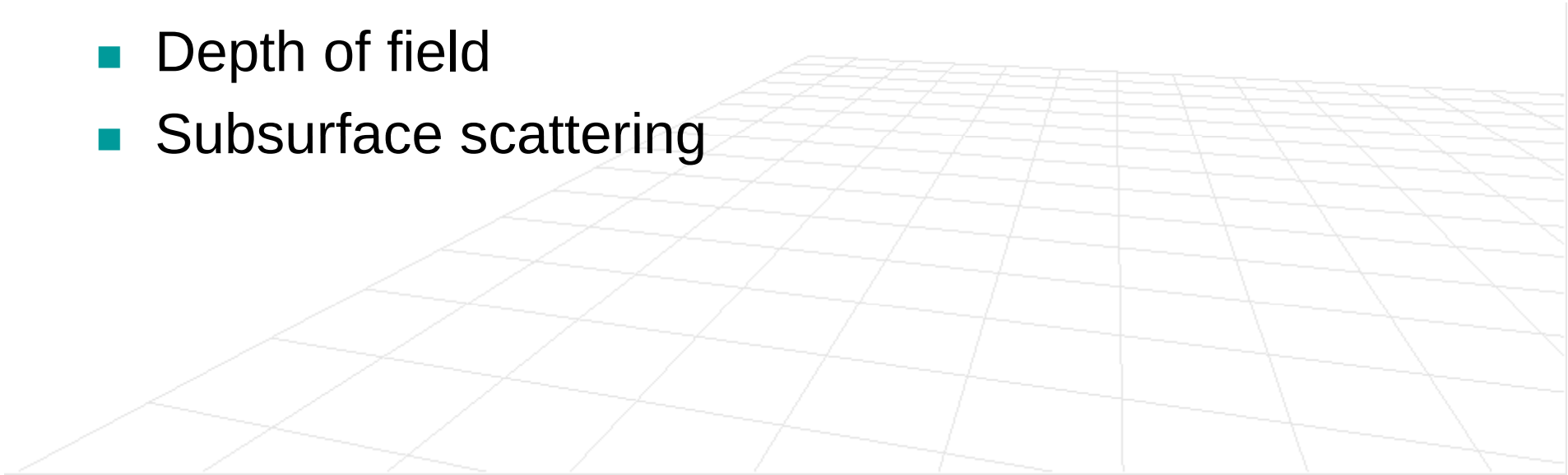
Distribution Ray Tracing

- In **Whitted Ray Tracing** we computed lighting very crudely
  - Phong + specular global lighting

- In **Distributed Ray Tracing** we want to compute the lighting as accurately as possible
  - Use the formalism of Radiometry
  - **Compute irradiance at each pixel** (by integrating all the incoming light)
  - Since integrals are can not be done analytically, we will employ **numeric approximations**

Benefits of Distribution Ray Tracing

- Better global diffuse lighting
  - Color bleeding
  - Bouncing highlights
- Extended light sources
- Anti-aliasing
- Motion blur
- Depth of field
- Subsurface scattering



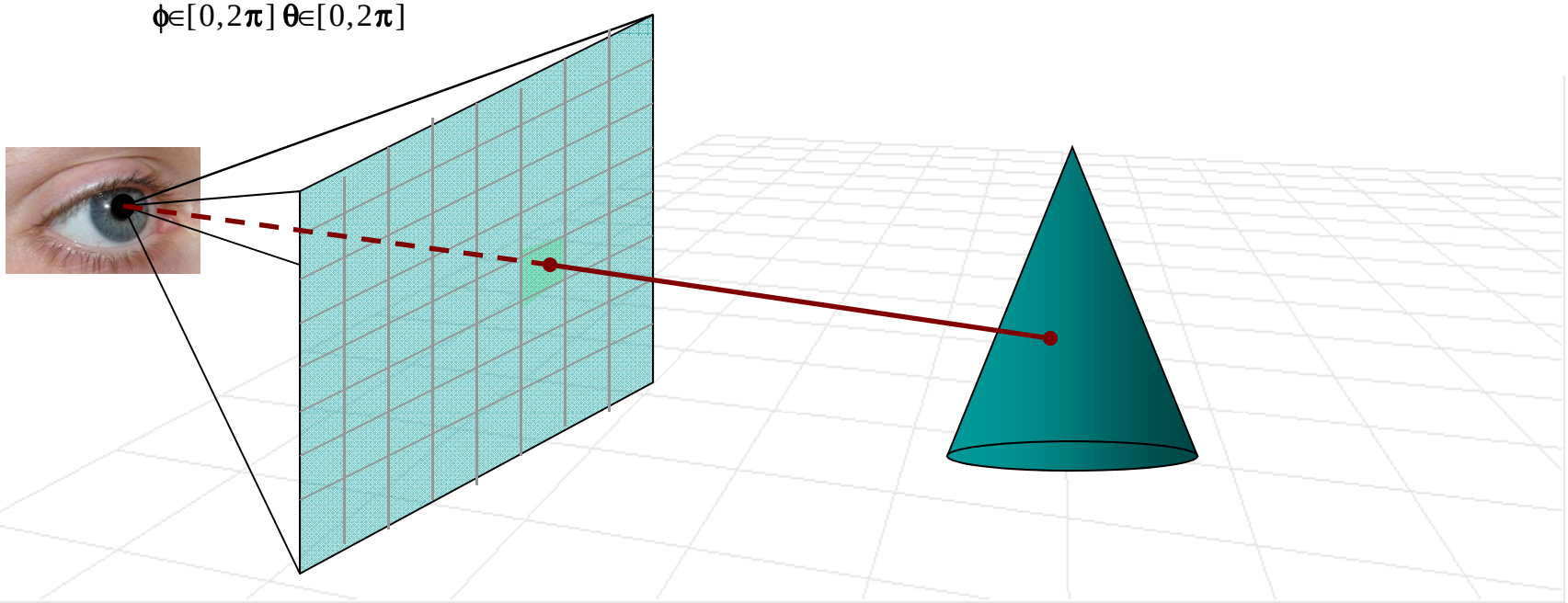
Radiance at a Point

- Recall that radiance (shading) at a surface point is given by

$$\mathbf{L}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_e) = \int_{\Omega} \rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i)\mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i)\, d\omega$$

- If we parameterize directions in spherical coordinates and assume small differential solid angle, we get

$$\mathbf{L}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_e) = \int_{\phi \in [0,2\pi]} \int_{\theta \in [0,2\pi]} \rho\left(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i(\phi,\theta)\right)\mathbf{L}\left(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i(\phi,\theta)\right)\left(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i(\phi,\theta)\right)\sin\theta\, d\theta\, d\phi$$



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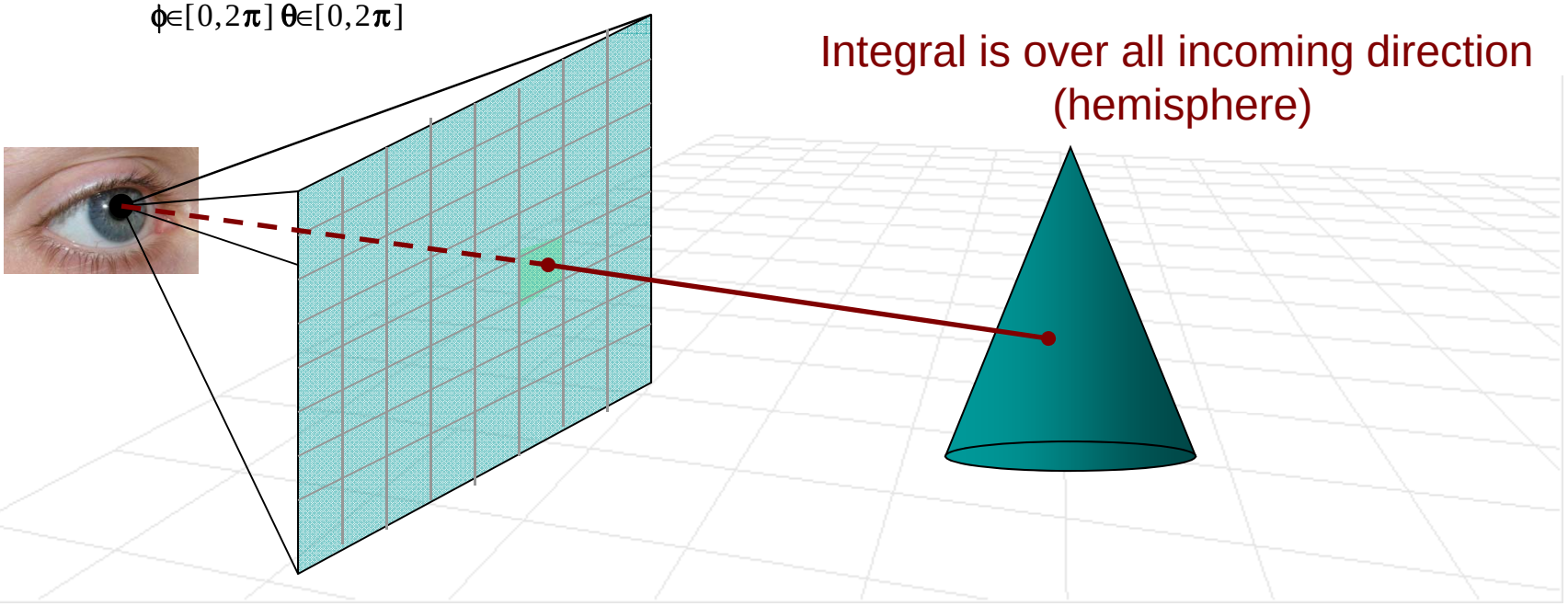
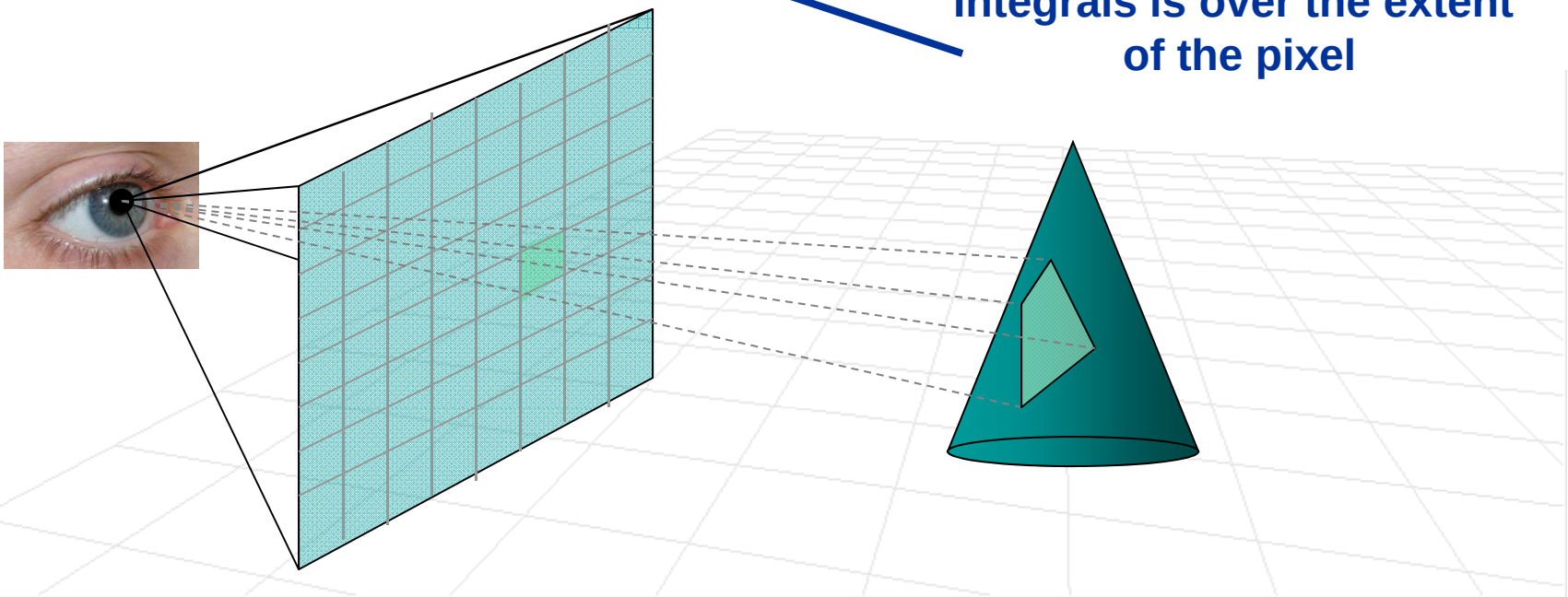


Irradiance at a Pixel

- To compute the color of the pixel, we need to compute **total light energy** (flux) **passing through the pixel** (rectangle) (i.e. we need to compute the total irradiance at a pixel)

