Today’s Topics

9. Lighting & Reflection models

10. Shading

Topic 13:

Radiometry

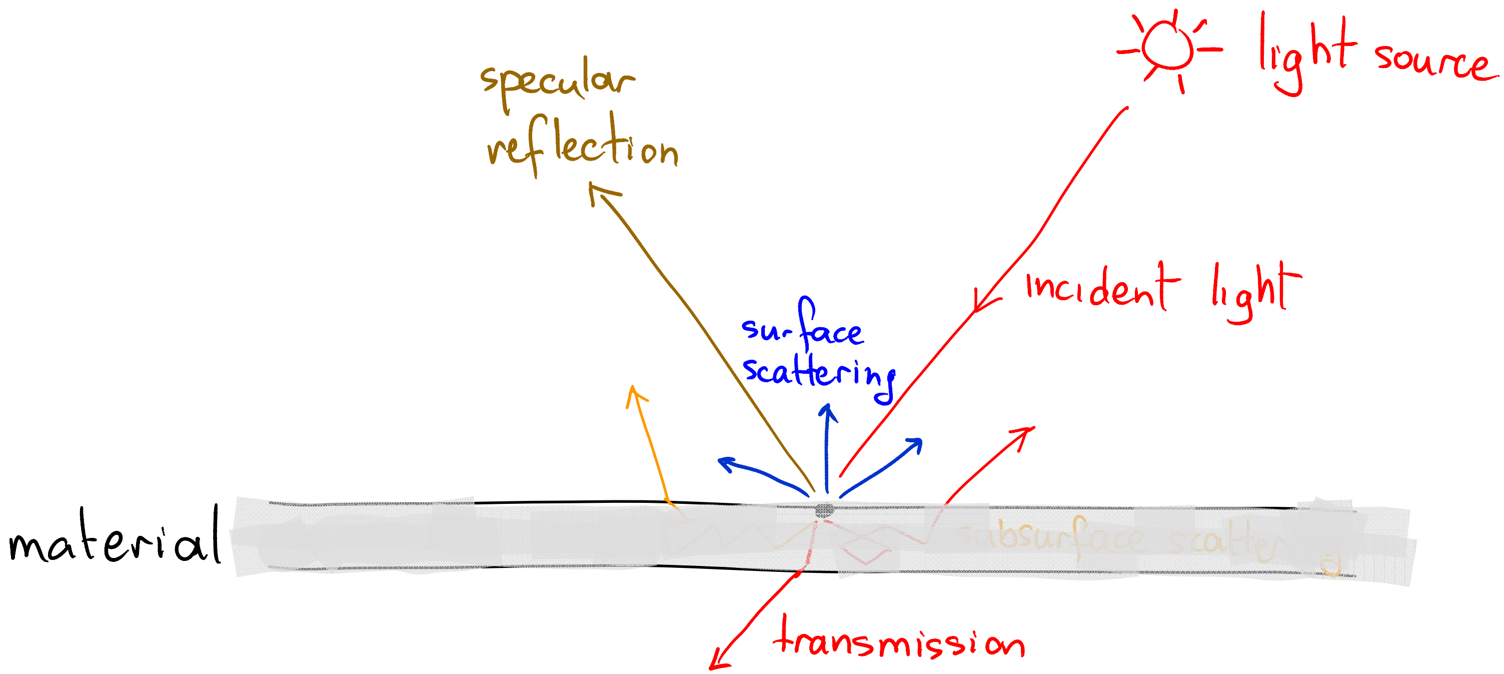
- The big picture
- Measuring light coming from a light source
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Light Transport Cycle
- The Bidirectional Reflectance Distribution Function

www.povray.org/community/hof

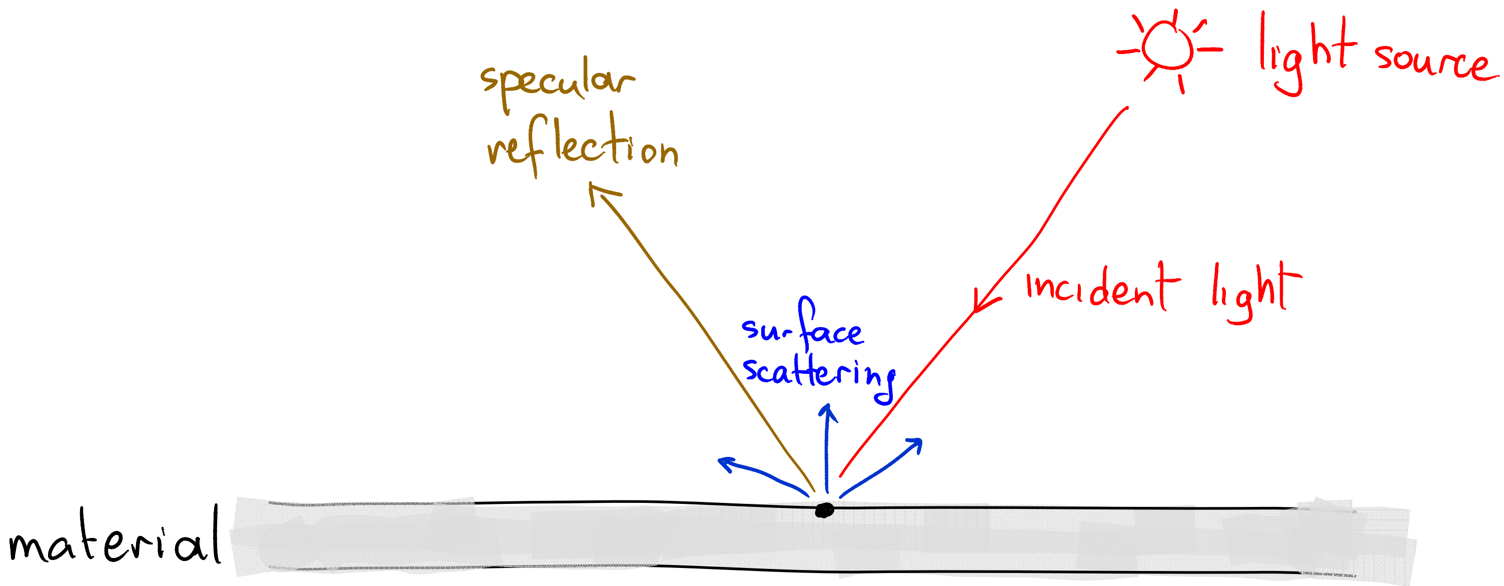
" Main steet (blue) "

" The Cool Cows "

The Common Modes of “Light Transport”



The Phong Reflectance Model

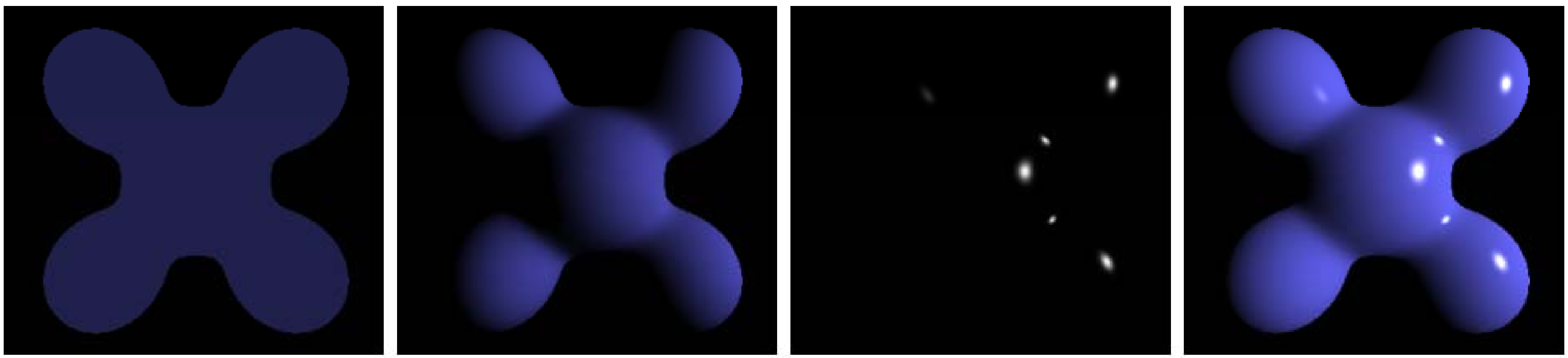


Phong model: A simple, computationally-efficient model that has 3 components:

