array is large enough
for (int i = N - 2; i >= 0; i--)
{
    a[i + 1] = a[i];
}
a[0] = newItem;
```

1	2	4	8
---	---	---	---

a[0] a[1] a[2] a[3]

N-4 N-3 N-2 N-1 N

# Insert at Beg. of Linked List

Insert at End of Linked List:

- Find the end of the list

- Update that node's pointer to point at the new node

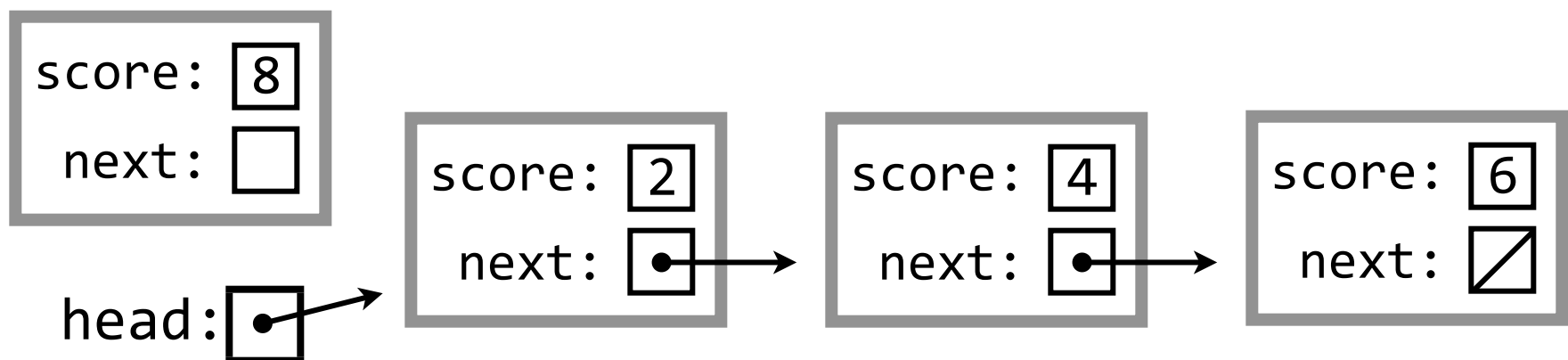
- Make sure the new node's pointer is NULL

Insert at Beginning of Linked List:

- Find the beginning of the list

- Point the new node's pointer at that node

- Make sure head points to the new node



# Insert at Beg. of Linked List

Insert at End of Linked List:

- Find the end of the list

- Update that node's pointer to point at the new node

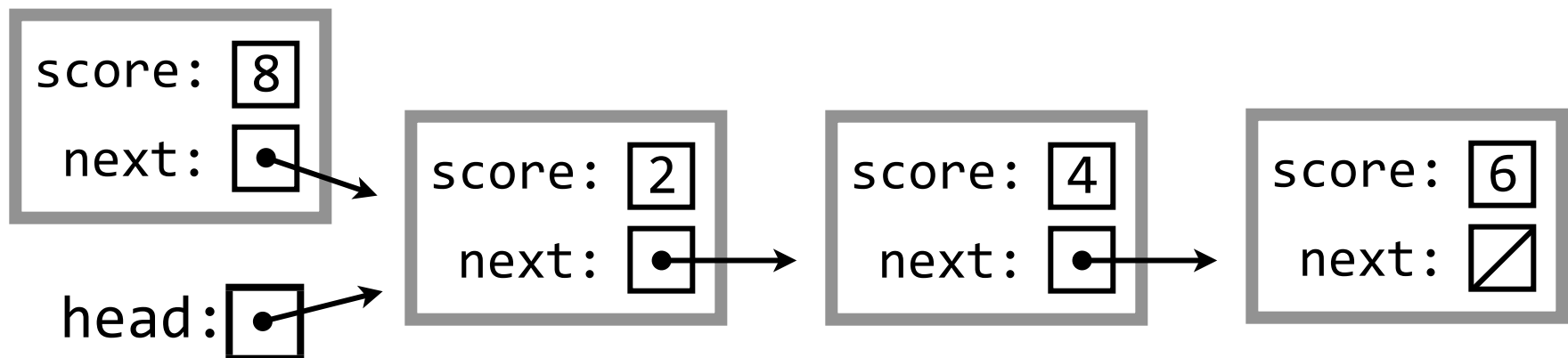
- Make sure the new node's pointer is NULL

Insert at Beginning of Linked List:

- Find the beginning of the list

- Point the new node's pointer at that node

- Make sure head points to the new node



# Insert at Beg. of Linked List

Insert at End of Linked List:

- Find the end of the list

- Update that node's pointer to point at the new node

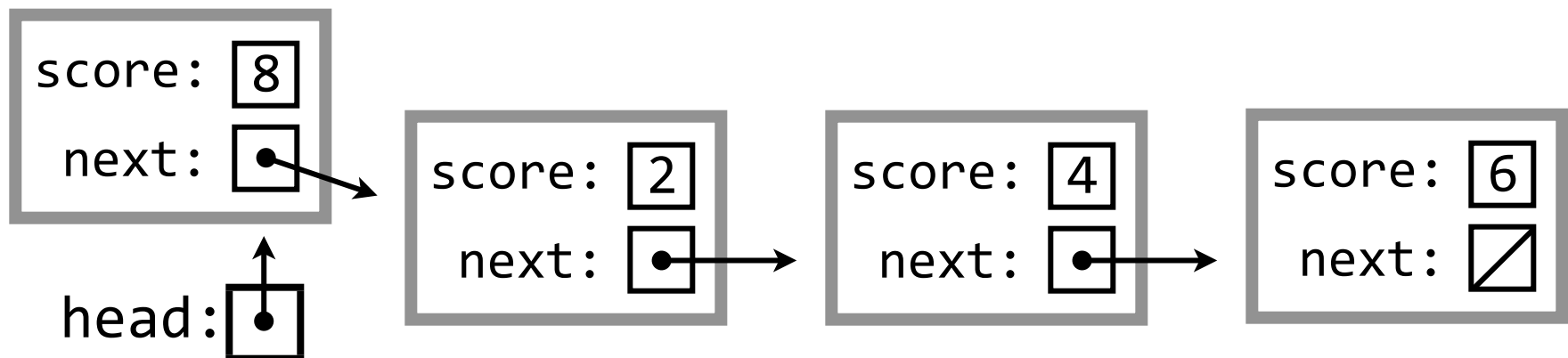
- Make sure the new node's pointer is NULL

Insert at Beginning of Linked List:

- Find the beginning of the list

- Point the new node's pointer at that node

- Make sure head points to the new node



# Insert at Beg. of Linked List

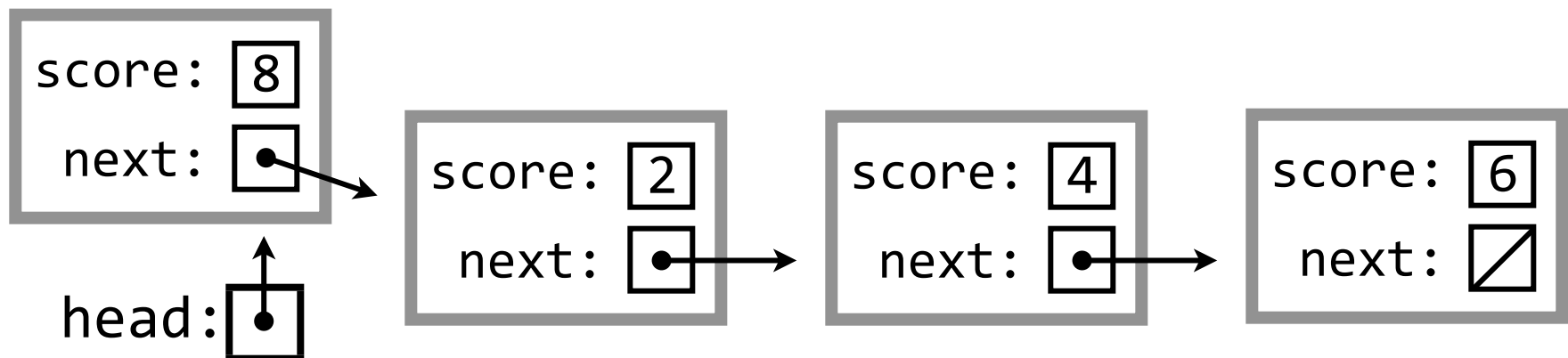
```
newNode->next = head;  
head = newNode;
```

Insert at Beginning of Linked List:

Find the beginning of the list

Point the new node's pointer at that node

Make sure head points to the new node



# Insert at Beg. of Linked List