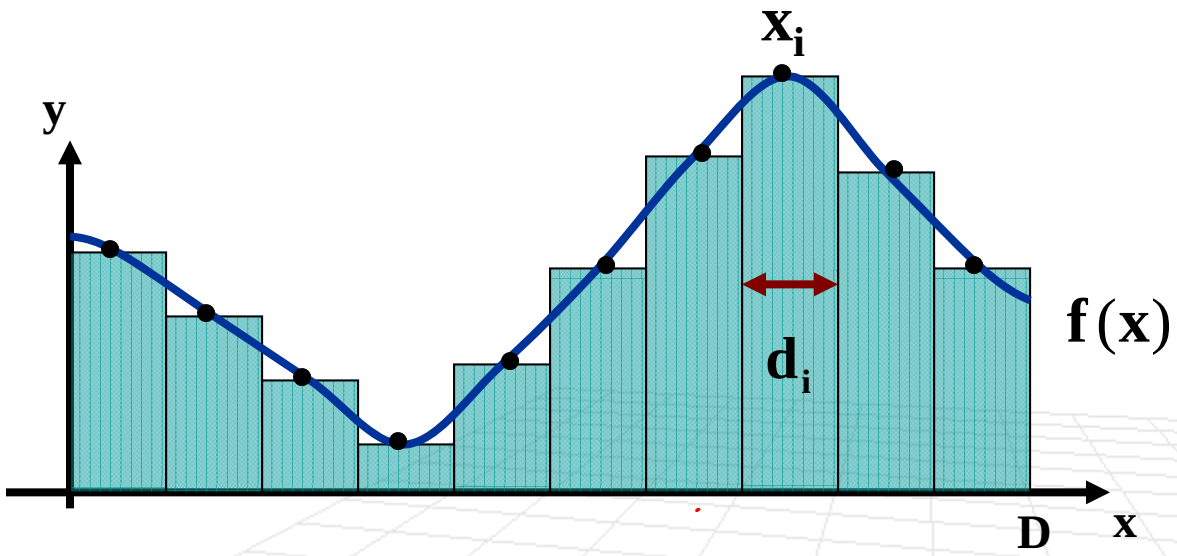
$$\Phi_{i,j} = \int_{\alpha_{min} \le \alpha \le \alpha_{max}} \int_{\beta_{min} \le \beta \le \beta_{max}} H(\alpha, \beta)\, d\alpha\, d\beta$$

Integrals is over the extent of the pixel



Numerical Integration (1D Case)

- **Remember:** integral is an area under the curve
- We can approximate any integral numerically as follows



$$\sum_{i=1}^{N} d_i f(x_i) \xrightarrow[N\to\infty]{} \int_0^D f(x)\,dx$$

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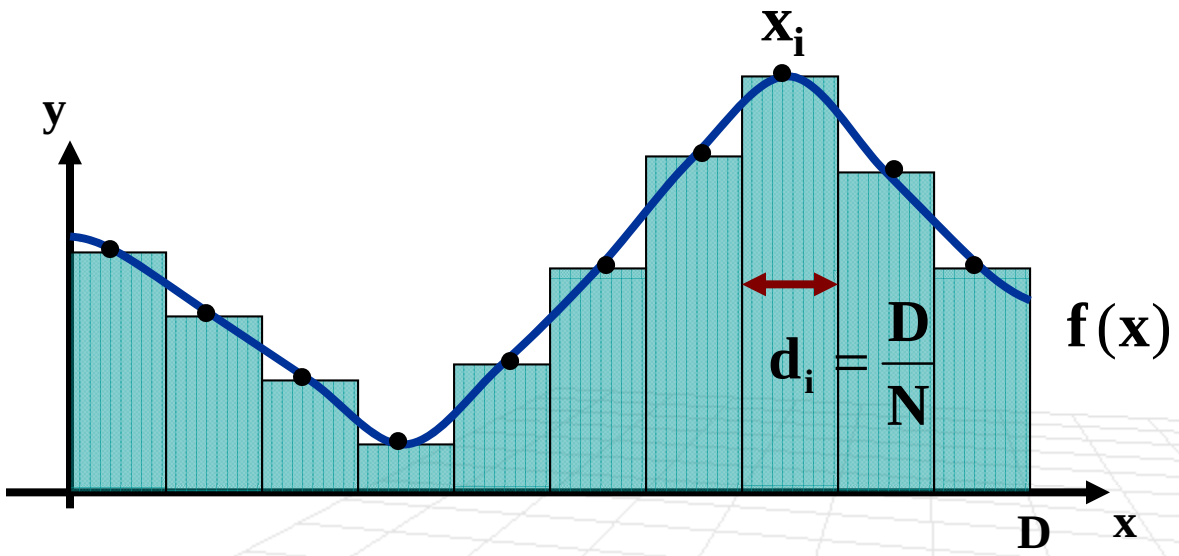
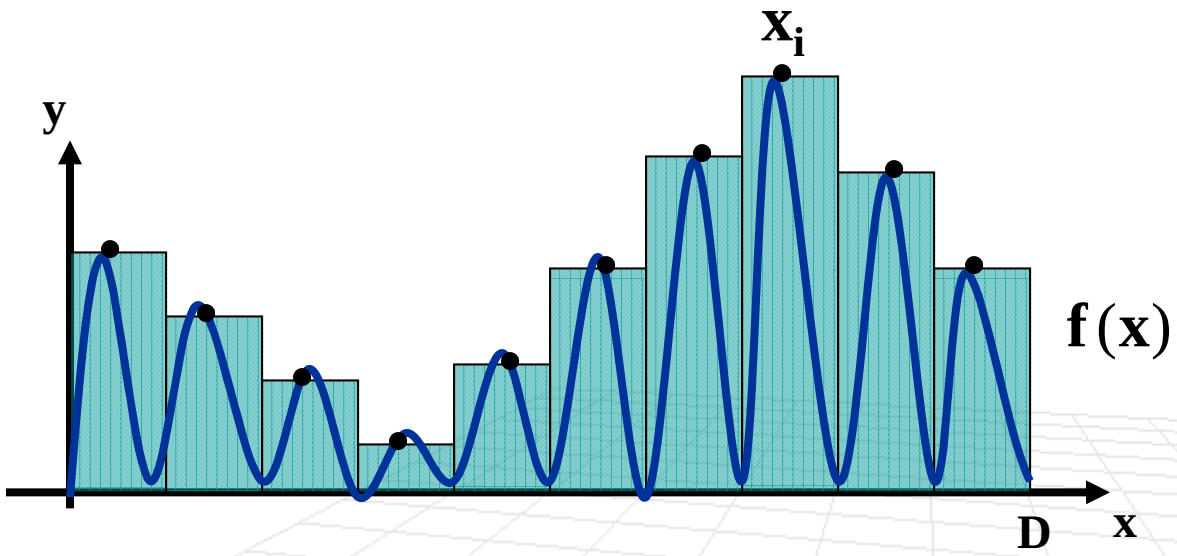


$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Numerical Integration (1D Case)

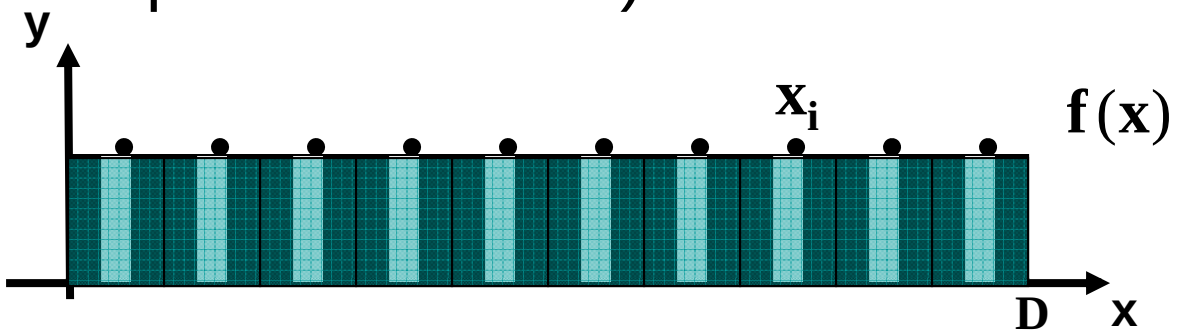
- **Problem:** what if we are really unlucky and our signal has the same structure as sampling?



$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Monte Carlo Integration

- **Idea:** randomize points $\mathbf{x_i}$ to avoid structured noise (e.g. due to periodic texture)

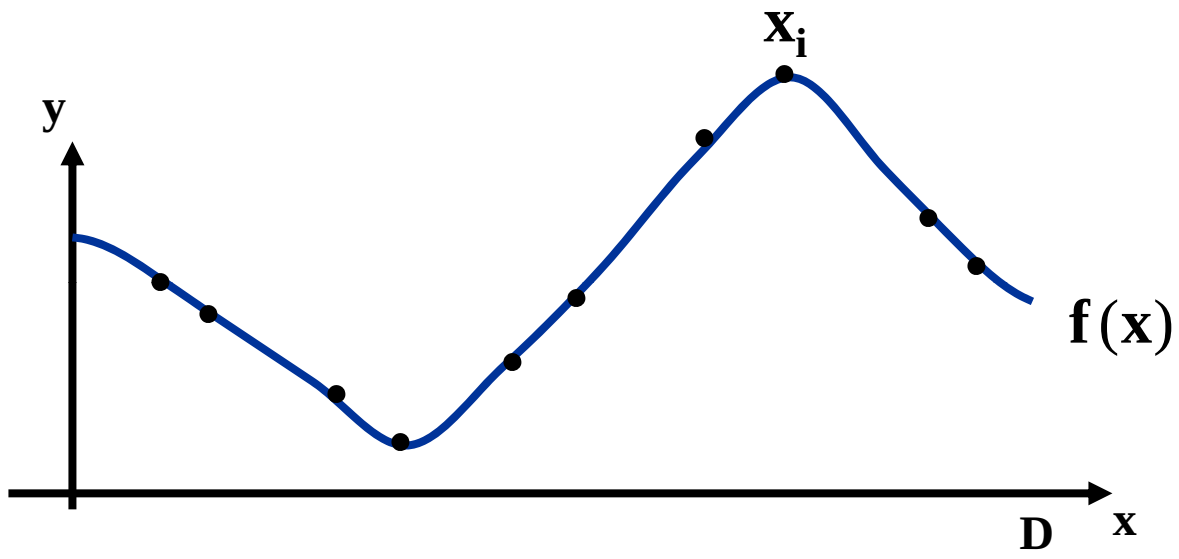


- Draw $\mathbf{N}$ random samples $\mathbf{x_i}$ independently from uniform distribution $\mathbf{Q(x)=U[0,D]}$ (i.e. $\mathbf{Q(x) = 1/D}$ is the uniform probability density function)
- Then approximation to the integral becomes

$$\frac{1}{N}\sum w_i f(x_i) \approx \int f(x)\,dx \quad \text{, for } w_i = \frac{1}{Q(x_i)}$$

- **We can also use other Q's for efficiency !!!** (a.k.a. importance sampling)

Monte Carlo Integration



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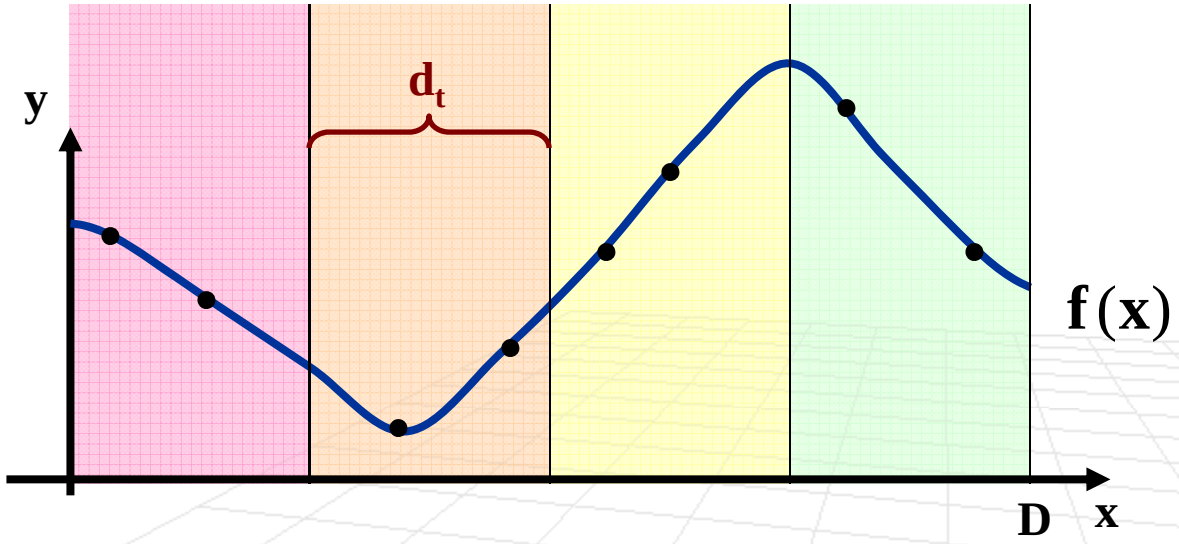
- **We can also use other Q's for efficiency !!!** (a.k.a. importance sampling)