- Diffuse
- Ambient
- Specular

Phong Reflection: The General Equation

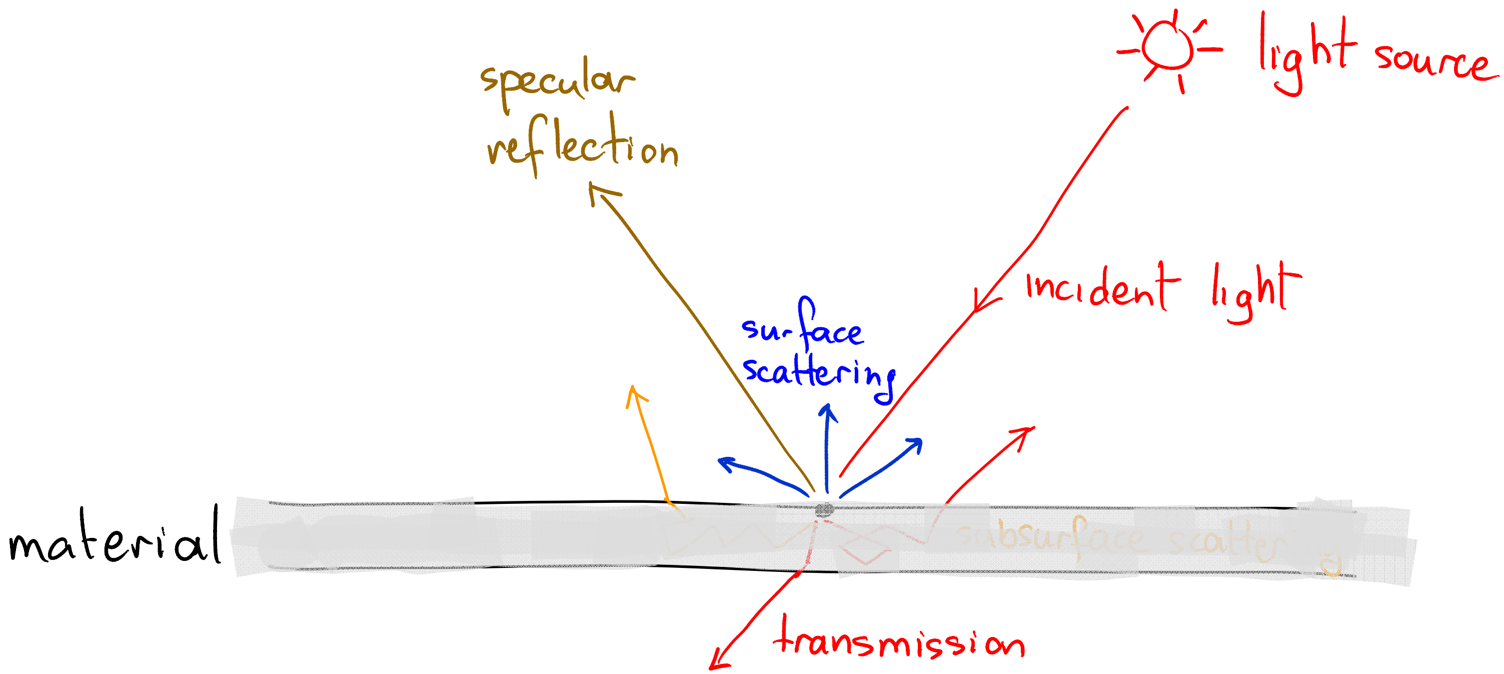
Brad Smith, Wikipedia



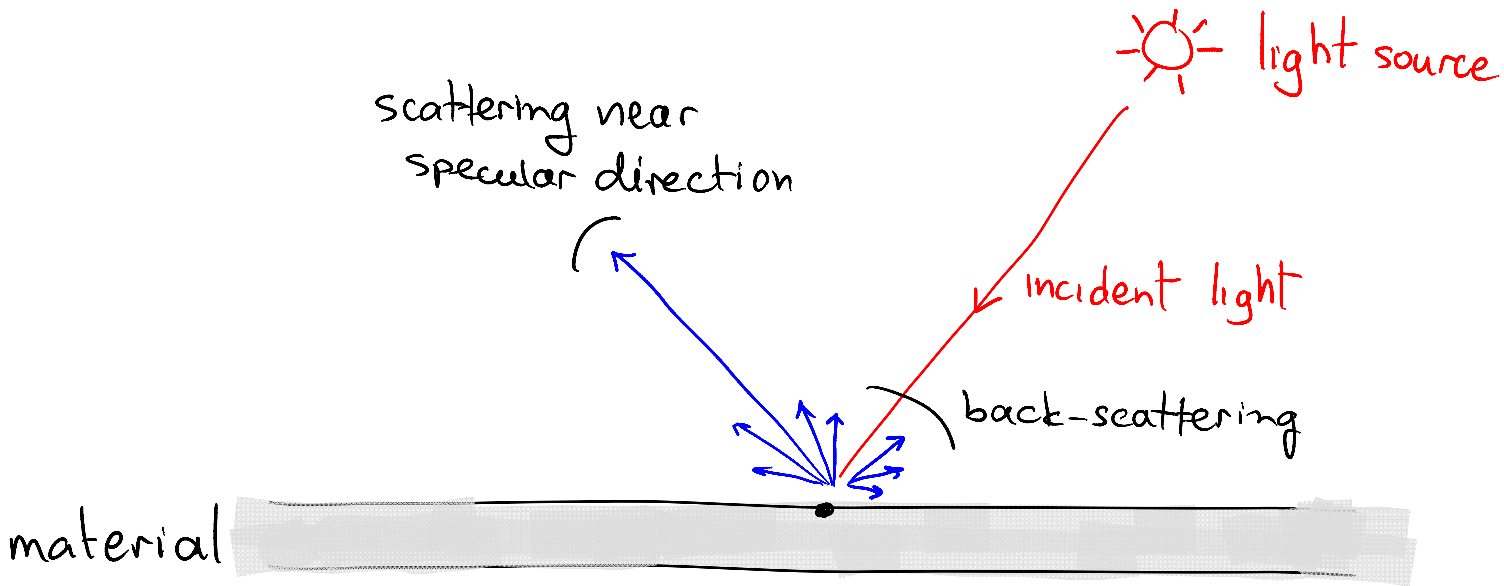
Ambient + **Diffuse** + **Specular** = **Phong Reflection**

$$\underbrace{L(\vec{b},\vec{n},\vec{s})}_{\text{intensity at projection of point } \bar{p}} = \underbrace{r_a I_\alpha}_{\text{ambient}} + \underbrace{r_d I_d \max(0,\vec{n}\cdot\vec{s})}_{\text{diffuse}} + \underbrace{r_s I_s \max(0,\vec{r}\cdot\vec{b})^{\alpha}}_{\text{specular}}$$

The Common Modes of “Light Transport”

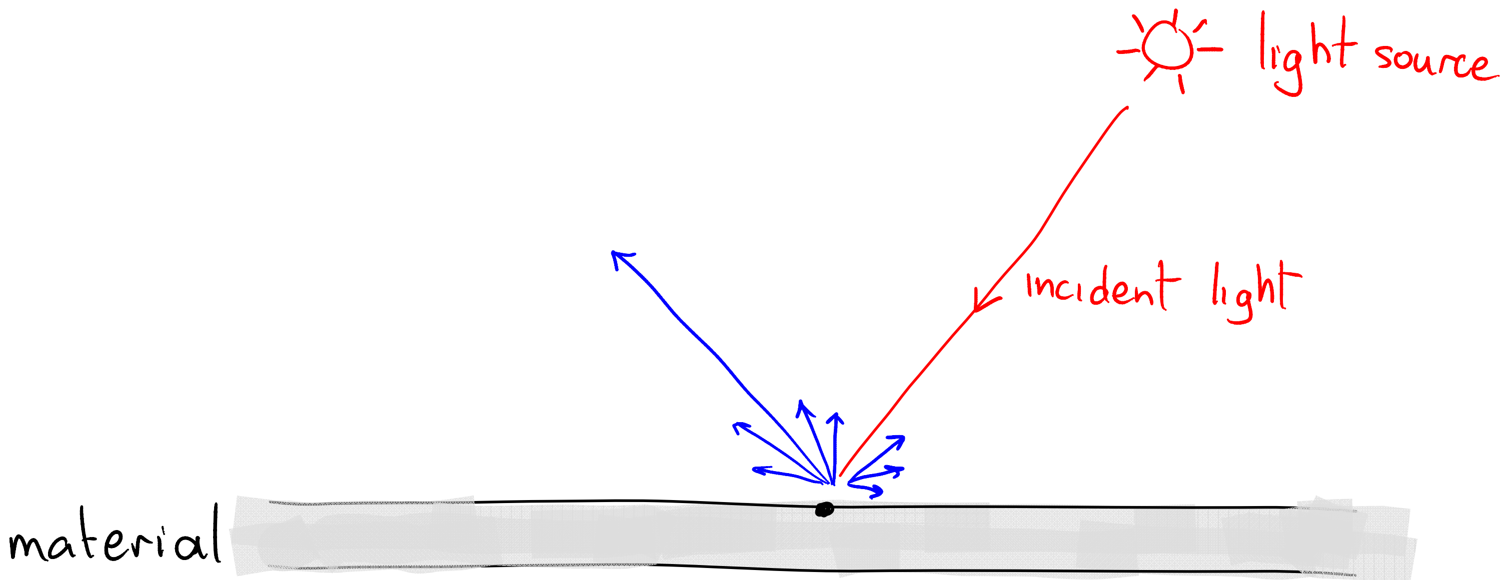


Generalizing the Phong Model



- All reflected light can be thought of as a form of scattering
- For most real materials, the Phong-based distinction into specular + diffuse reflection is a crude approximation

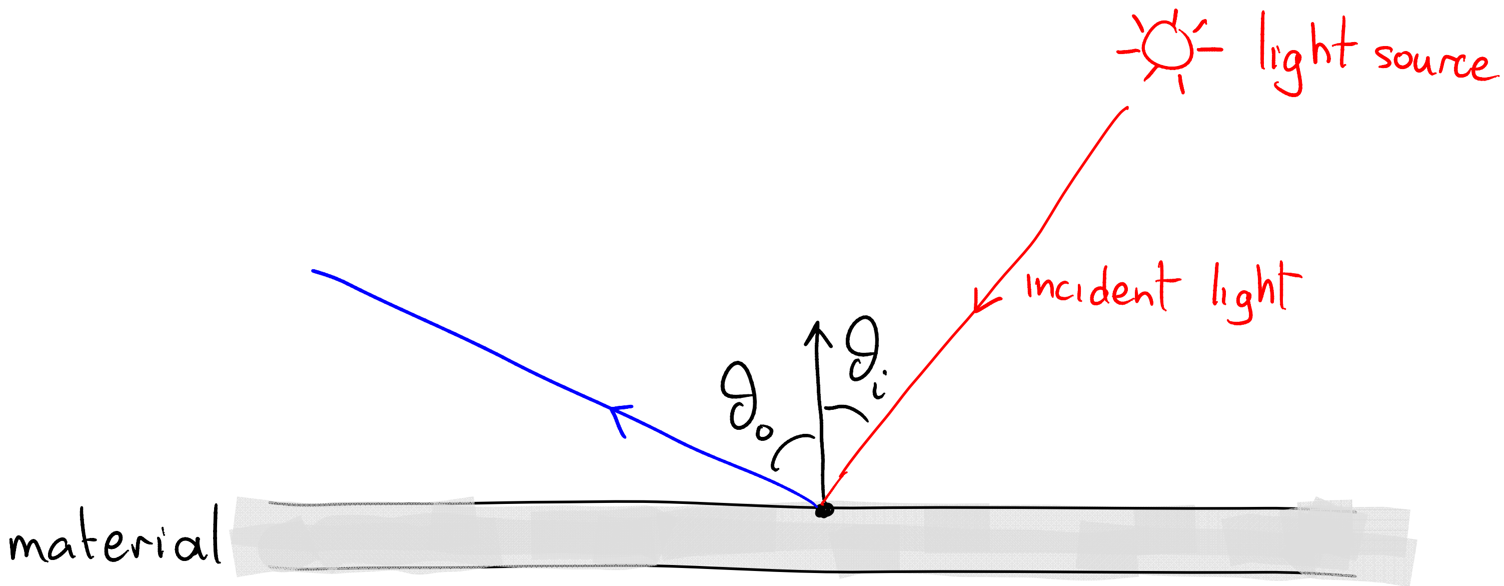
Generalizing the Phong Model: How?



. Seek to answer the following question:

given a specific incident direction
how much light is reflected along a
specific outgoing direction?