```
Node *addToFront(Node *head, Node *newNode)
{
    newNode->next = head;

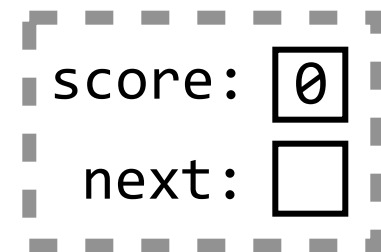
    return newNode;
}
```




# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToFront(head, newNode);
}
```

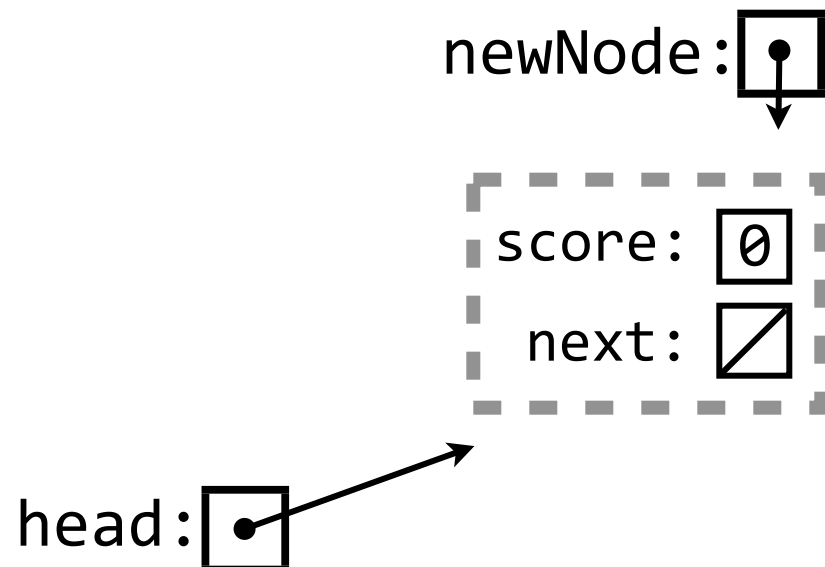
newNode: 



head: 