Stratified Sampling

- **Idea:** combination of uniform sampling plus random jitter
- Break domain into $\mathbf{T}$ intervals of widths $\mathbf{d_t}$ and $\mathbf{N_t}$ samples in interval $\mathbf{t}$



- Integral approximated using the following:

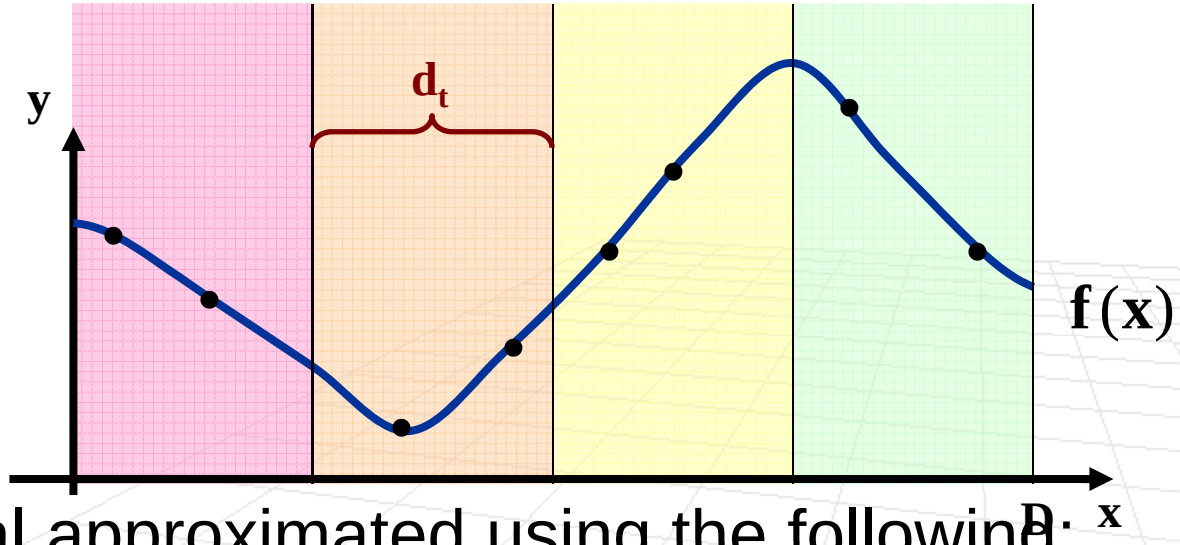
$$\sum_{t=1}^{T} \frac{1}{N_t} \sum_{j=1}^{N_t} d_t f(x_{t,j})$$

Stratified Sampling

- If intervals are uniform $\mathbf{d_t = D/T}$ and there are same number of samples in each interval $\mathbf{N_t = N/T}$ then this approximation reduces to:

$$\sum_{t=1}^{T}\sum_{j=1}^{N_t}\frac{D}{N}f(x_{t,j})$$

- **The interval size and the # of samples can vary !!!**



- Integral approximated using the following:

$$\sum_{t=1}^{T}\frac{1}{N_t}\sum_{j=1}^{N_t}d_t f(x_{t,j})$$

Back to Distribution Ray Tracing

- Based on one of the approximate integration approaches we need to compute
  - Let's try uniform sampling

$$\mathbf{L}(\bar{\mathbf{p}},\vec{\mathbf{d}}_e) = \int_{\phi\in[0,2\pi]}\int_{\theta\in[0,2\pi]} \rho\left(\vec{\mathbf{d}}_e,\vec{\mathbf{d}}_i(\phi,\theta)\right)\mathbf{L}\left(\bar{\mathbf{p}},-\vec{\mathbf{d}}_i(\phi,\theta)\right)\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i(\phi,\theta)\right)\sin\theta\,\mathbf{d}\theta\,\mathbf{d}\phi$$

$$\approx \sum_{m=1}^{M}\sum_{n=1}^{N} \rho\left(\vec{\mathbf{d}}_e,\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\mathbf{L}\left(\bar{\mathbf{p}},-\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\sin\theta\,\Delta\theta\,\Delta\phi$$

where

$$\theta_n = \left(n-\frac{1}{2}\right)\Delta\theta \qquad \Delta\theta = \frac{\pi/2}{M}$$

$$\phi_m = \left(m-\frac{1}{2}\right)\Delta\phi \qquad \Delta\phi = \frac{2\pi}{N}$$

midpoint of the interval (sample point)

Interval width

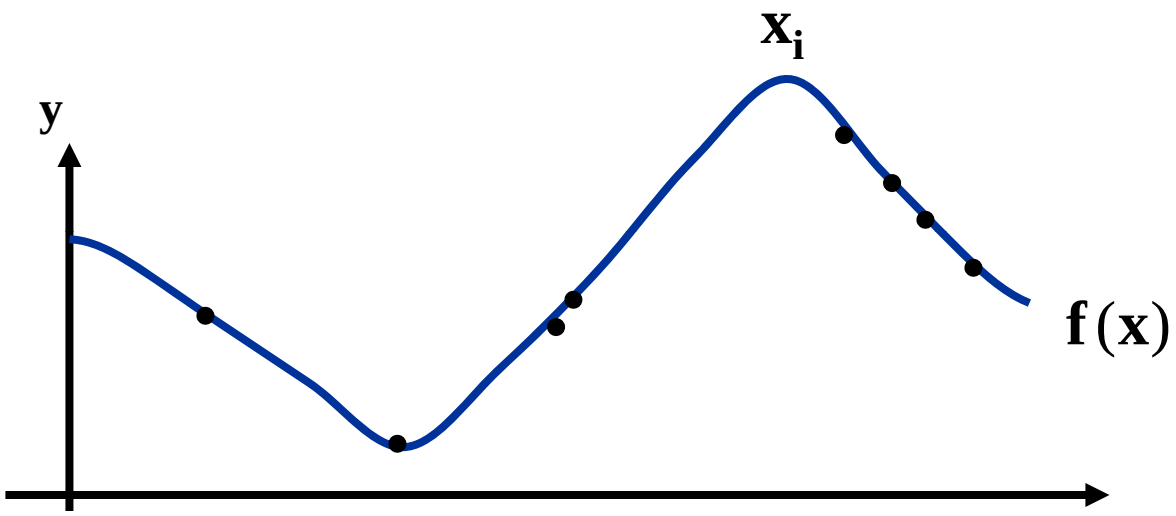
Importance Sampling in Distribution Ray Tracing

- **Problem:** Uniform sampling is too expensive (e.g. **100** samples/hemisphere with depth of ray recursion of **4** => $\mathbf{100^4=10^8}$ samples per pixel … with $\mathbf{10^5}$ pixels =>$\mathbf{10^{15}}$ **samples**)

- **Solution:** Sample more densely (using **importance sampling**) where we know that effects will be most significant
  - Direction toward point or extended light source are significant
  - Specular and off-axis specular are significant
  - Texture/lightness gradients are significant
  - Sample less with greater depth of recursion

Importance Sampling

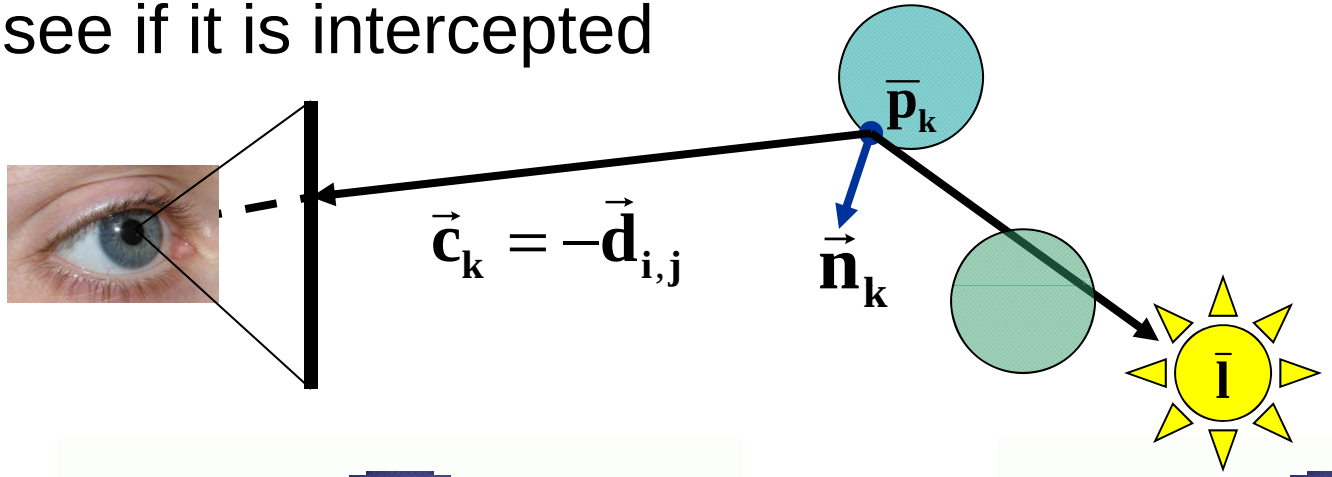
- **Idea:** randomize points $\mathbf{x_i}$ to avoid structured noise (e.g. due to periodic texture)

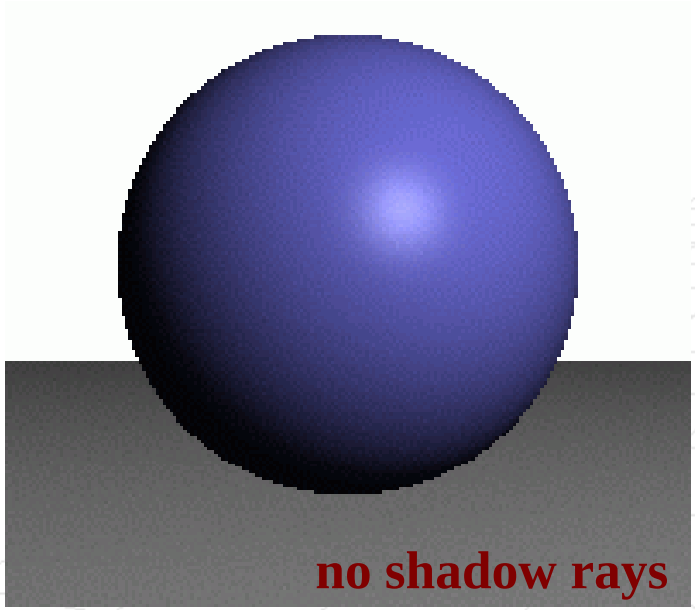


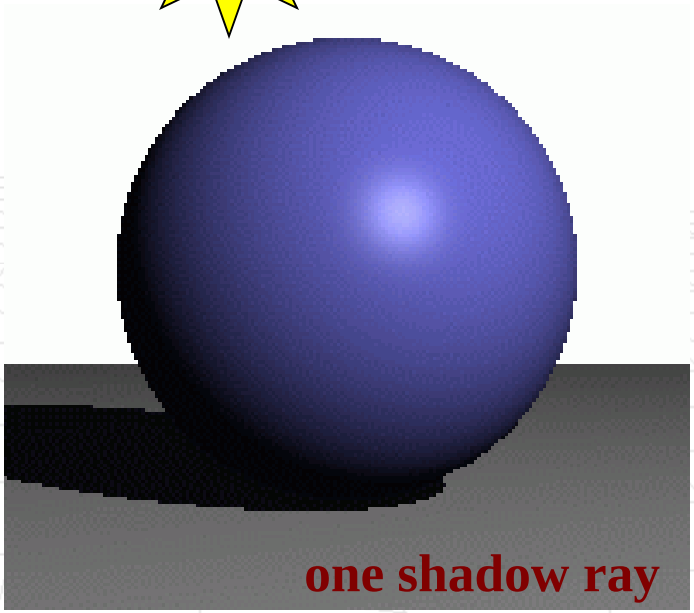
$$\frac{1}{N}\sum \mathbf{w_i f(x_i)} \approx \int \mathbf{f(x)\,dx} \quad \text{, for } \mathbf{w_i} = \frac{1}{\mathbf{Q(x_i)}}$$