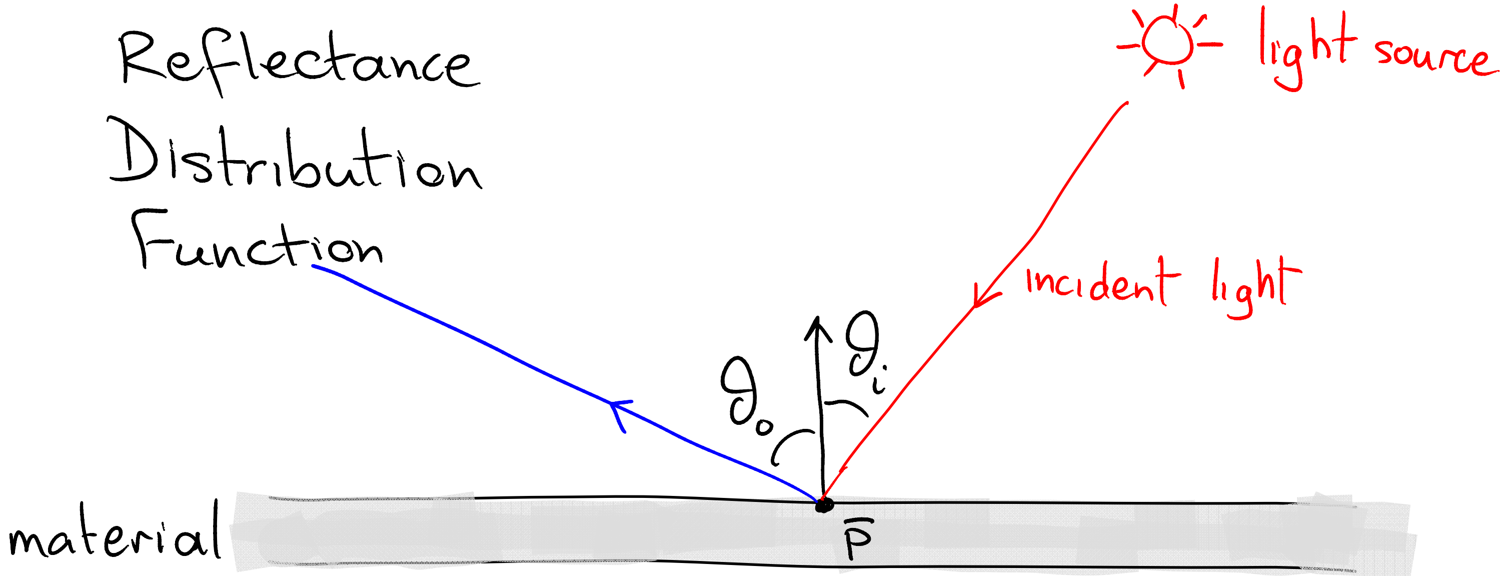
Generalizing the Phong Model: How?



- Seek to answer the following question:
  given a specific incident direction how much light is reflected along a specific outgoing direction?

The BRDF of a Surface Point (in 2D)

BRDF = Bidirectional
Reflectance
Distribution
Function



- It is a function $\rho_{\bar{p}} : [-\frac{\pi}{2}, \frac{\pi}{2}) \times [-\frac{\pi}{2}, \frac{\pi}{2}) \longrightarrow [0,1]$

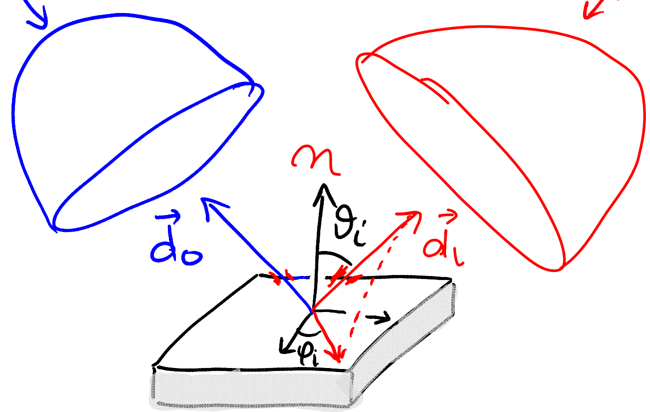
$$\rho_{\bar{p}}(\vartheta_i, \vartheta_o) = \frac{\text{emitted light in direction } \vartheta_o}{\text{incident light in direction } \vartheta_i}$$

ϑ_i: incoming direction; ϑ_o: outgoing direction

The BRDF of a Surface Point (in 3D)

hemisphere of all possible outgoing directions

(parameterized by two angles ϑ_o, φ_o)



hemisphere of all possible incoming directions

(parameterized by 2 angles ϑ_i, φ_i)

swap the range of ϑ_o, φ_o

$$\rho : [-\pi, \pi) \times [0, \tfrac{\pi}{2}) \times [-\pi, \pi) \times [0, \tfrac{\pi}{2}) \to [0,1]$$

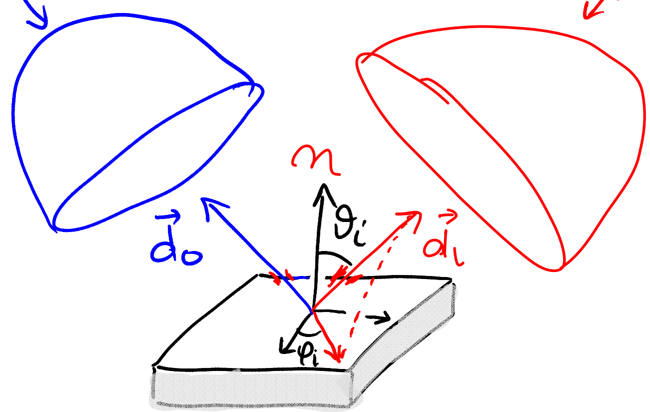
$$\rho(\vec{d}_i, \vec{d}_o) = \frac{\text{emitted light in direction } \vec{d}_o}{\text{incident light in direction } \vec{d}_i}$$

incoming direction   outgoing direction

The BRDF of a Surface Point (in 3D)

hemisphere of all possible outgoing directions

(parameterized by two angles ϑ_o, φ_o)



hemisphere of all possible incoming directions

(parameterized by 2 angles ϑ_i, φ_i)

$$\rho : [-\pi,\pi) \times [0, \tfrac{\pi}{2}) \times [-\pi,\pi) \times [0, \tfrac{\pi}{2}) \to [0,1]$$

$$\rho(\vartheta_i, \varphi_i, \vartheta_o, \varphi_o) = \frac{\text{emitted light in direction } \vartheta_o, \varphi_o}{\text{incident light in direction } \vartheta_i, \varphi_i}$$

incoming direction outgoing direction

Measuring BRDFs with a Gonioreflectometer

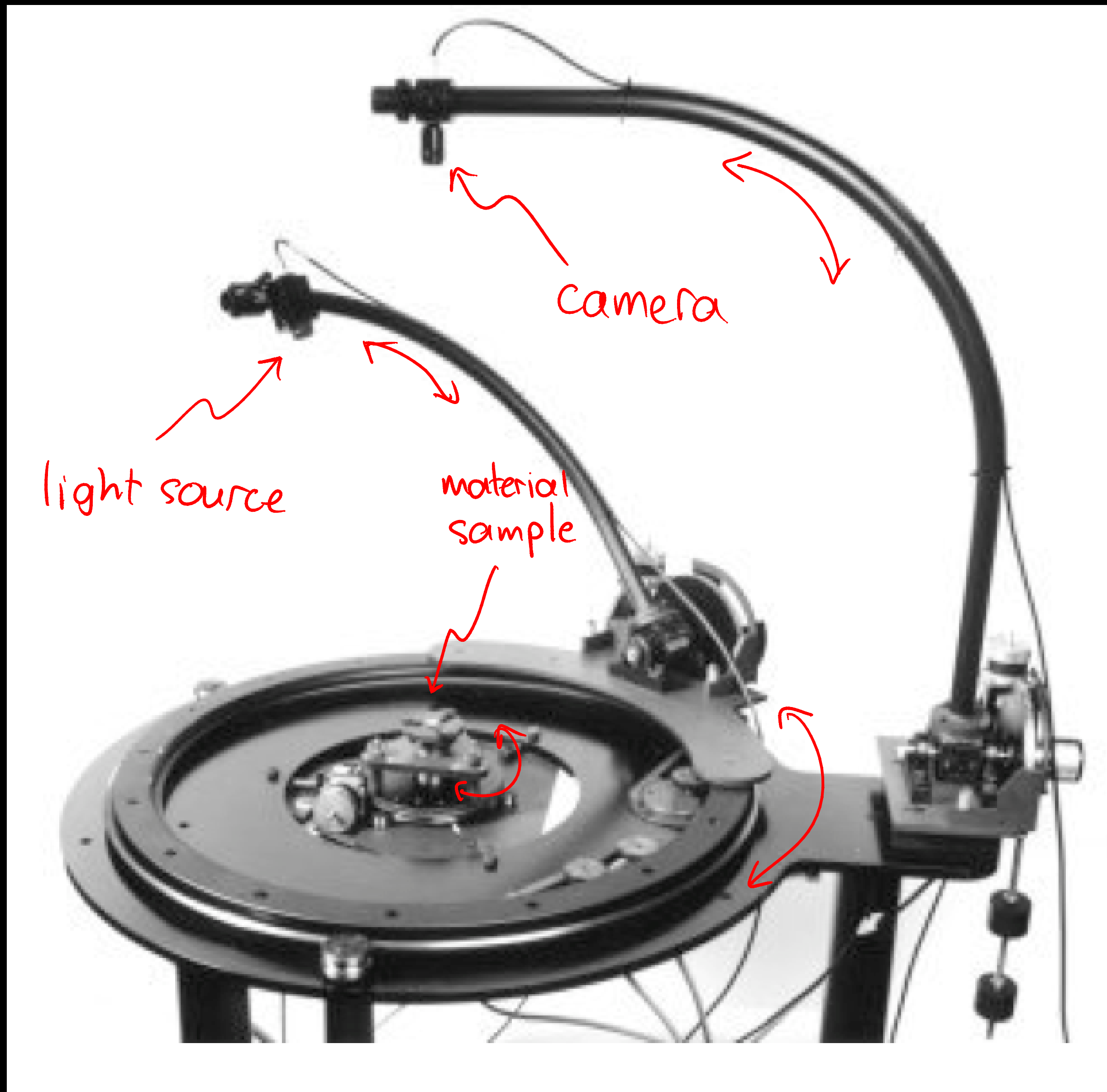
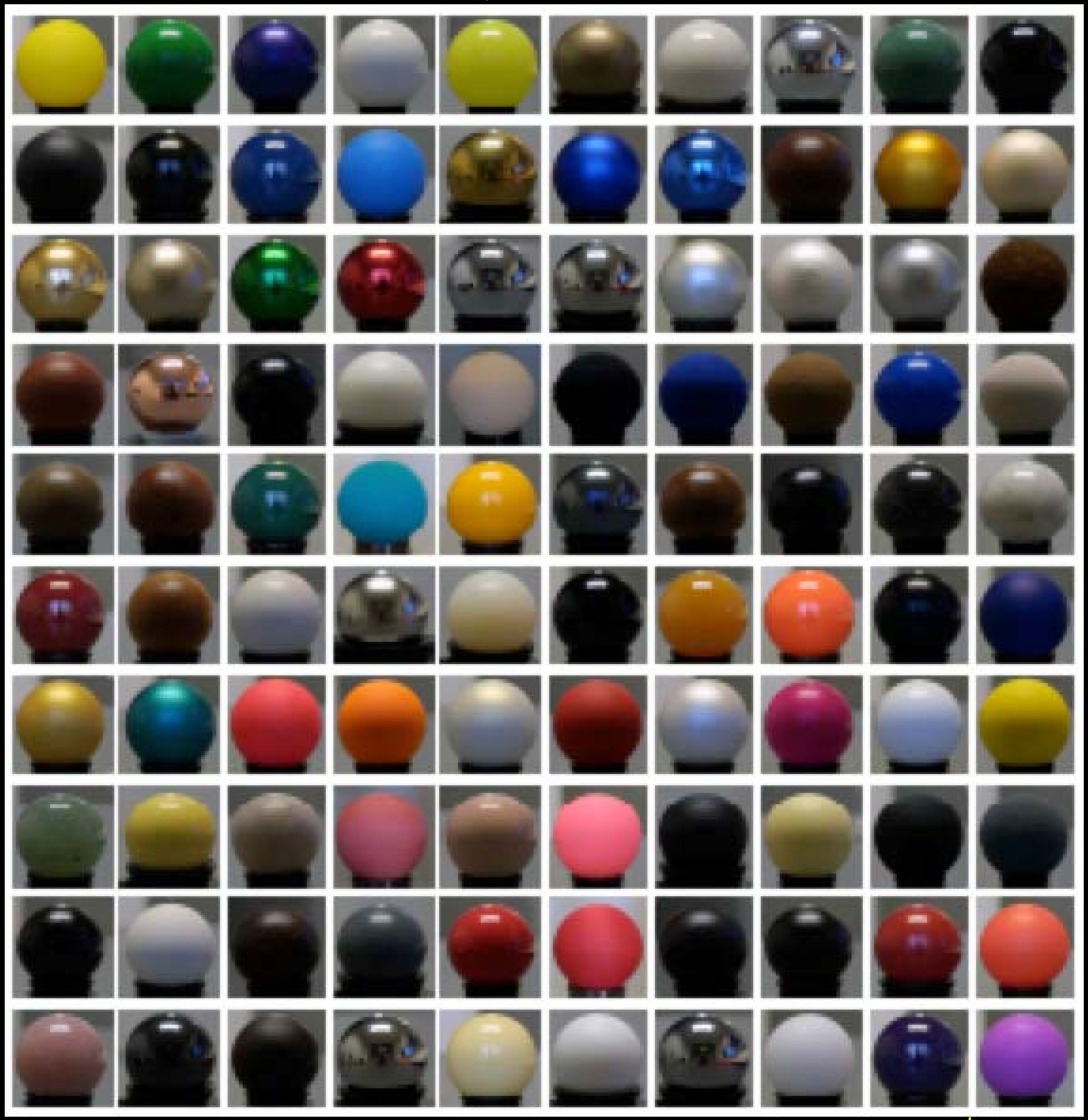