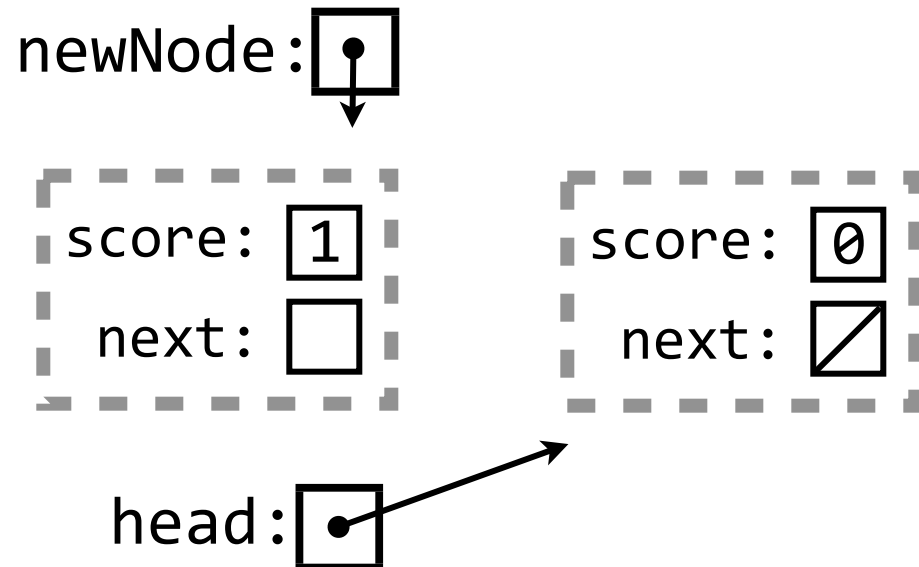
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToFront(head, newNode);
}
```



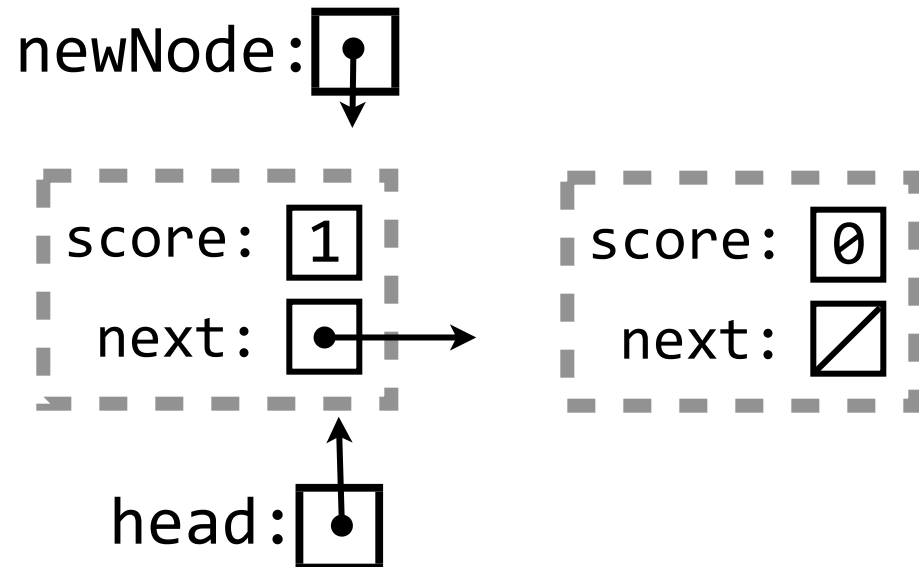
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToFront(head, newNode);
}
```