Shadows in Ray Tracing

- Recall, we shoot a ray towards a light source and see if it is intercepted



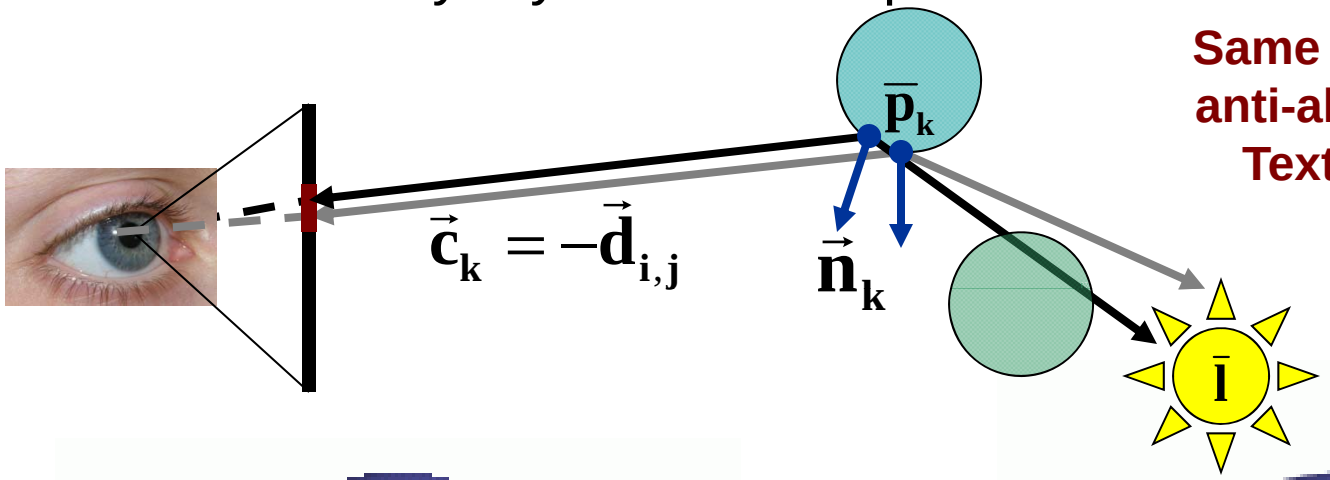




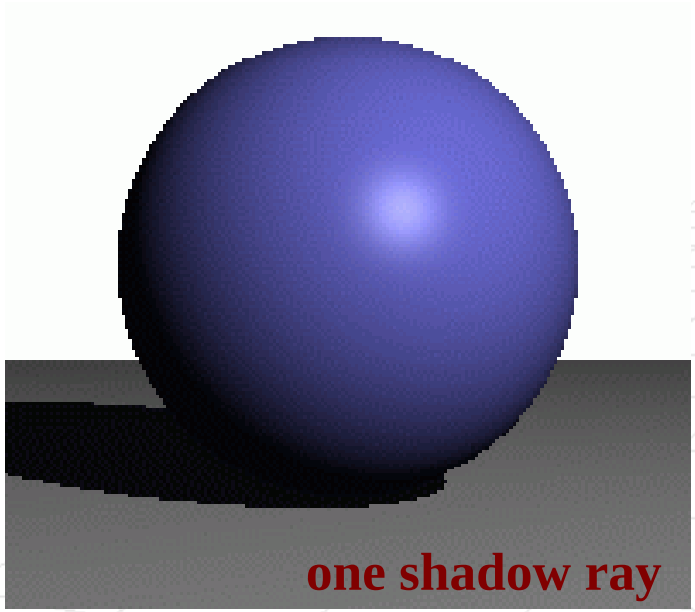
Images from the slides by Durand and Cutler

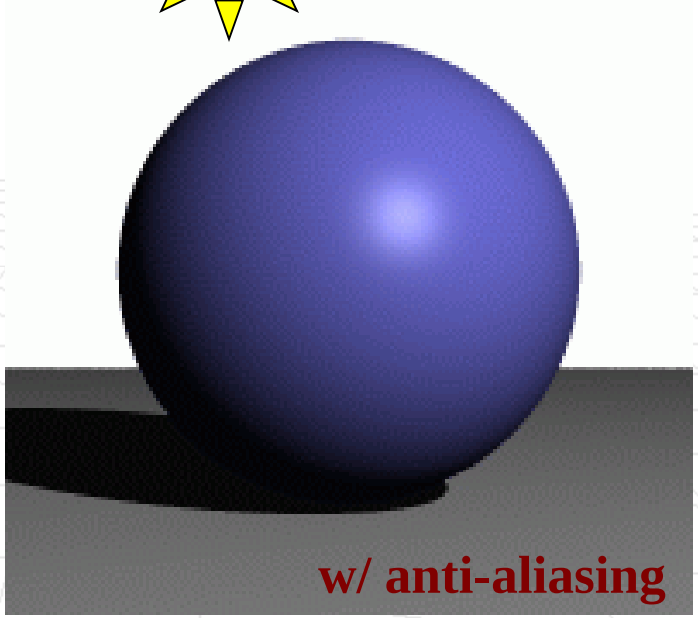
Anti-aliasing in Distribution Ray Tracer

- Lets shoot multiple rays from the same point and attenuate the color based on how many rays are intercepted



Same works for anti-aliasing of Textures !!!





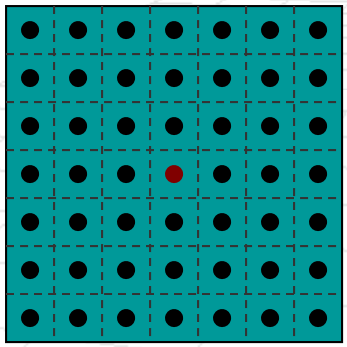
Images from the slides by Durand and Cutler

Anti-aliasing by Deterministic Integration

- **Idea:** Use multiple rays for every pixel

- **Algorithm**
  - Subdivide pixel ***(i,j)*** into squares
  - Cast ray through square centers
  - Average the obtained light

- Susceptible to structured noise, repeating textures

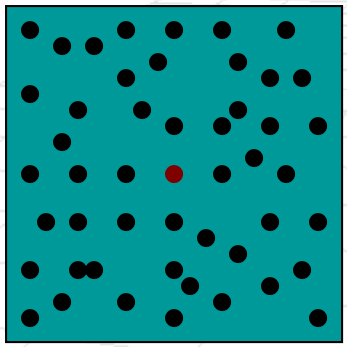


Anti-aliasing by Monte Carlo Integration

- **Idea:** Use multiple rays for every pixel

- **Algorithm**
  - Randomly sample point inside the pixel ***(i,j)***
  - Cast ray through square centers
  - Average the obtained light

- **Does not** suffer from structured noise, repeating textures



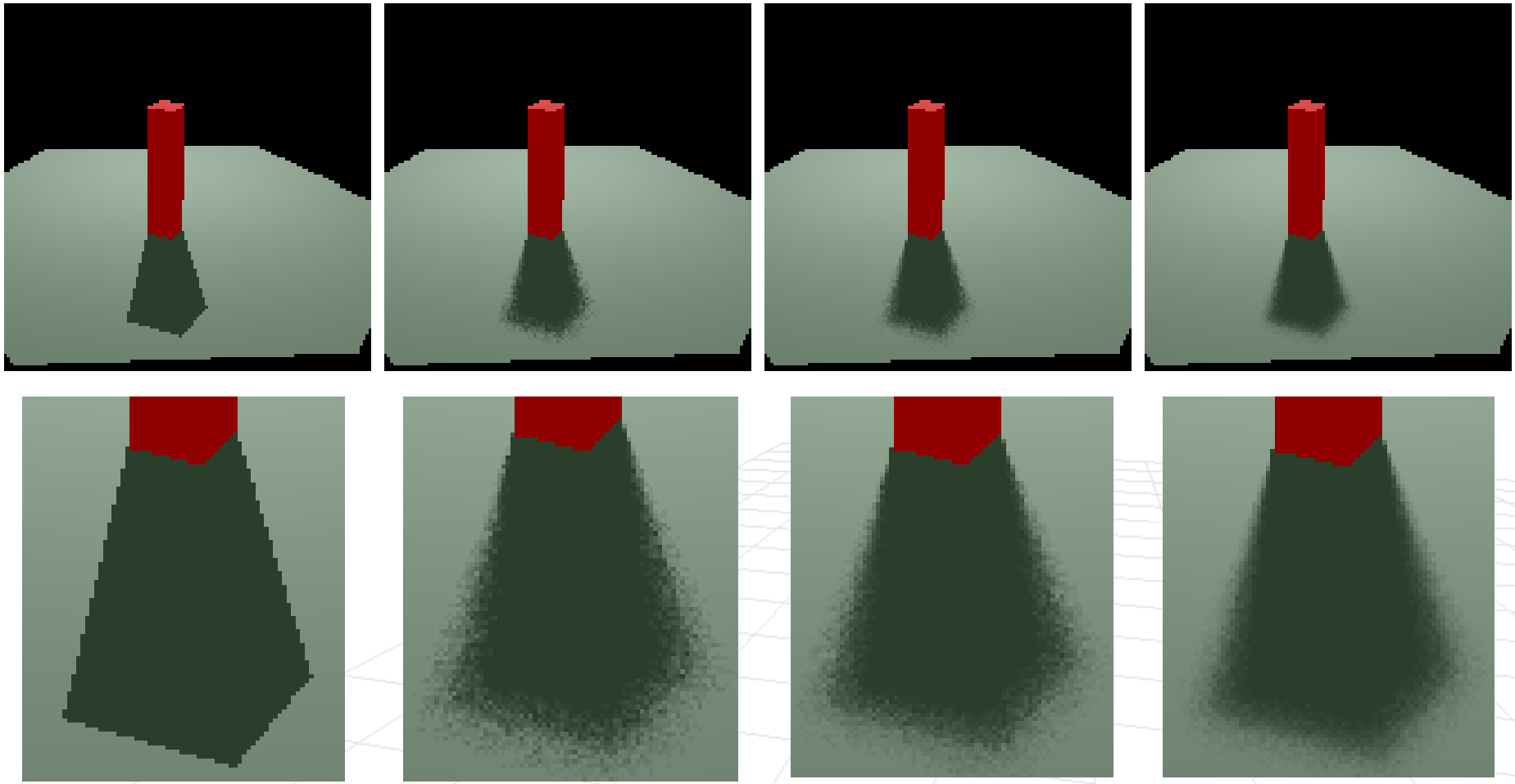
How many rays do you need?

1 ray/light

10 ray/light

20 ray/light

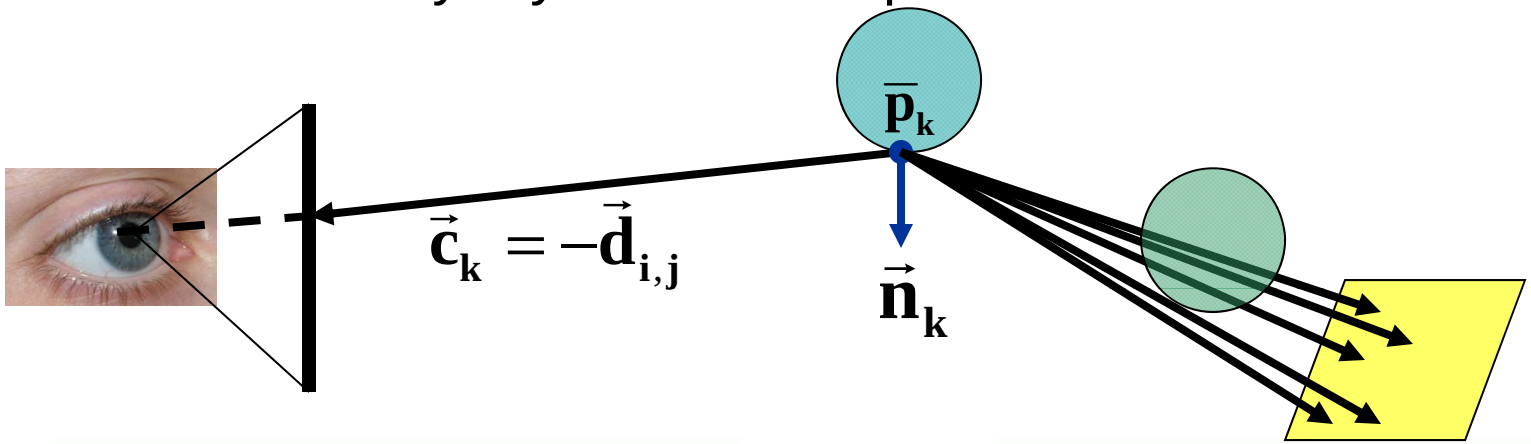
50 ray/light

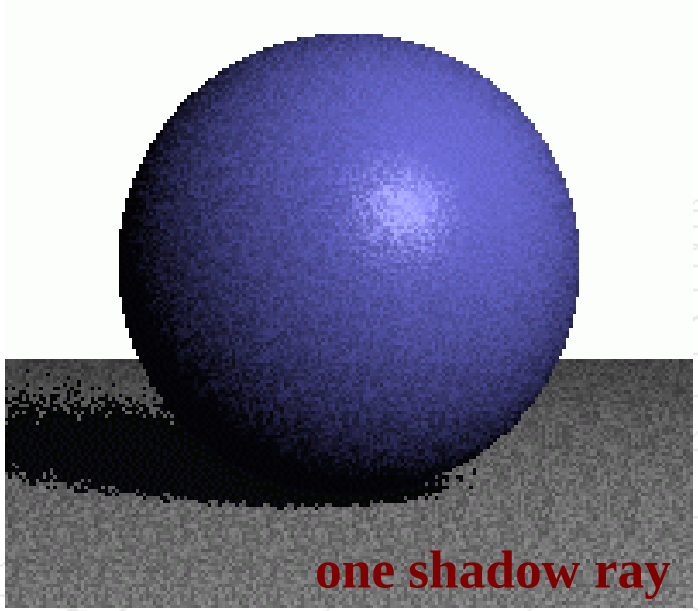


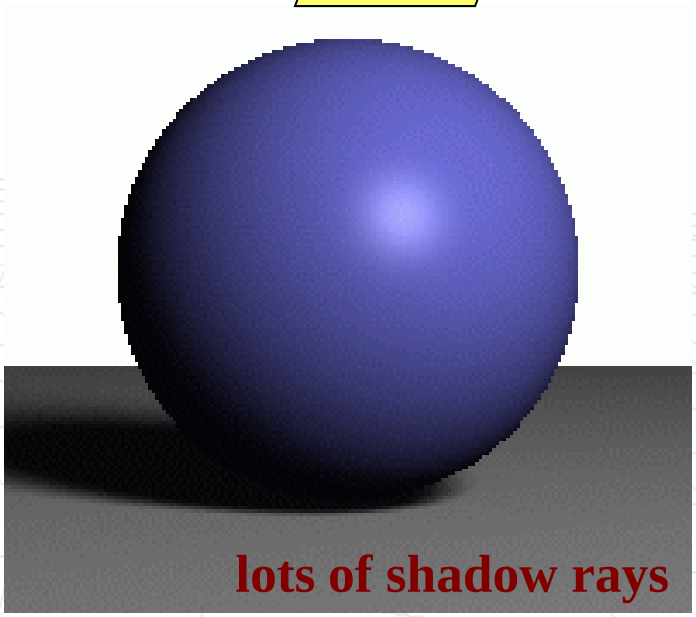
Images taken from http://web.cs.wpi.edu/~matt/courses/cs563/talks/dist_ray/dist.html