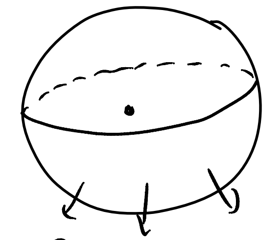
Photo: MSL New Zealand

Visualizing BRDFs

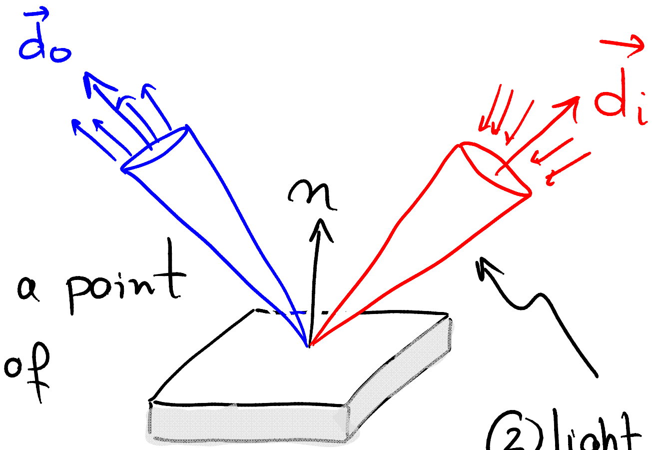


The MERL BRDF database

Five Issues We Have Ignored So Far...



① light sources emit light along a continuum of directions



③ light is reflected from a point to a continuous "cone" of directions

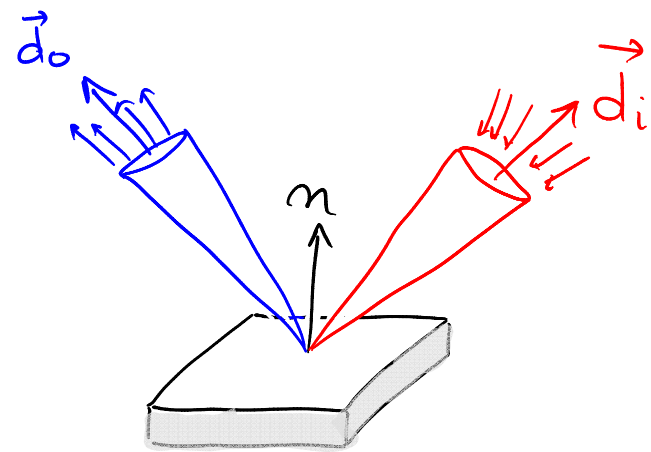
② light converges onto a point from a continuous "cone" of directions

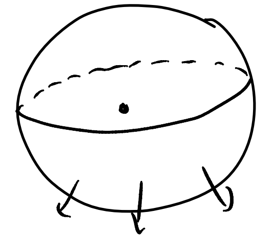
④ real objects are made of surfaces, not isolated points

$$\rho(\vec{d}_i, \vec{d}_o) = \frac{\text{emitted light in direction } \vec{d}_o}{\text{incident light in direction } \vec{d}_i}$$

⑤ must be expressed in terms of physical quantities

Radiometry: Getting the Physics Right





Radiometry: measurement of electromagnetic radiation

Physics:
- Joules
- Watts

Geometry:

differential
- patches
- directions
- solid angles
- foreshortened area

Radiometry:
- radiant energy
- radiant flux
- irradiance
- radiance
- radiant exitance
- BRDF

"Radiometrically-Correct" Ray Tracing

Basic loop:

for each pixel $\bar{q}$

① cast ray r through $\bar{q}$

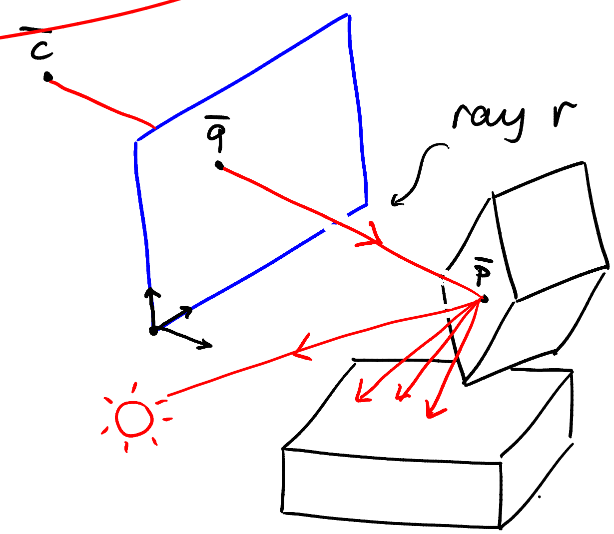
② find 1st intersection of $\bar{q}$ with scene (i.e. point $\bar{p}$)

③ estimate amount of light reaching $\bar{p}$

④ estimate amount of light travelling from $\bar{p}$ to $\bar{q}$ along ray r

Implemented by

- spawning a large set of rays at each step
- computing integrals of radiometric quantities



Topic 13:

Radiometry

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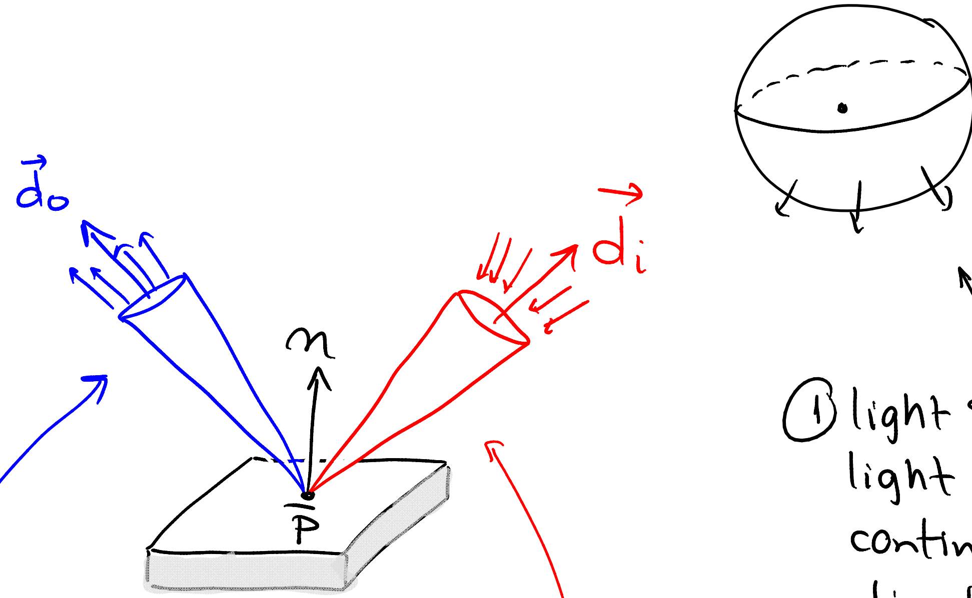
The Basic "Light Transport" Path



① light sources emit light along a continuoum of directions

③ light is reflected from a point to a continuous "cone" of directions

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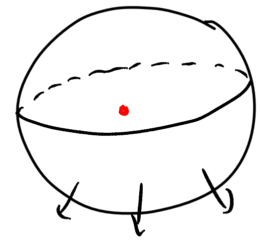
The Basic “Light Transport” Path



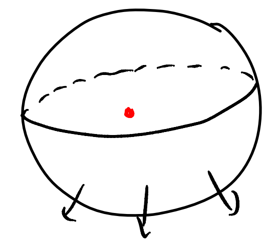
light sources emit light along a continuum of directions

Radiant Energy & Radiant Flux

- Light is energy (i.e. photons)

  $\Rightarrow$ measured in Joules (J)
  (4×10^{-19} J per photon)

  $\Rightarrow$ energy emitted by a light source is called radiant energy



light source emits radiant flux Φ (in Watts)

- We are interested in "steady-state" conditions (energy per unit time interval)

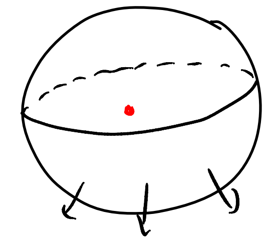
  $\Rightarrow$ energy per unit time (a.k.a power)

  $\Rightarrow$ measured in Watts (= J/sec)

  $\Rightarrow$ called radiant flux Φ

Measuring Light Emitted from a (Point) Source

In rendering, light emitted by the light source falls onto an object surface.



light source emits radiant flux Φ (in Watts)

Q: How do we measure emission along specific directions?