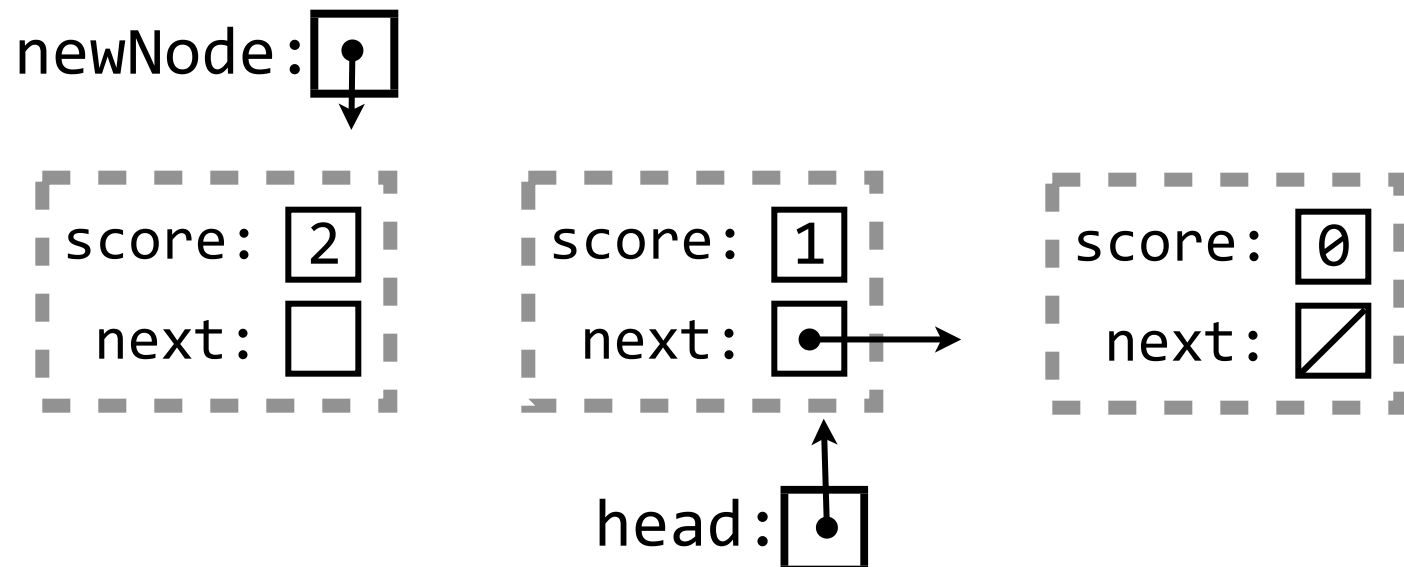
# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToFront(head, newNode);
}
```



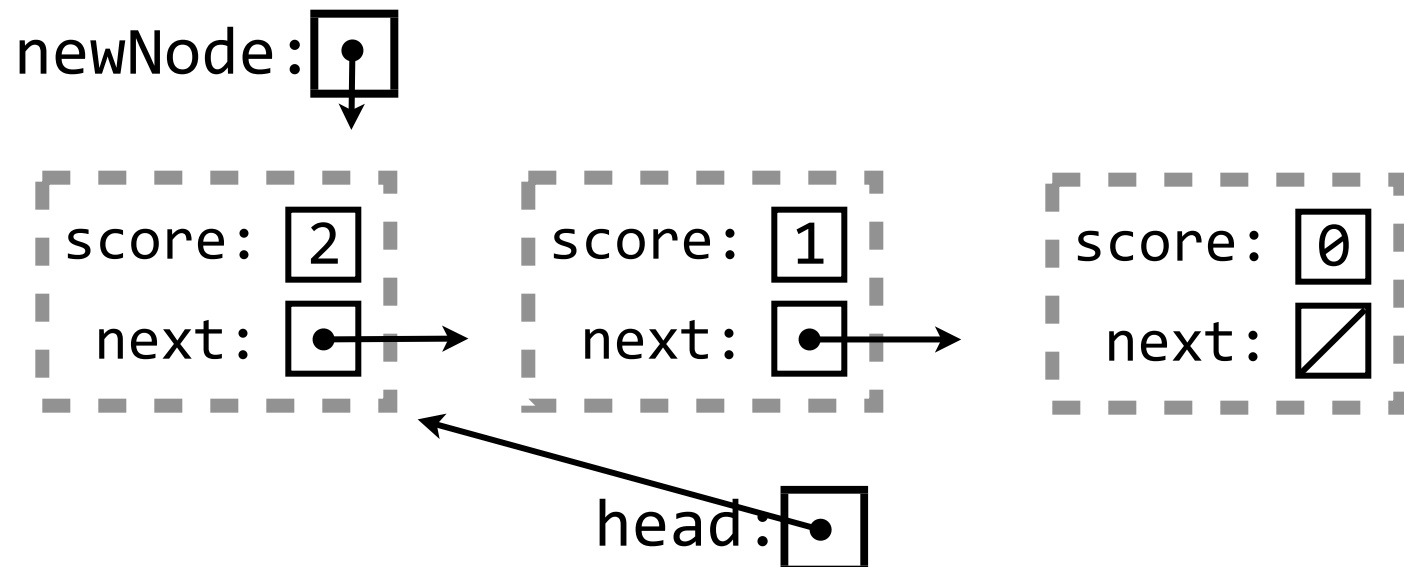
# Building a List