Soft Shadows with Distribution Ray Tracing

- Lets shoot multiple rays from the same point and attenuate the color based on how many rays are intercepted

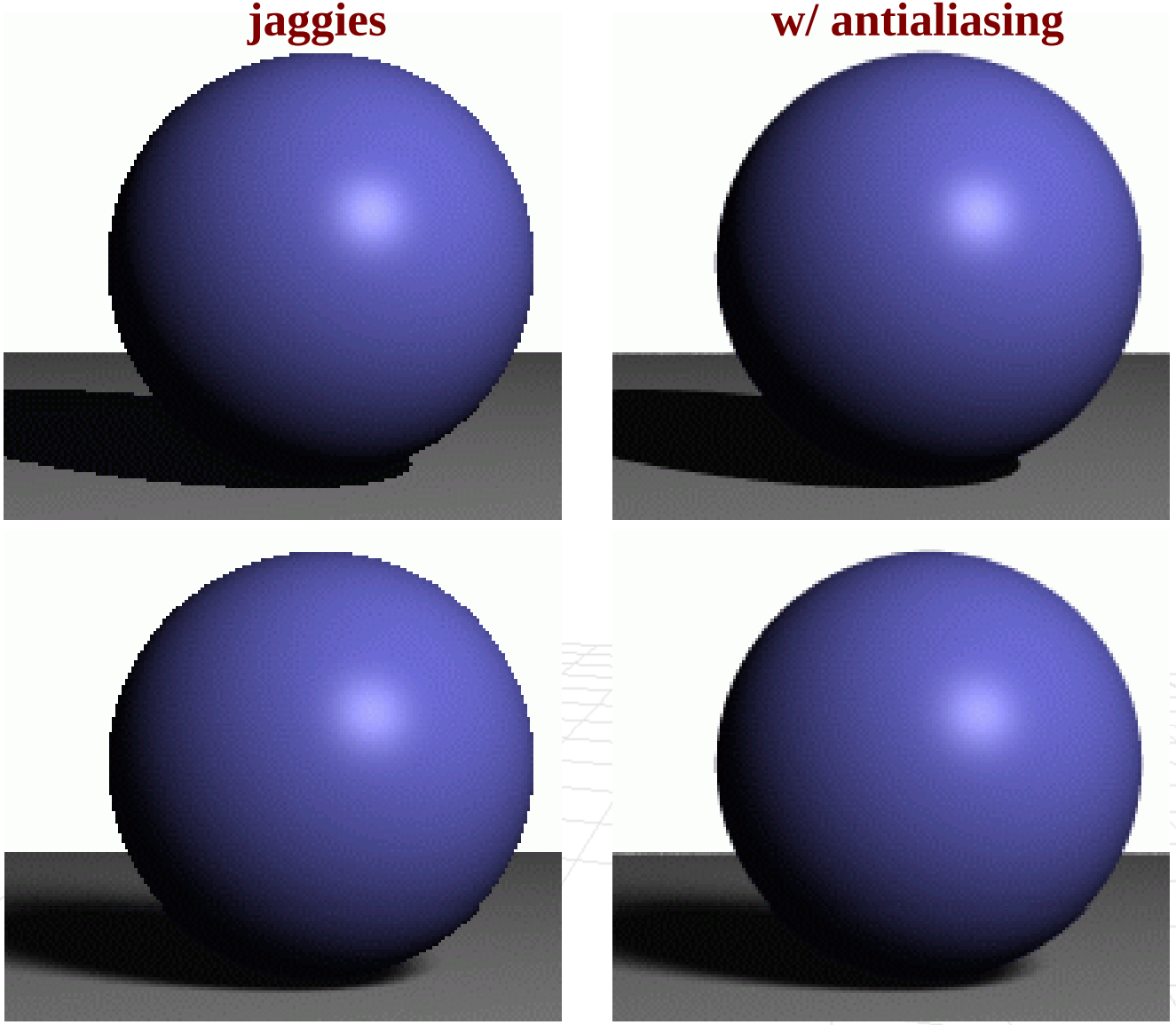






Images from the slides by Durand and Cutler

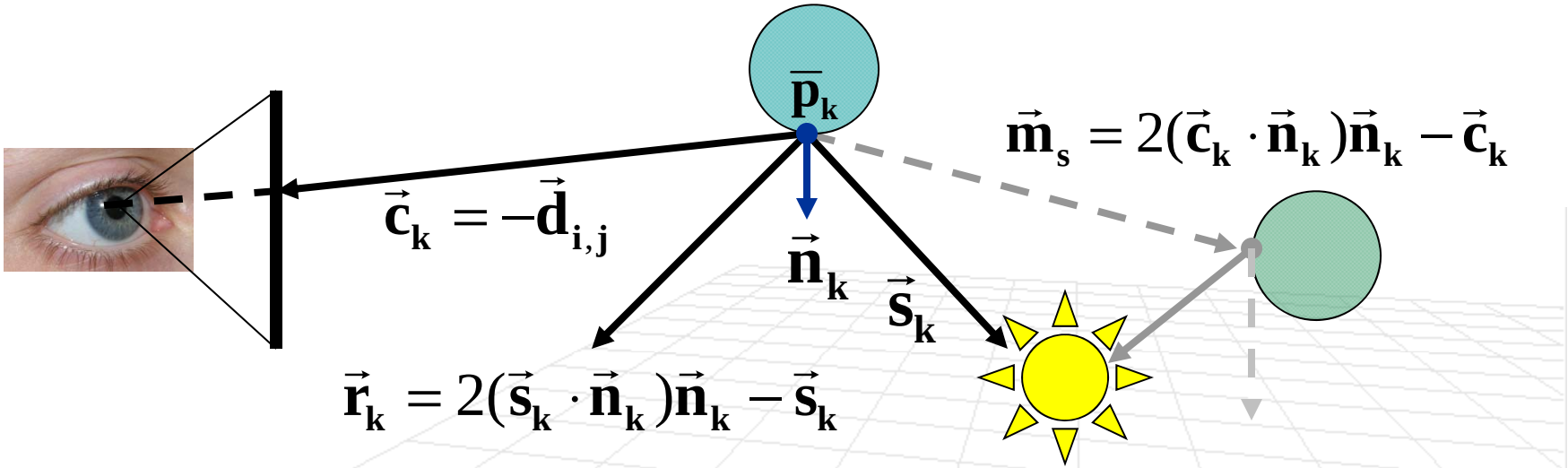
Antialiasing – Supersampling



Images from the slides by Durand and Cutler

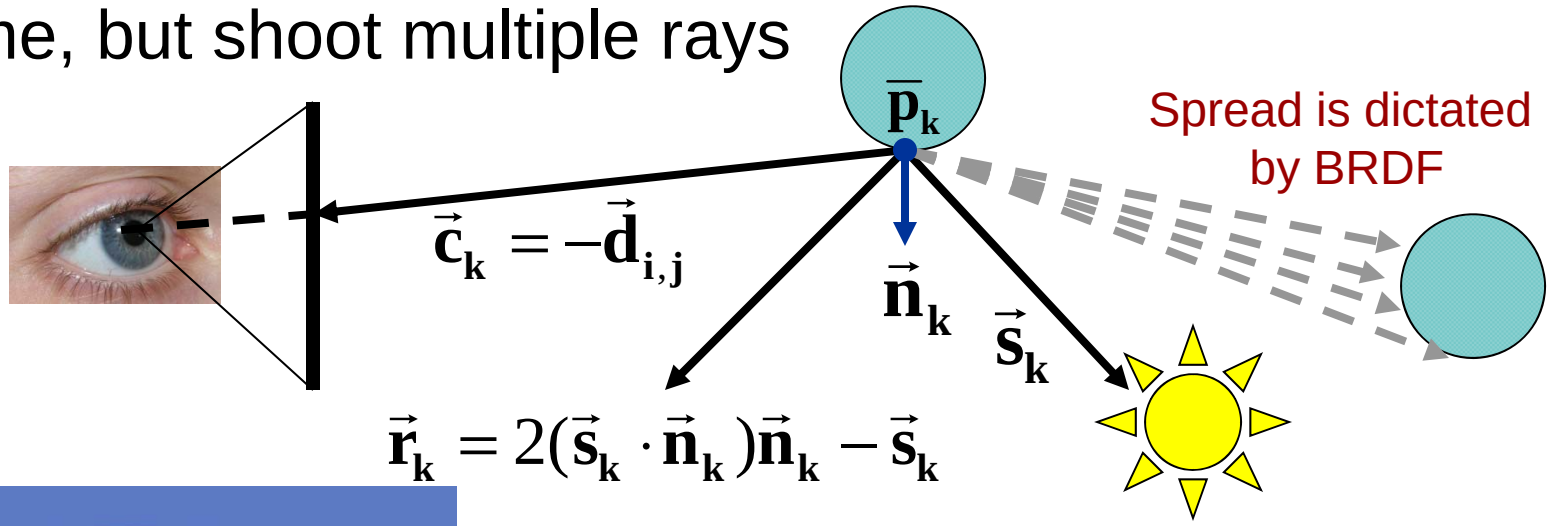
Specular Reflections

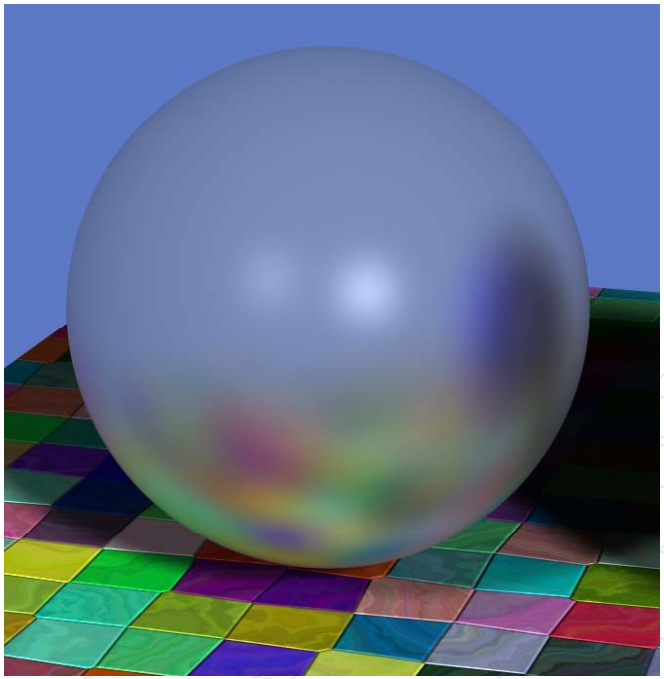
- Recall, we had to shoot a ray in a perfect specular reflection direction (with respect to the camera) and get the radiance at the resulting hit point