Q: How do we measure light received at a point on a surface?

Q: What units should we use?

Flux Through an Arc (for 2D, Uniform Source)

- Suppose source emits uniformly in all directions

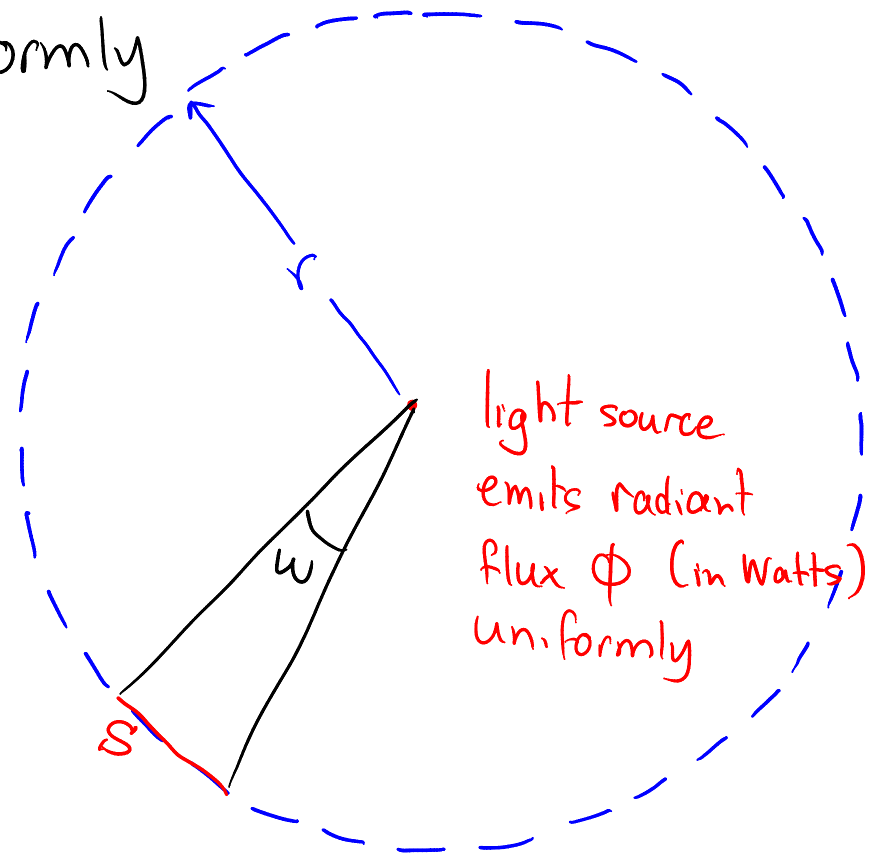
- How much flux flows through arc S'?

$$\Phi \; \frac{\text{length of } S'}{\text{perimeter}} =$$

$$\Phi \; \frac{\omega \cancel{r}}{2\pi \cancel{r}} = \omega \; \frac{\Phi}{2\pi}$$

measured in radians (rad)

measured in Watts per radian (W/rad)



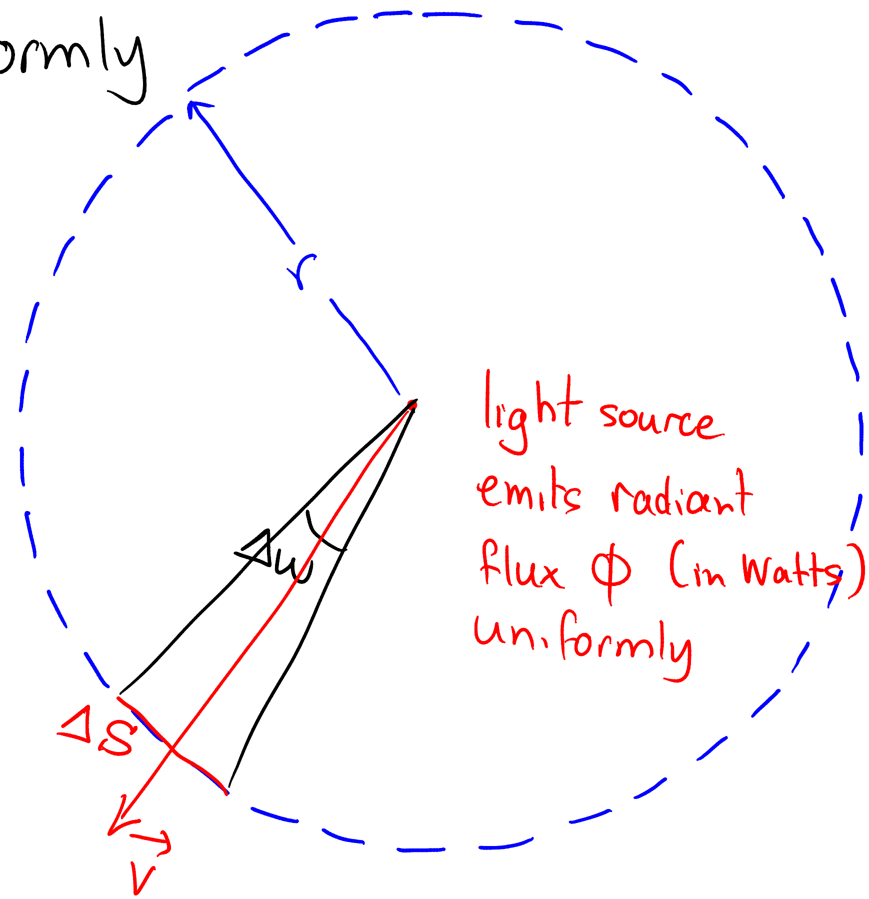
Flux Along a Direction (for 2D, Uniform Source)

- Suppose source emits uniformly in all directions
- How much flux flows along direction v?
  - Consider a small arc ΔS centered at $\vec{v}$

$$\Delta\Phi = \Phi \frac{\text{length } \Delta S}{\text{perimeter}} = \Delta\omega \frac{\Phi}{2\pi}$$

  - Define flux along $\vec{v}$ to be the limit of $\Delta\Phi$ as $\Delta\omega \to 0$:

$$d\Phi = d\omega \frac{\Phi}{2\pi} \overset{\text{def}}{=} \lim_{\Delta\omega \to 0} \Delta\omega \frac{\Phi}{2\pi}$$



Flux Along a Direction (for 2D, Uniform Source)

- Suppose source emits uniformly in all directions
- How much flux flows along direction $\vec{v}$?

Ans: A differential flux $d\Phi$:

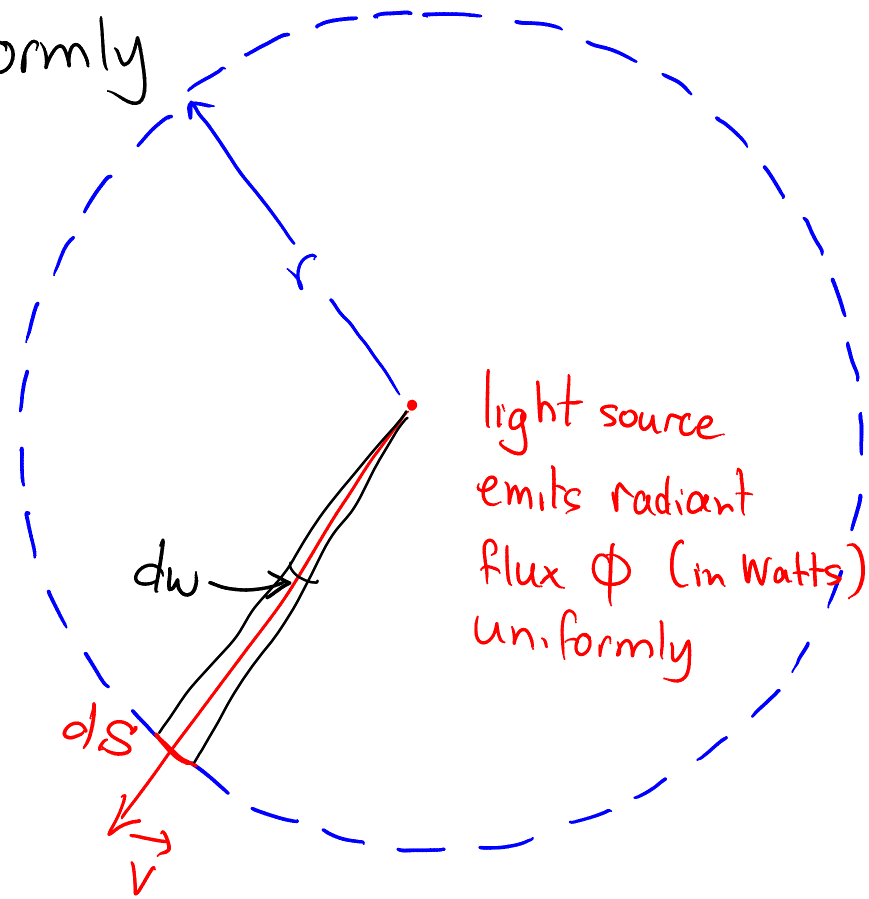
$$d\Phi = d\omega \, \frac{\Phi}{2\pi}$$