```
Node *head = NULL;  
for (int i = 0; i < N; i++)  
{  
    Node *newNode = malloc(sizeof(Node));  
    newNode->score = i;  
    head = addToFront(head, newNode);  
}
```



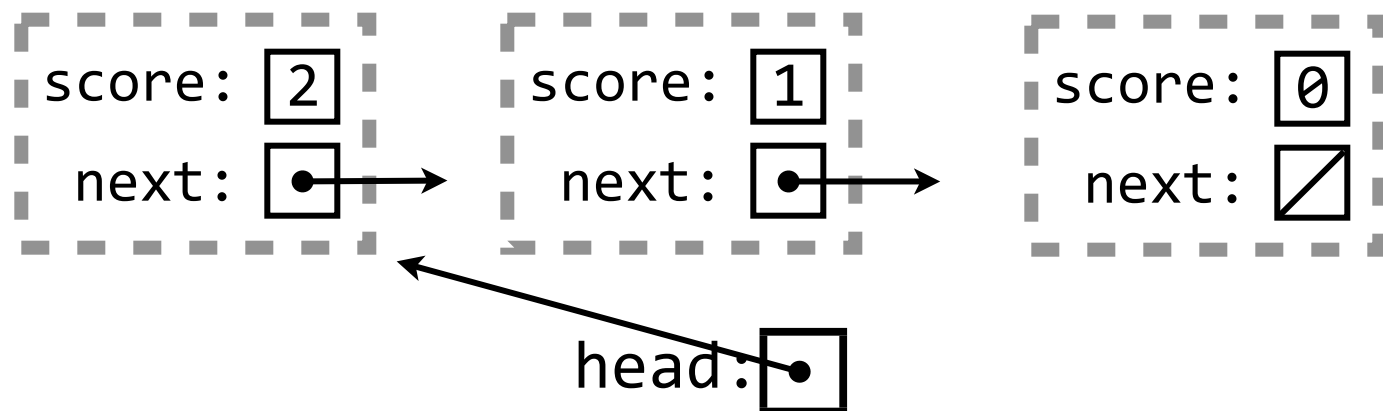
# Building a List

```
Node *head = NULL;  
for (int i = 0; i < N; i++)  
{  
    Node *newNode = malloc(sizeof(Node));  
    newNode->score = i;  
    head = addToFront(head, newNode);  
}
```



# Building a List

```
Node *head = NULL;
for (int i = 0; i < N; i++)
{
    Node *newNode = malloc(sizeof(Node));
    newNode->score = i;
    head = addToFront(head, newNode);
}
```



# Verdict?

- Array time consuming, if it has space
  - Gets more time consuming as the array gets larger
- Array (much?) harder, if it doesn't
- Linked list never runs out of space
- Linked list is very fast
  - Always takes the same time, no matter the length



# Insert in Middle of Array

Want to insert 1 at  $a[3]$

2	4	8	16	32	64	
---	---	---	----	----	----	--

# Insert in Middle of Array

Want to insert 1 at  $a[3]$

2	4	8	16	32		64
---	---	---	----	----	--	----

# Insert in Middle of Array

Want to insert 1 at  $a[3]$

2	4	8	16		32	64
---	---	---	----	--	----	----

# Insert in Middle of Array

Want to insert 1 at  $a[3]$

2	4	8		16	32	64
---	---	---	--	----	----	----

# Insert in Middle of Array

Want to insert 1 at  $a[3]$

2	4	8	1	16	32	64
---	---	---	---	----	----	----

# Insert in Middle of Array

```
// Assuming the array is large enough
```

```
for (int i = N - 2; i >= 3; i--)
```

```
{
```

```
    a[i + 1] = a[i];
```

```
}
```

```
a[3] = 1;
```

Want to insert 1 at a[3]

2	4	8	1	16	32	64
---	---	---	---	----	----	----

a[0] a[1] a[2] a[3] a[4] a[5] a[6]

N-2 N-1

# Insert in Middle of Array

```
// Assuming the array is large enough
```

```
for (int i = N - 2; i >= k; i--)
```

```
{
```

```
    a[i + 1] = a[i];
```

```
}
```

```
a[k] = x;
```

Want to insert 1 at a[3]

Want to insert x at a[k]

2	4	8	1	16	32	64
---	---	---	---	----	----	----

a[0] a[1] a[2] a[3] a[4] a[5] a[6]

N-2 N-1