Specular Reflections with DRT

- Same, but shoot multiple rays





Justin Legakis

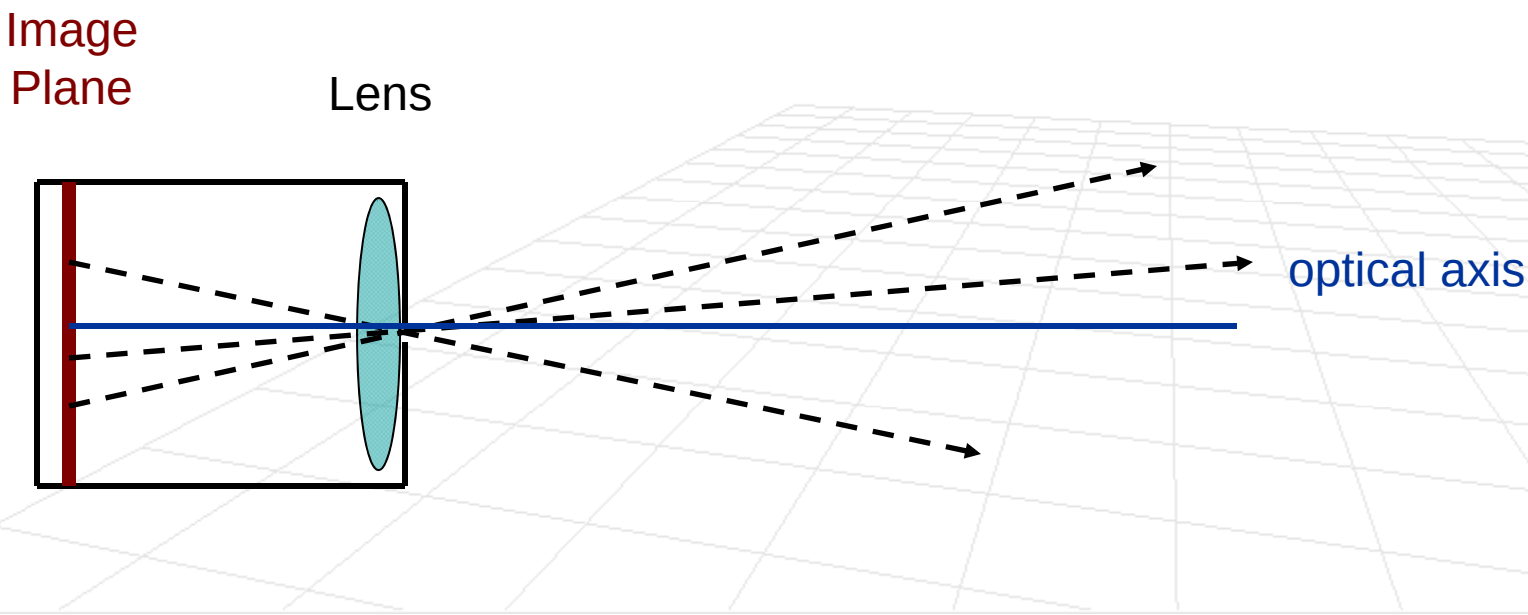
Perfect Reflections (Metal)

Perfect Reflections (glossy polished surface)

[Jensen]

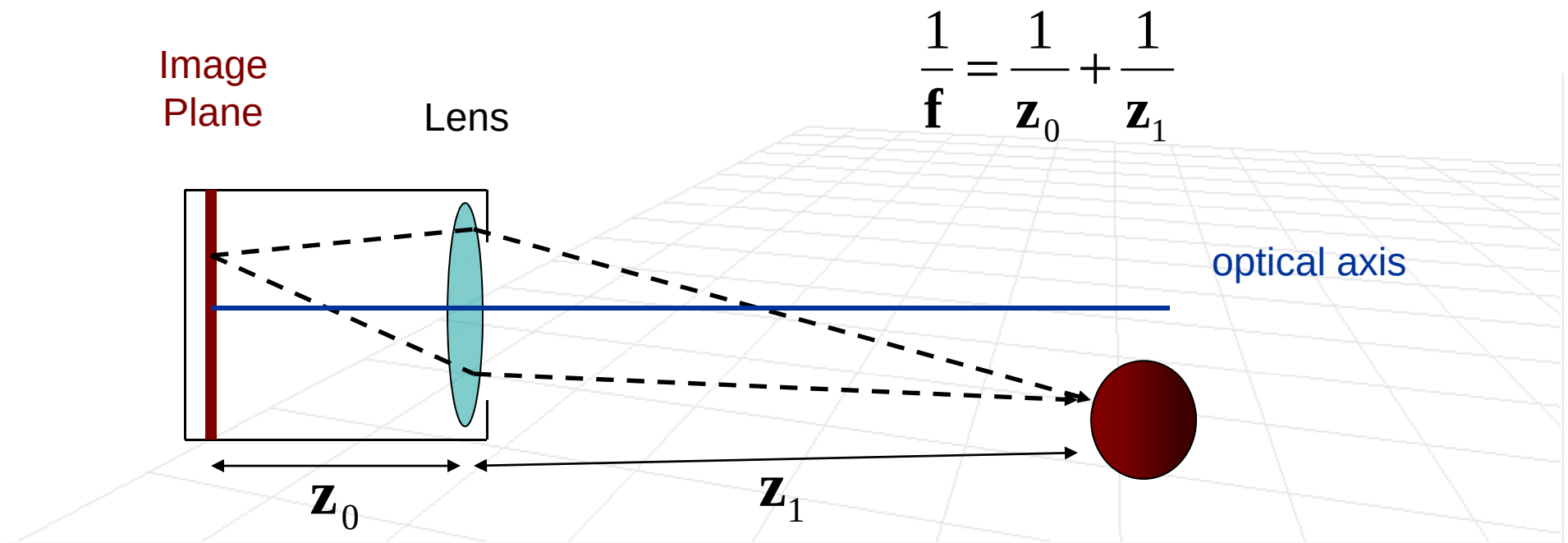
Depth of Field

- So far with our Ray Tracers we only considered **pinhole camera model** (no lens)
  - or alternatively, lens, but tiny aperture



Depth of Field

- So far with our Ray Tracers we only considered pinhole camera model (no lens)
  - or alternatively, lens, but tiny aperture
- What happens if we put a lens into our “camera”
  - or increase the aperture
- Remember the thin lens equation?



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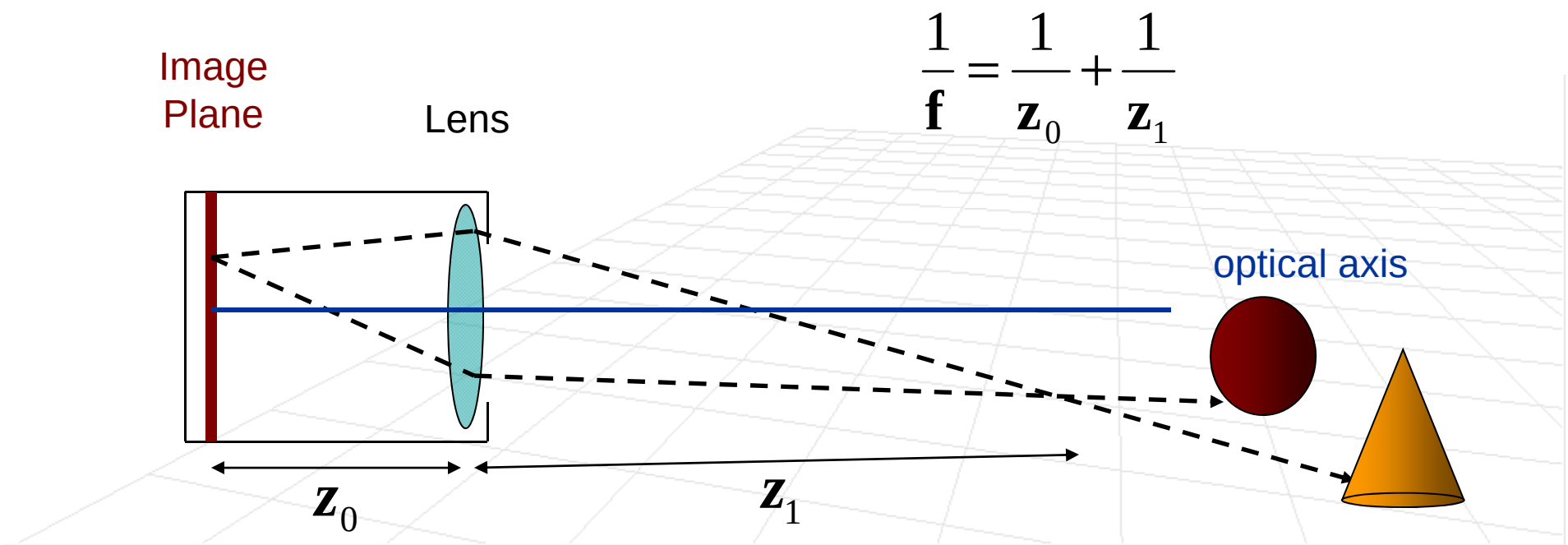
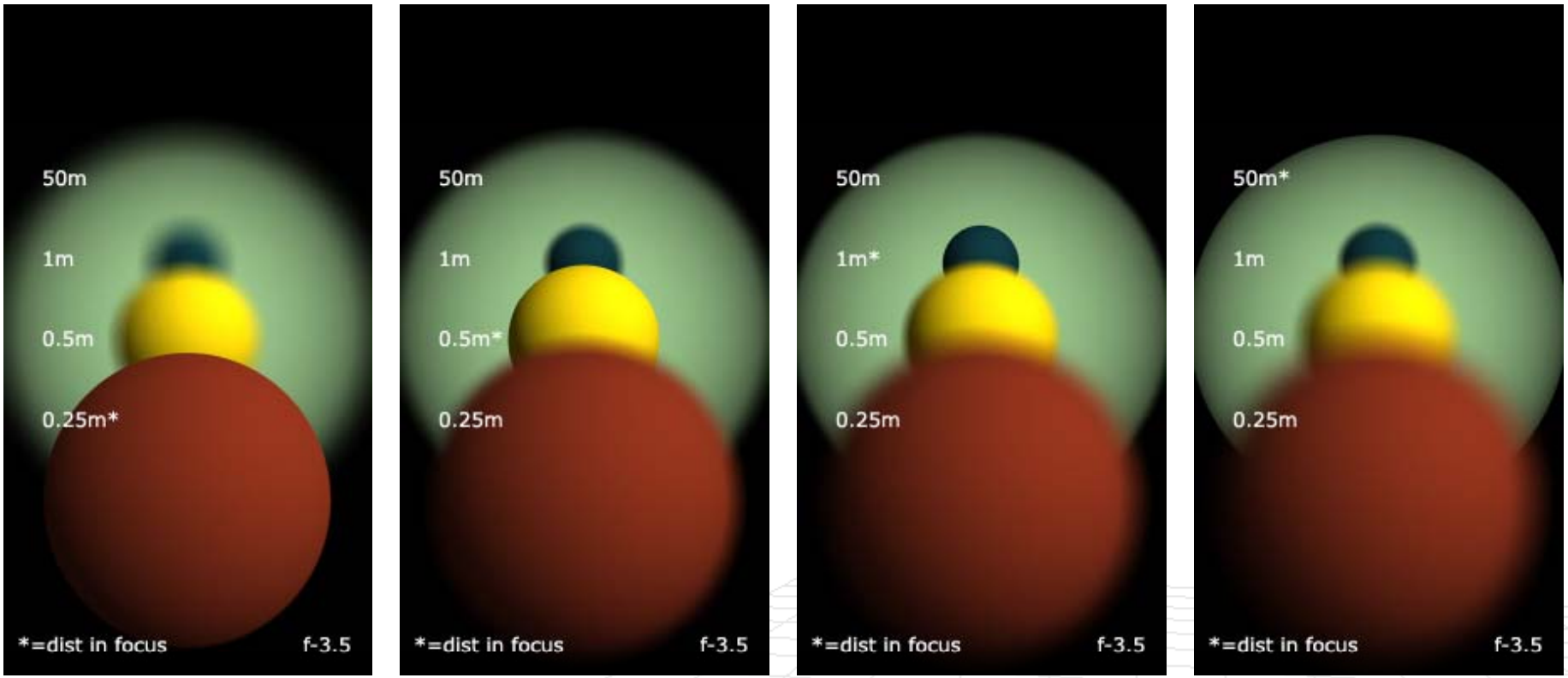
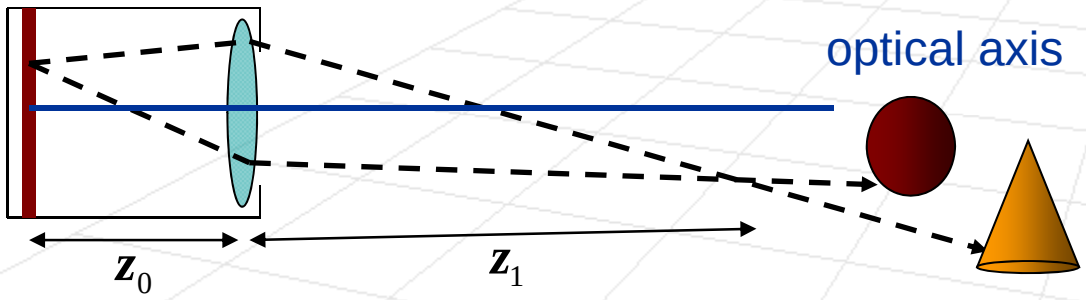