differential flux

differential angle

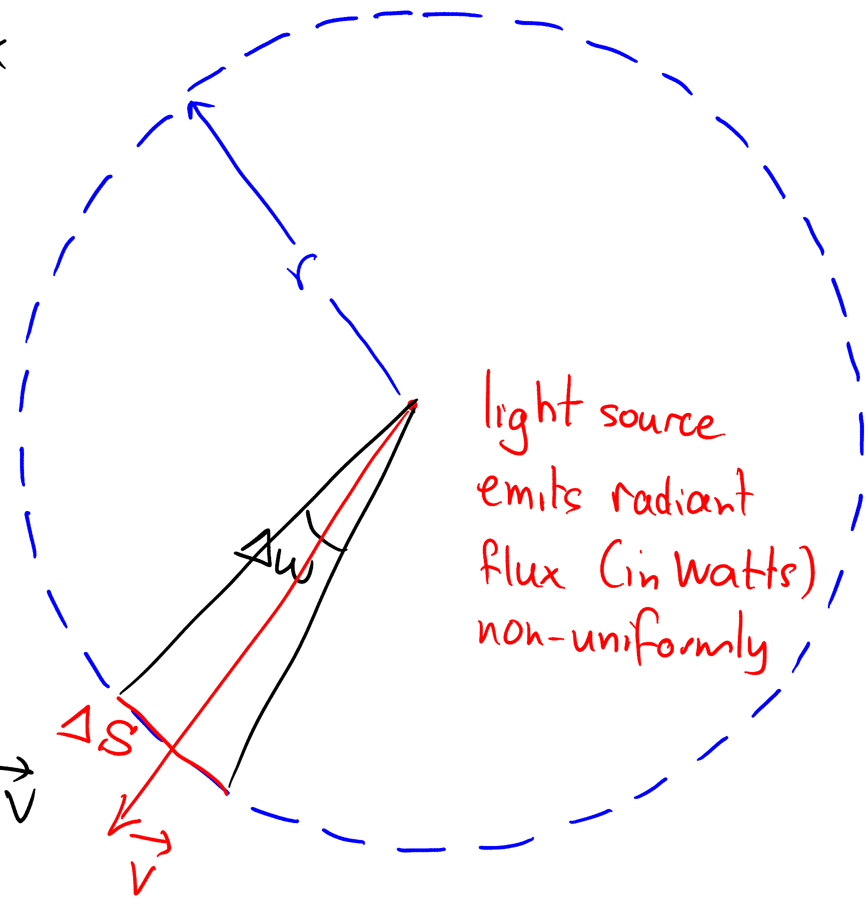
measured in Watts per radian (W/rad)

these are both infinitesimal quantities!



Flux Along a Direction (2D, Non-Uniform Src)

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the fraction



$$\frac{\text{Watts}}{\text{radian}} \left\{ \frac{\Delta\Phi}{\Delta\omega} \right.$$

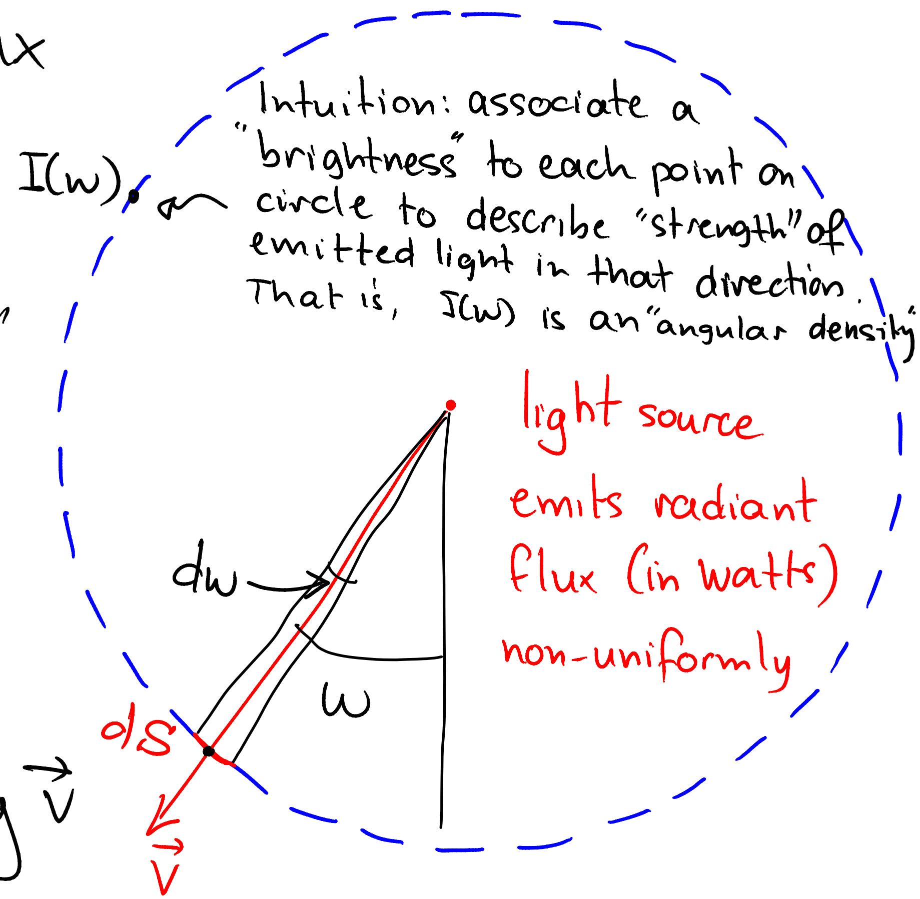
$\Delta\Phi$ ← flux "sent" through angle $\Delta\omega$

$\Delta\omega$ ← angle $\Delta\omega$

as $\Delta\omega \to 0$

Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the limit



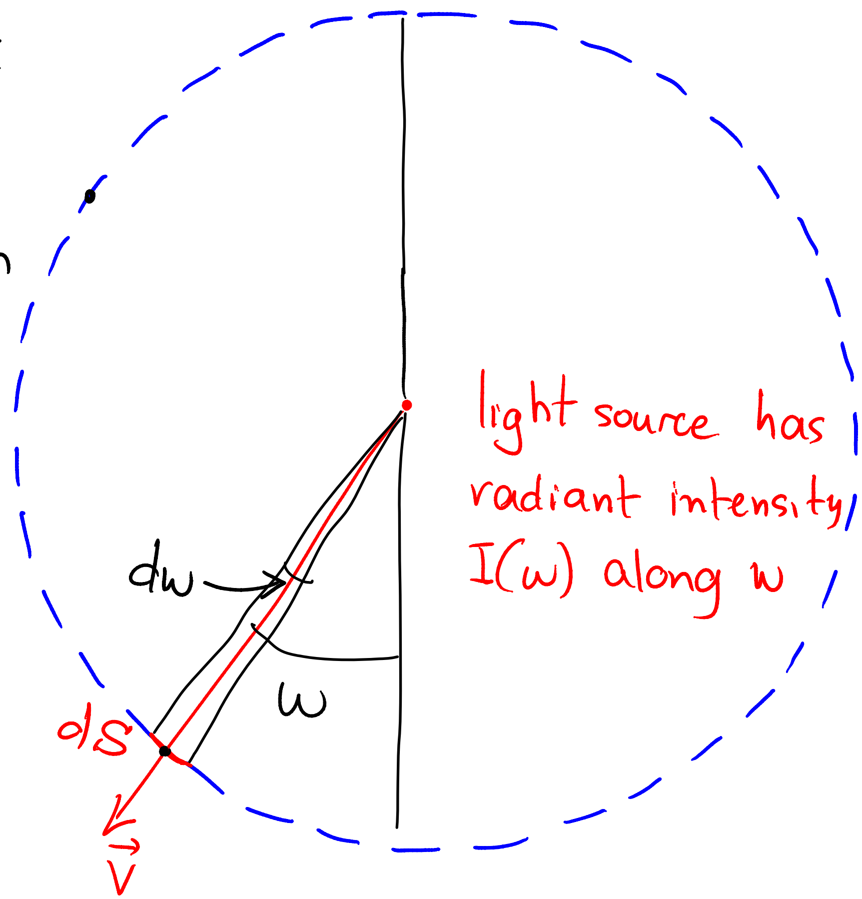
$$\lim_{\Delta\omega \to 0} \frac{\Delta\Phi}{\Delta\omega} = \boxed{\frac{d\Phi}{d\omega} = I(\omega)}$$

Radiant intensity of source in direction ω $\left(\frac{\text{watts}}{\text{radian}}\right)$

Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- Its emission strength given by radiant intensity:

$$I(\omega) = \frac{d\Phi}{d\omega} \quad \left(\frac{\text{watts}}{\text{radian}}\right)$$



- Differential flux given by

$$d\Phi = d\omega \cdot I(\omega)$$

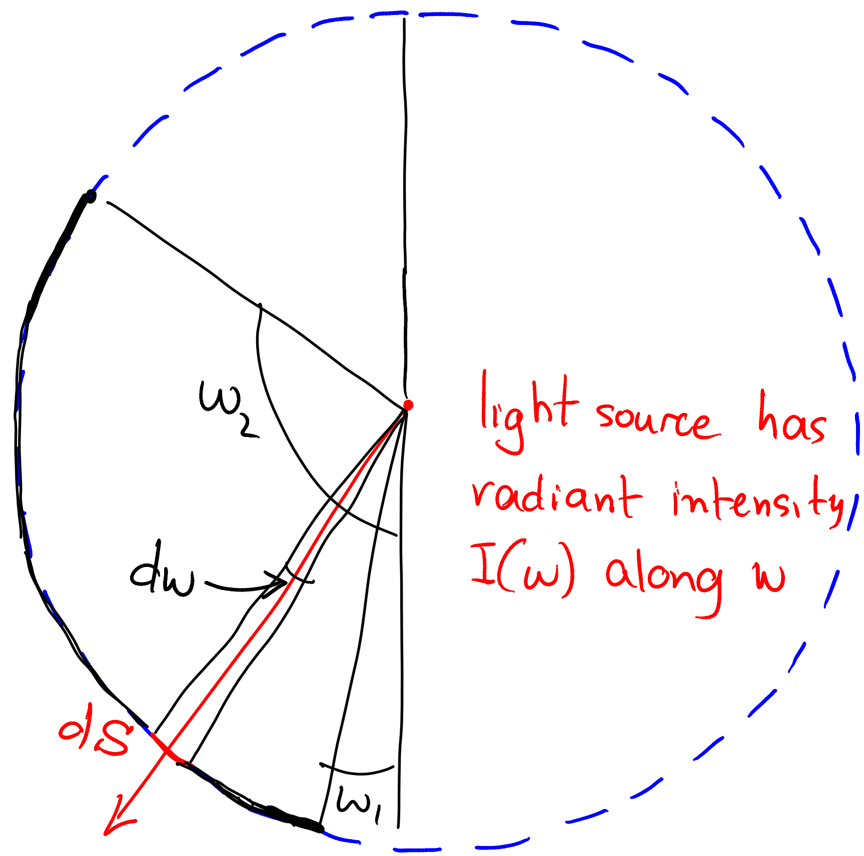
- Compare to a uniformly-emitting source of flux Φ

$$d\Phi = d\omega \cdot \frac{\Phi}{2\pi}$$

i.e. $I(\omega) = \frac{\Phi}{2\pi} = \text{const}$ for a uniform source

Flux Through a General Arc (2D, Non-Uniform)

- What is the total flux through the arc $[\omega_1, \omega_2]$?