Changing the focal-length in DRT

increasing focal length

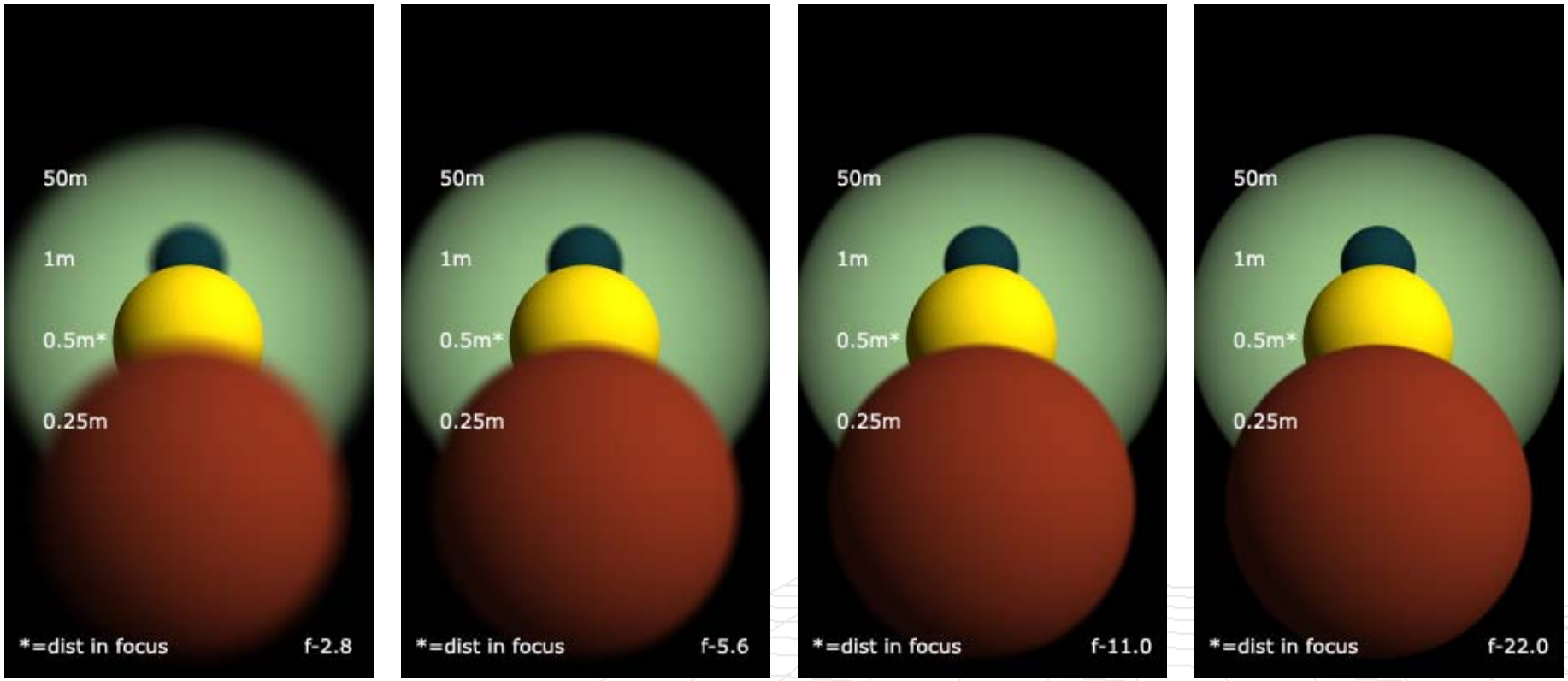


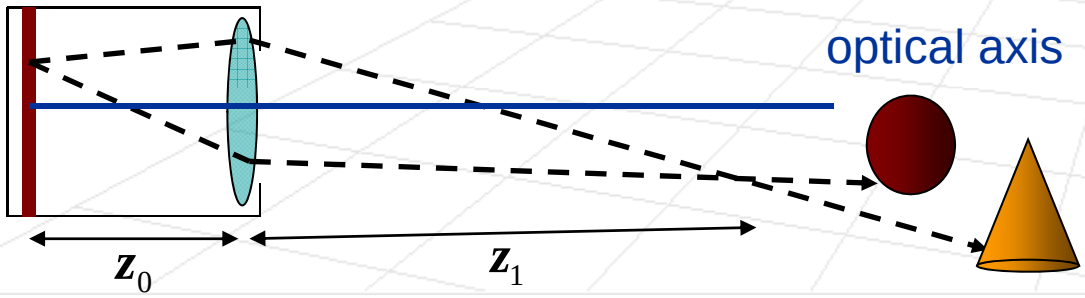


220x400 pixels
144 samples per pixel
~4.5 minutes to render

Changing the aperture in DRT

decreasing aperture





220x400 pixels
144 samples per pixel
~4.5 minutes to render

Depth of Field

P. Haeberli

Depth of Field

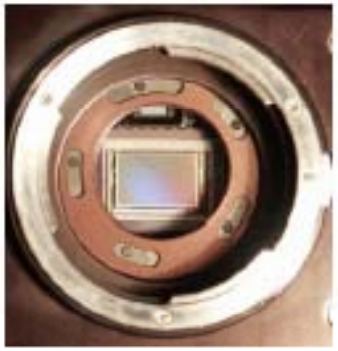
Depth of Field

Depth of Field

Depth of Field

Camera Shutter

Closed



Open

- We ignored the fact that **it takes time to form the image**
  - We ignored this for radiometry

- During that time the shutter is open and light is collected
  - We need to **integrate temporally**, not only spatially

$$\int_t \int_\alpha \int_\beta H(\alpha, \beta, t)\, d\alpha\, d\beta\, dt$$

Motion Blur

Cook, Porter & Carpenter

Motion Blur

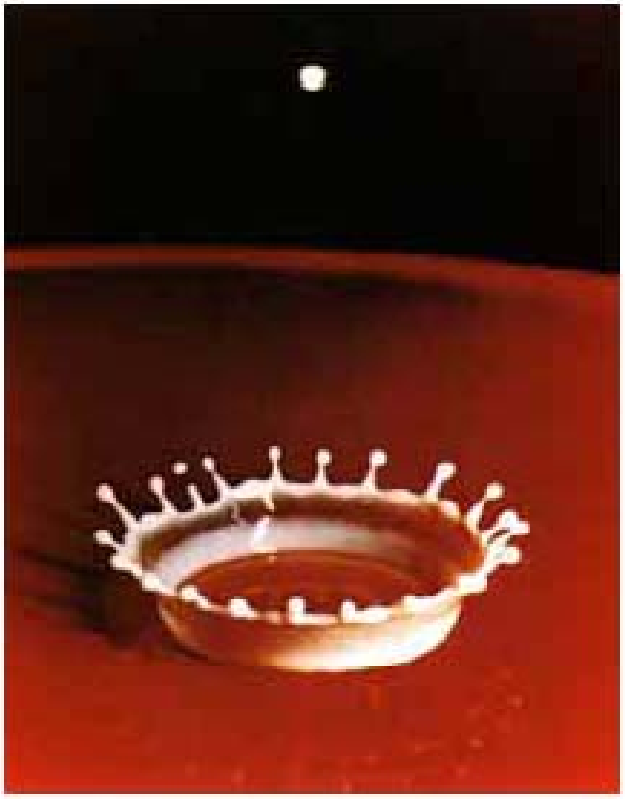
Long Exposure Photography

Motion Blur (long exposures)

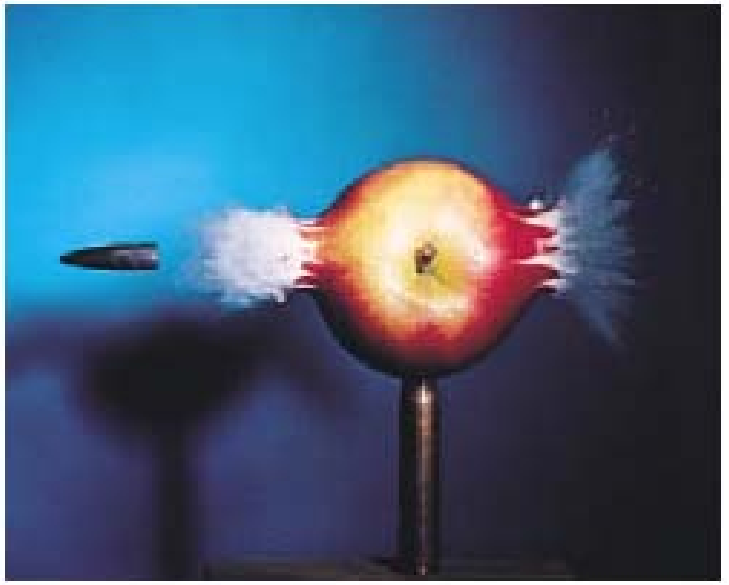
Golden Gate Bridge
30 sec. exposure @ f4

Bodie State Park
30 min. exposure @ f4

Motion Blur (short exposures)



Doc Edgerton, 1936



Sub-surface Scattering

H. W. Jensen

Sub-surface Scattering

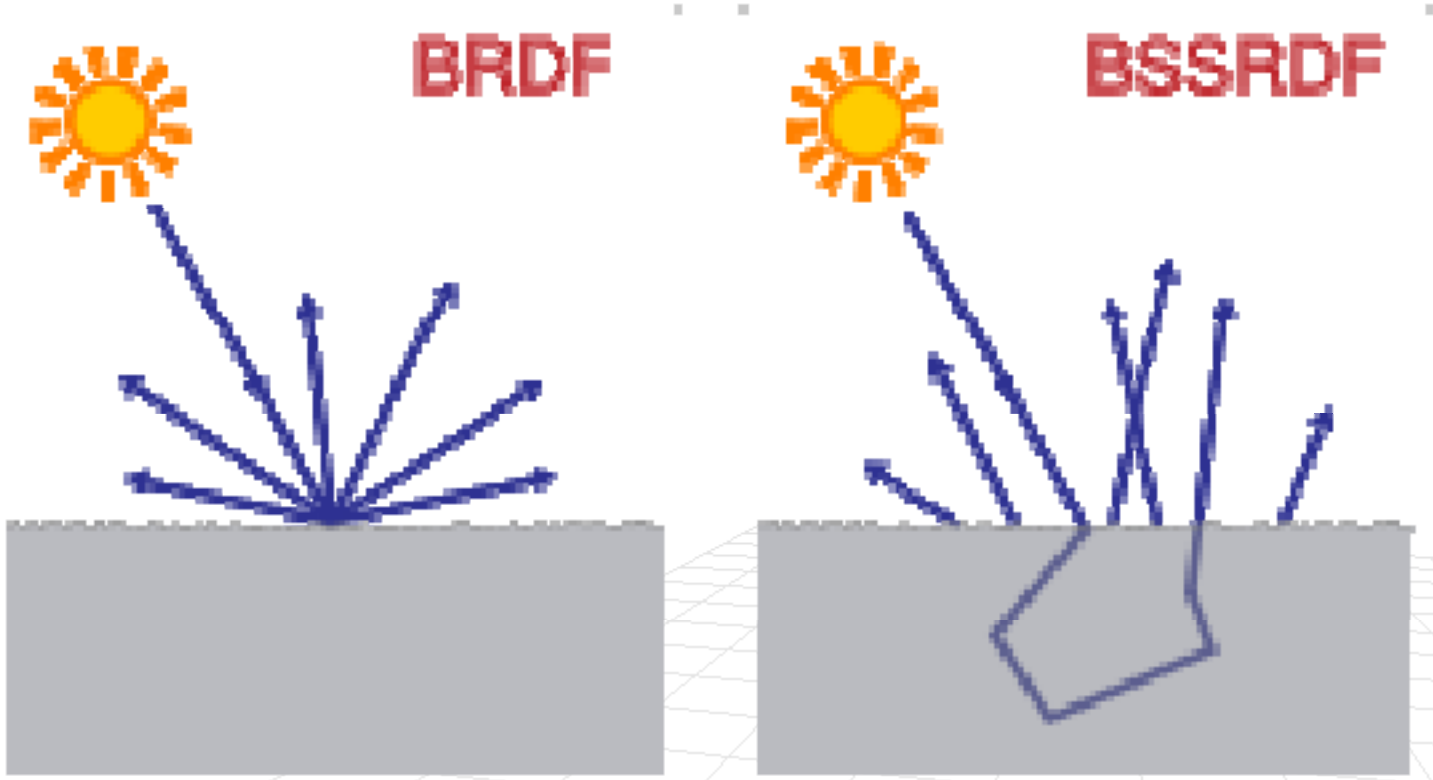
Bidirectional Surface Scattering Reflectance Distribution Function

Rendering with BRDF

Rendering with BSSRDF

H. W. Jensen

Bidirectional Surface Scattering Reflectance Distribution Function



[Images taken from Wikipedia]