- Differential flux given by

$$d\phi = d\omega \cdot I(\omega)$$

$\Rightarrow$ Compute the integral over $[\omega_1, \omega_2]$

$$\boxed{\phi_{\omega_1\omega_2} = \int_{\omega_1}^{\omega_2} d\phi = \int_{\omega_1}^{\omega_2} I(\omega)\, d\omega}$$

Topic 13:

Radiometry

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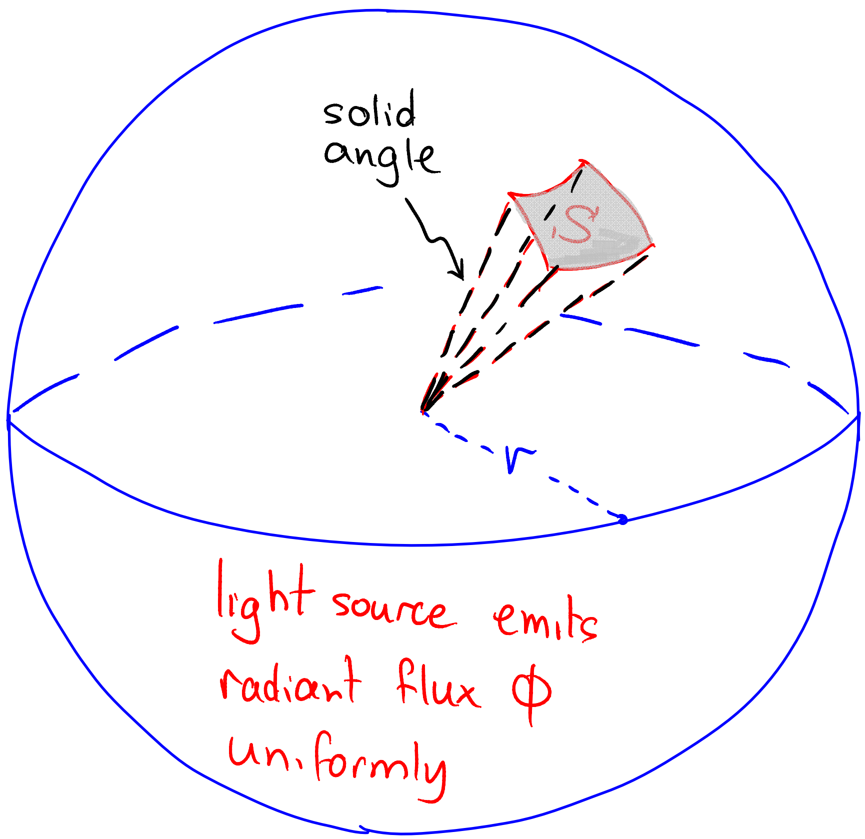
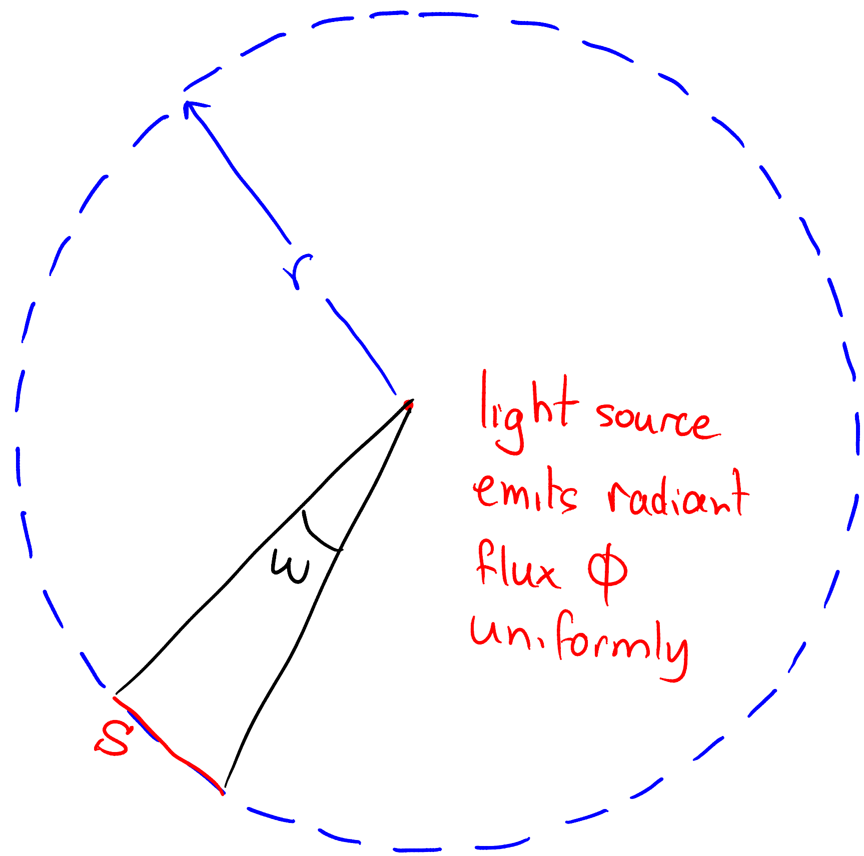
Flux Through an Arc (for 3D, Uniform Source)





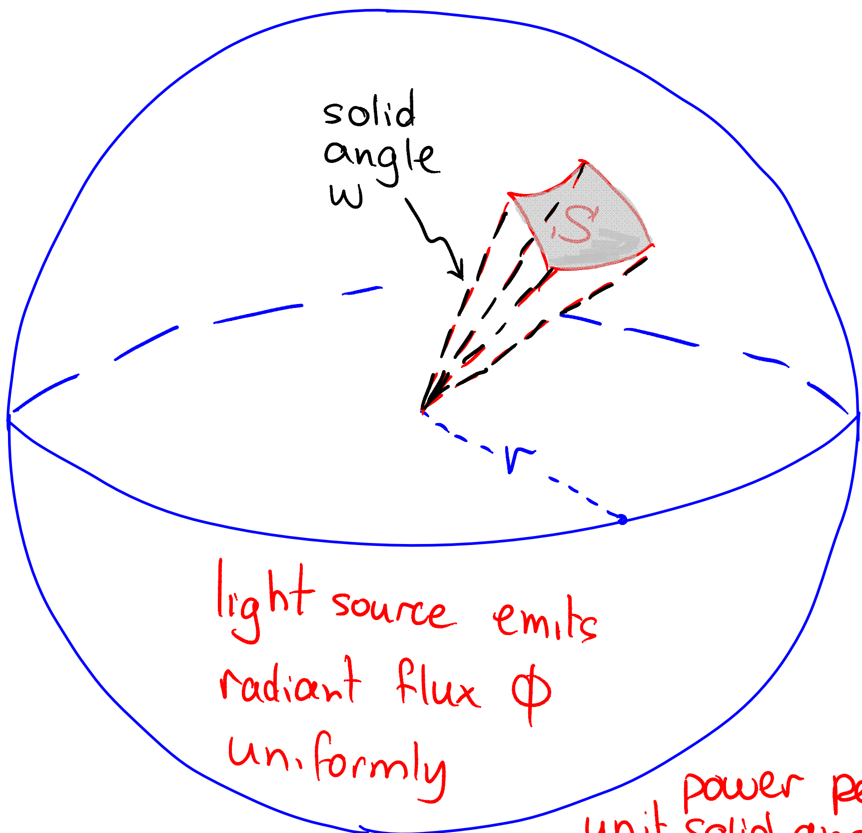
flux through S':

$$\Phi \frac{\text{area}(S')}{4\pi r^2} = \frac{\text{area}(S')}{r^2} \frac{\Phi}{4\pi}$$

flux through S:

$$\Phi \frac{\text{length}(S)}{\text{perimeter}} = \omega \cdot \frac{\Phi}{2\pi} \; (W/rad)$$

Arcs/Angles in 2D ⇔ Areas/Solid Angles in 3D



- <u>Definition</u>:

  Solid angle ω of a patch S on a sphere of radius r:

$$\boxed{\omega = \frac{\text{area}(S)}{r^2}}$$

- Solid angles are measured in steradians (sr)
- The solid angle of a full sphere is 4π

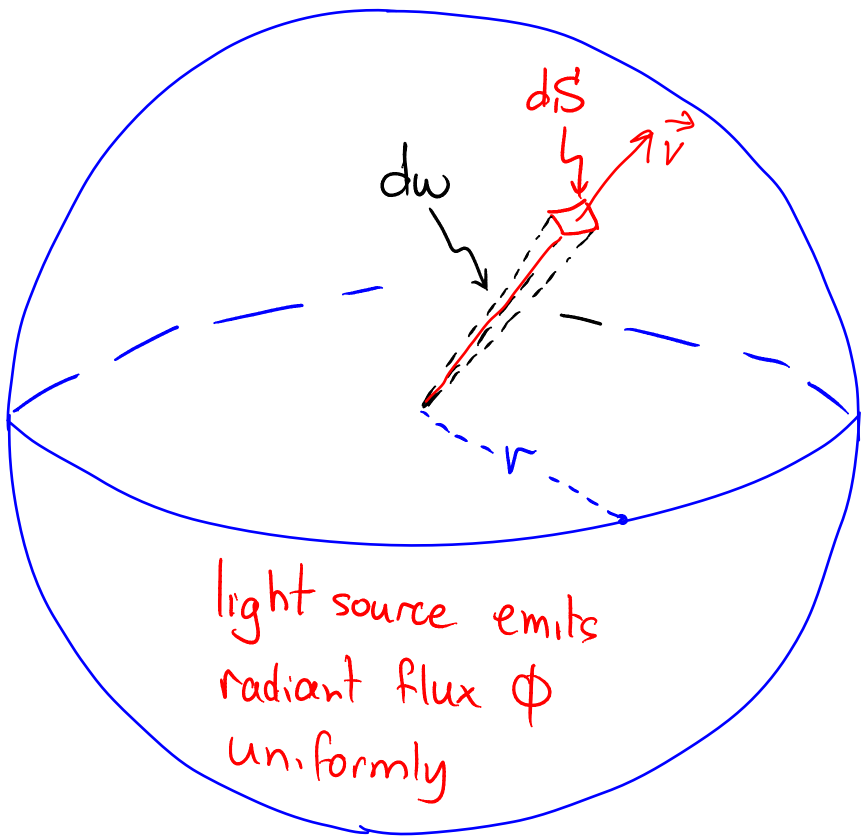
power per unit solid angle (W/sr)