Semi-Transparencies

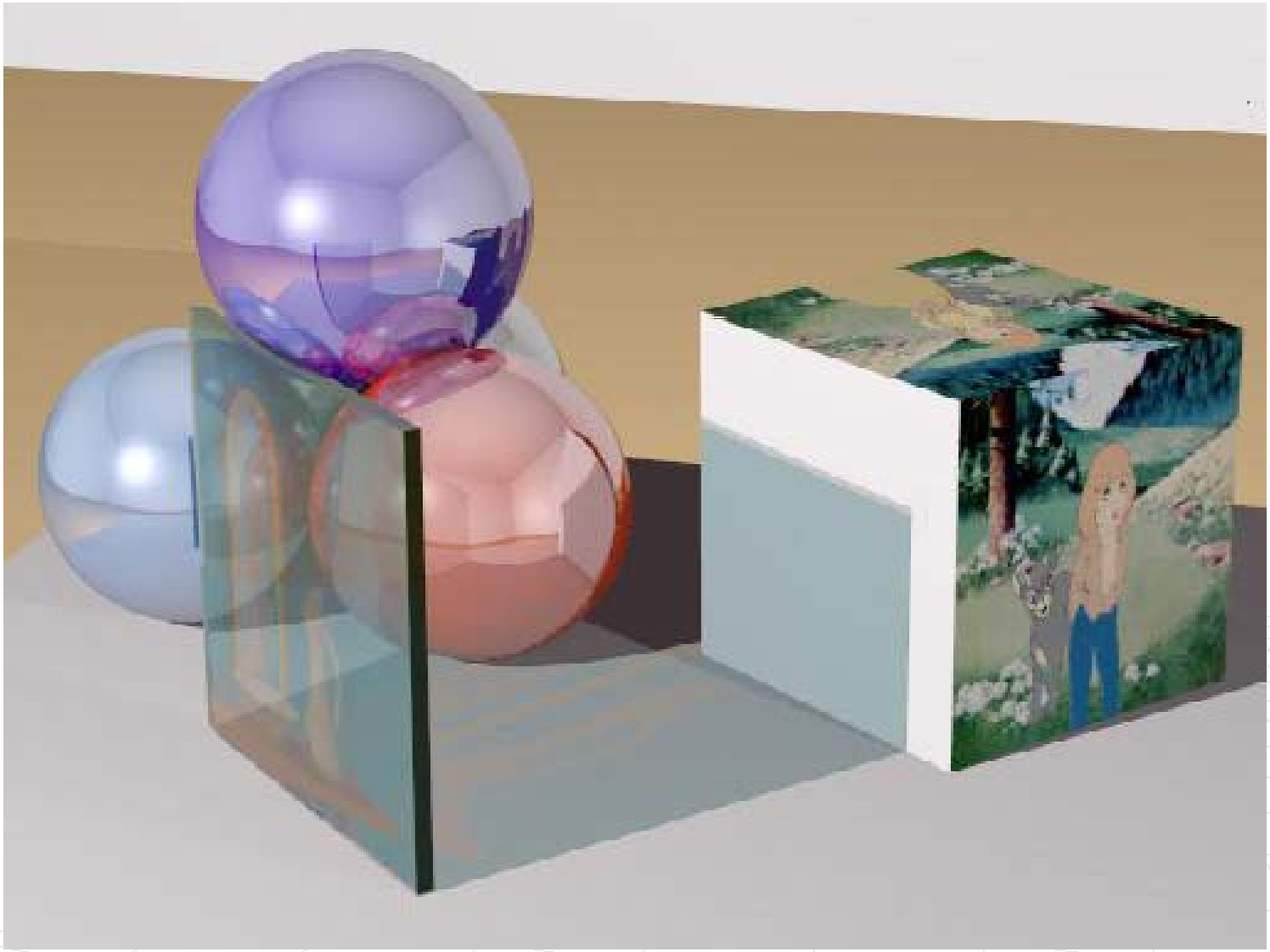


Image form http://www.graphics.cornell.edu/online/tutorial/raytrace/

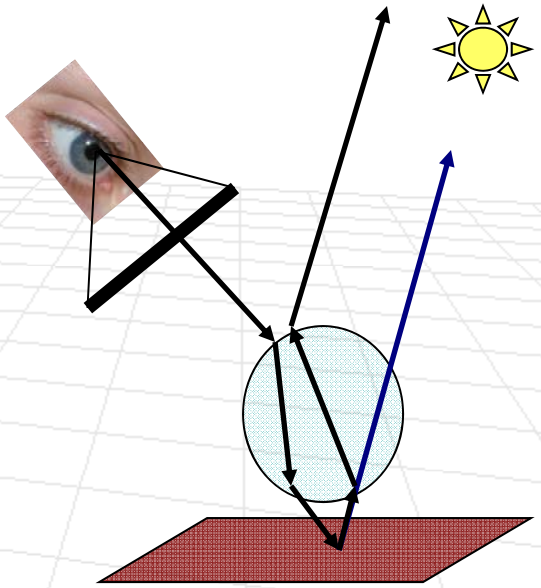
Caustics

- Hard to do in Distribution Ray Tracing
  - **Why?**

Caustics

- Hard to do in Distribution Ray Tracing
  - **Why?**

Hard to come up with a good importance function for sampling,
Hence, VERY VERY slow



Caustics

- Often done using bi-directional ray tracing (a.k.a. **photon mapping**)
  - Shoot light rays from light sources
  - Accumulate the amount of light (radiance) at each surface
  - Shoot rays through image plane pixels to “look-up” the radiance (and integrate irradiance over the area of the pixel)

Photon Mapping

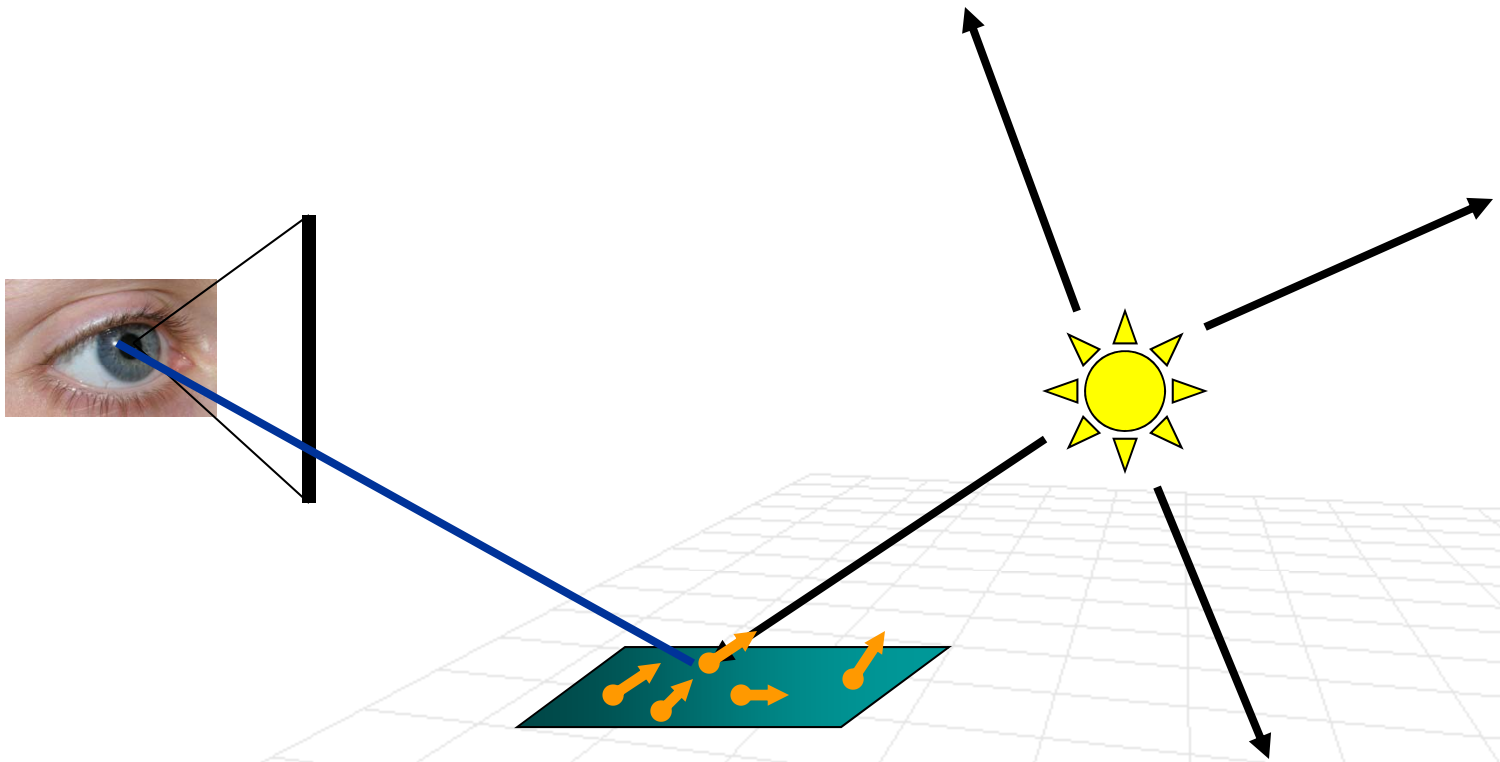
- Simulates **individual photons**
  - In DTR we were simulating radiance (flux)
- Photons are emitted from light sources
- Photons bounce off of specular surfaces
- Photons are **deposited on diffuse surfaces**
  - Held in a 3-D spatial data structure
  - Surfaces need not be parameterized
- Photons **collected by ray tracing** from eye

Photons

- A **photon** is a particle of light that carries flux, which is encoded as follows
  - magnitude (in Watts) and color of the flux it carries, stored as an RGB triple
  - location of the photon (on a diffuse surface)
  - the incident direction (used to compute irradiance)

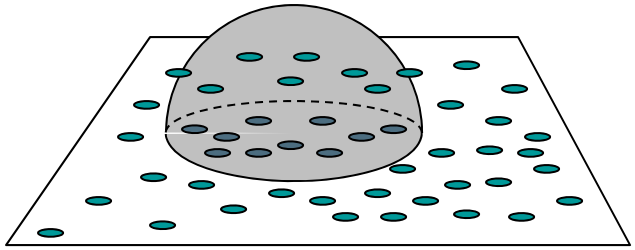
- **Example** (point light source, photons emitted uniformly)
  - Power of source (in Watts) distributed evenly among photons
  - Flux of each photon equal to source power divided by total # of photons
  - 60W light bulb would sending 100 photons, will result in 0.6 W per photon

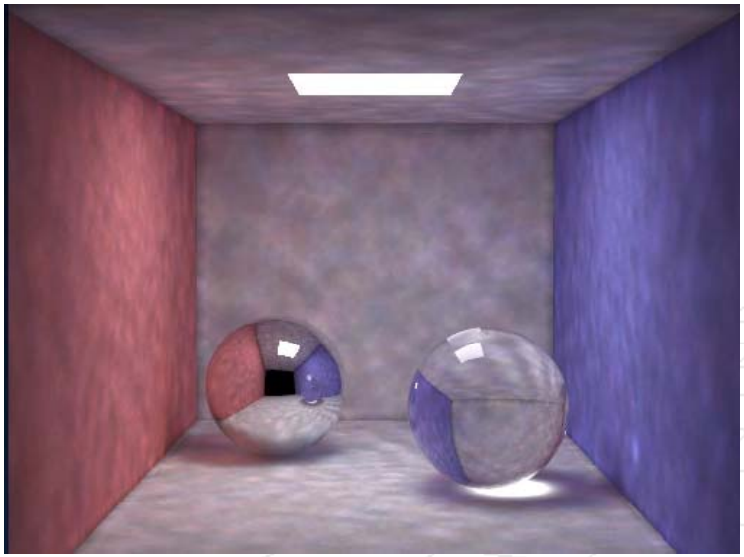
How does this actually work?



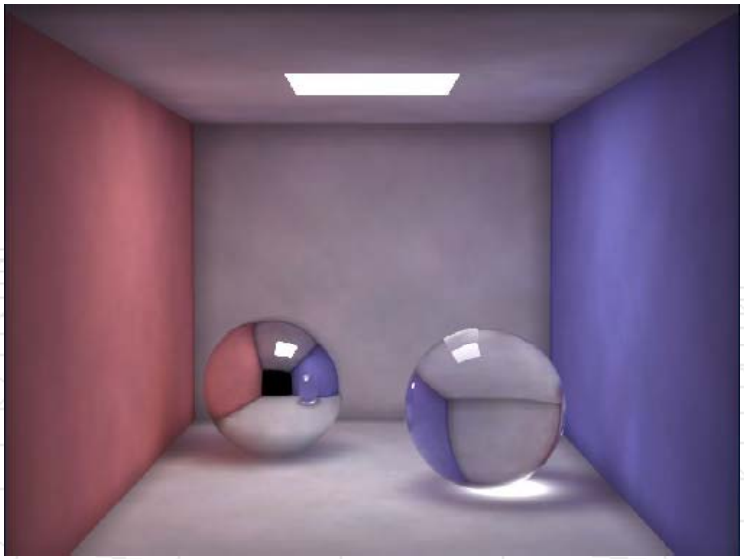
Special data structures are required to do fast look-up (KD-trees)

Photon Mapping Results





Radiance estimate using 50 photons



Radiance estimate using 500 photons