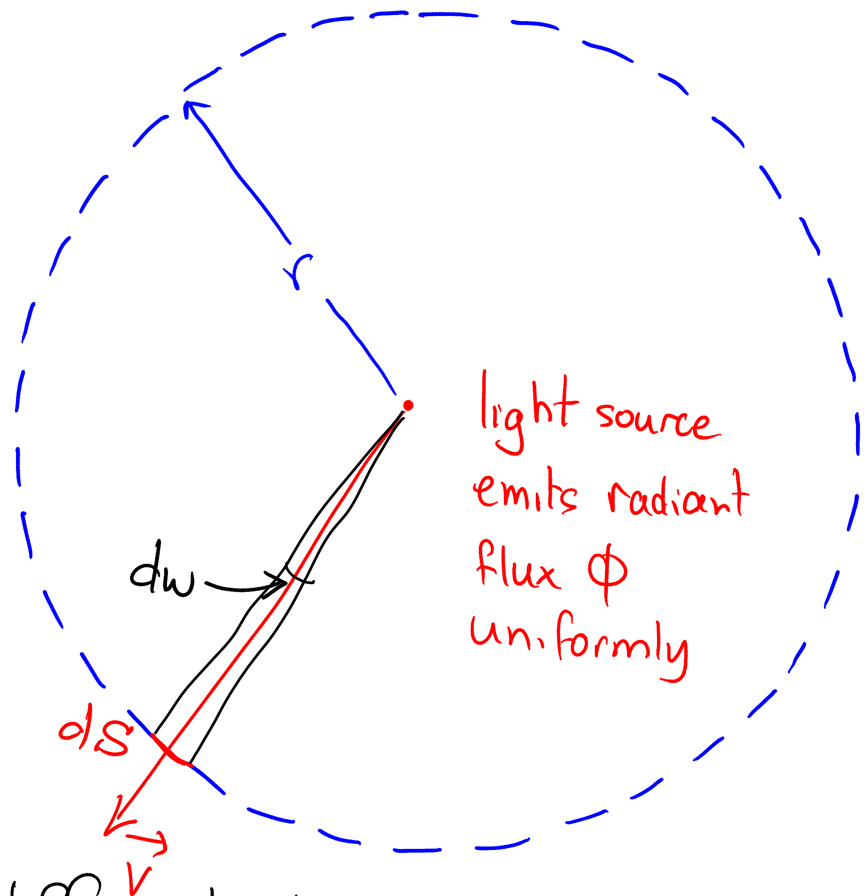
flux through S:

$$\phi \frac{\text{area}(S)}{4\pi r^2} = \omega \frac{\phi}{4\pi}$$

solid angle (sr)

Flux Along a Direction (for 3D, Uniform Source)





differential flux along direction $\vec{v}$:

$$d\phi = \underline{d\omega}\, \frac{\Phi}{4\pi}$$

differential solid angle

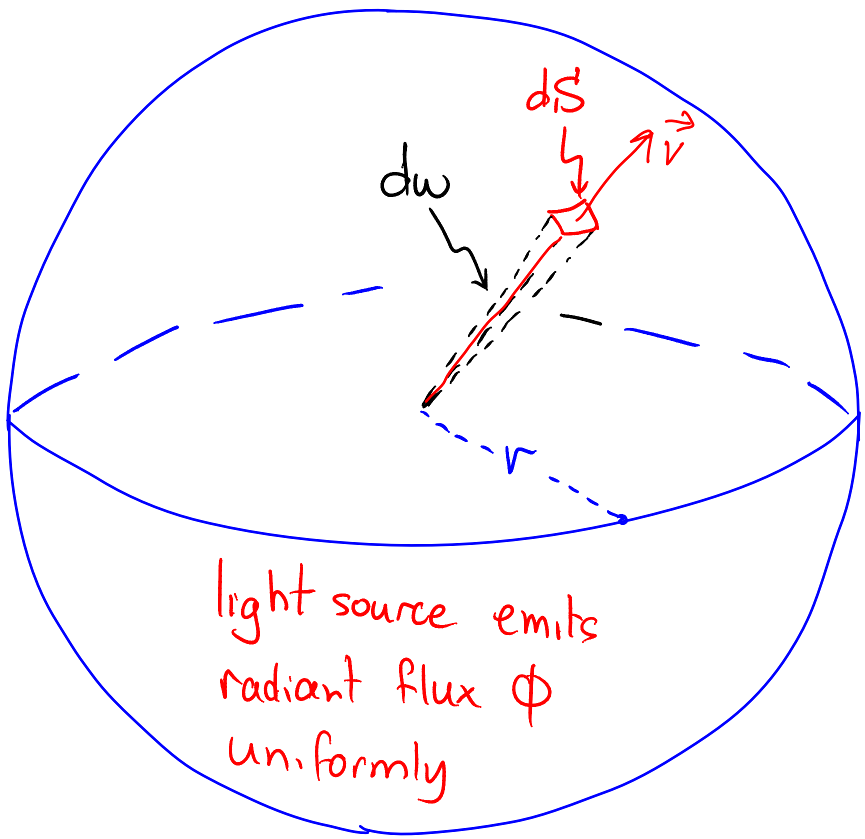
power per unit solid angle

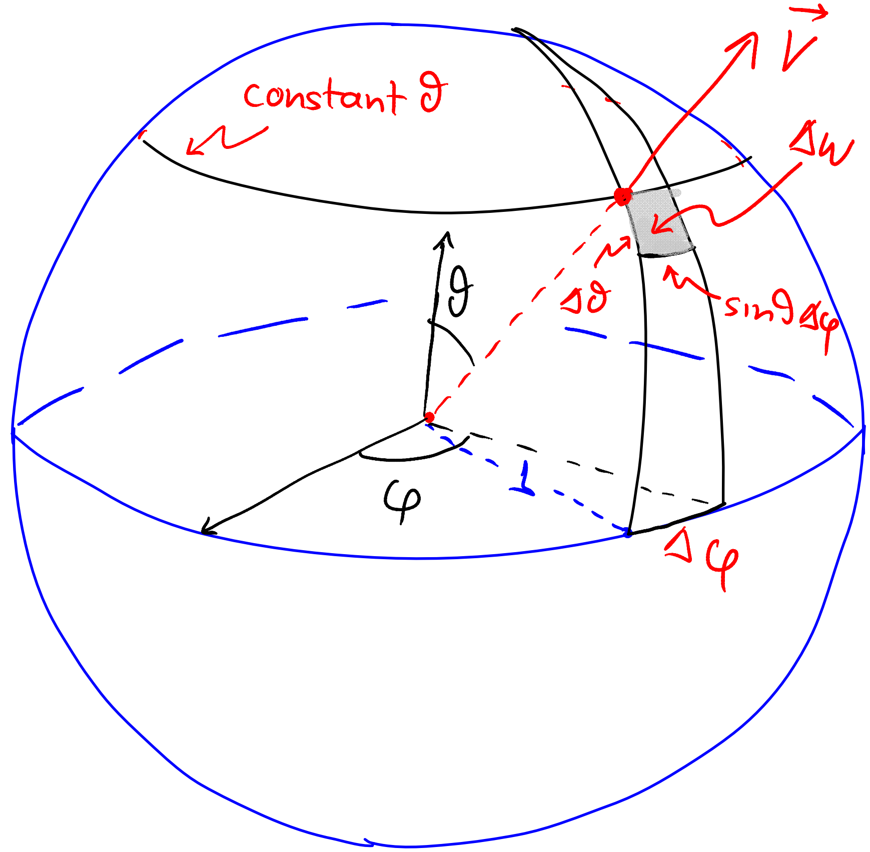
differential flux along direction $\vec{v}$:

$$d\phi = \underline{d\omega}\, \frac{\Phi}{2\pi}$$

differential angle

Differential Solid Angles ⇔ Spherical Coords





differential flux along direction $\vec{v}$:

$$d\Phi = dw \frac{\Phi}{4\pi}$$

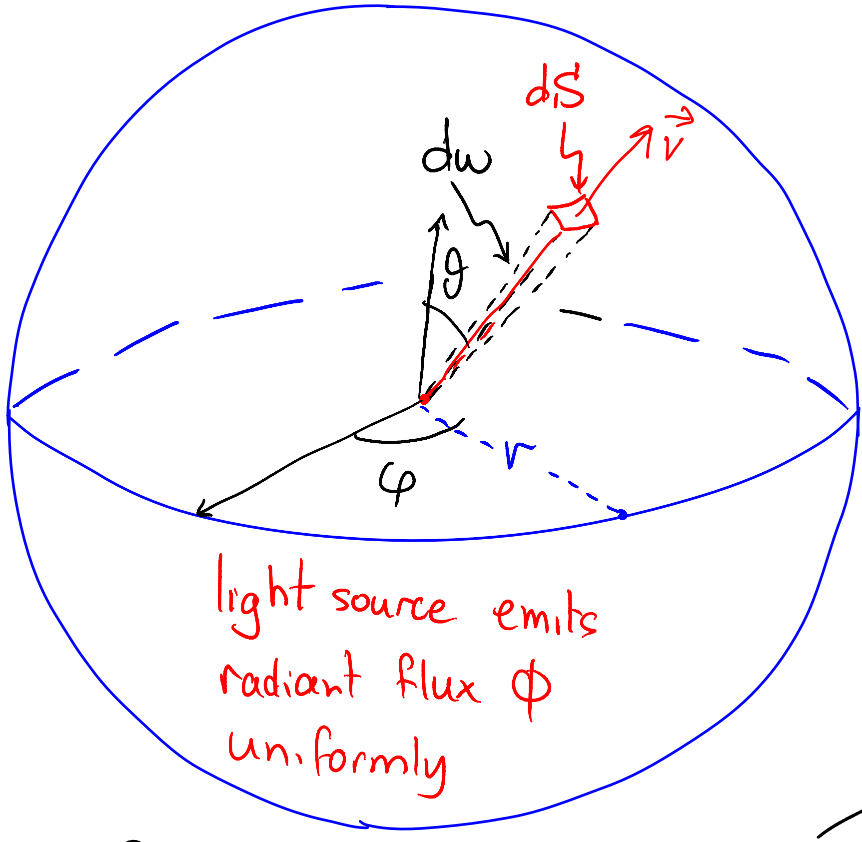
power per unit solid angle

if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

area(Δw) $\simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$

when $\Delta w \to 0$, $\boxed{dw = d\vartheta\, d\varphi \sin\vartheta}$

Flux Along a Direction (for 3D, Uniform Source)



Differential flux along direction (ϑ, φ) for a source emitting uniformly in all directions:

$$\boxed{d\phi = d\vartheta\, d\varphi \sin\vartheta \frac{\Phi}{4\pi}}$$

combining boxed expressions

differential flux along direction $\vec{v}$:

$$\boxed{d\phi = d\omega \frac{\Phi}{4\pi}}$$

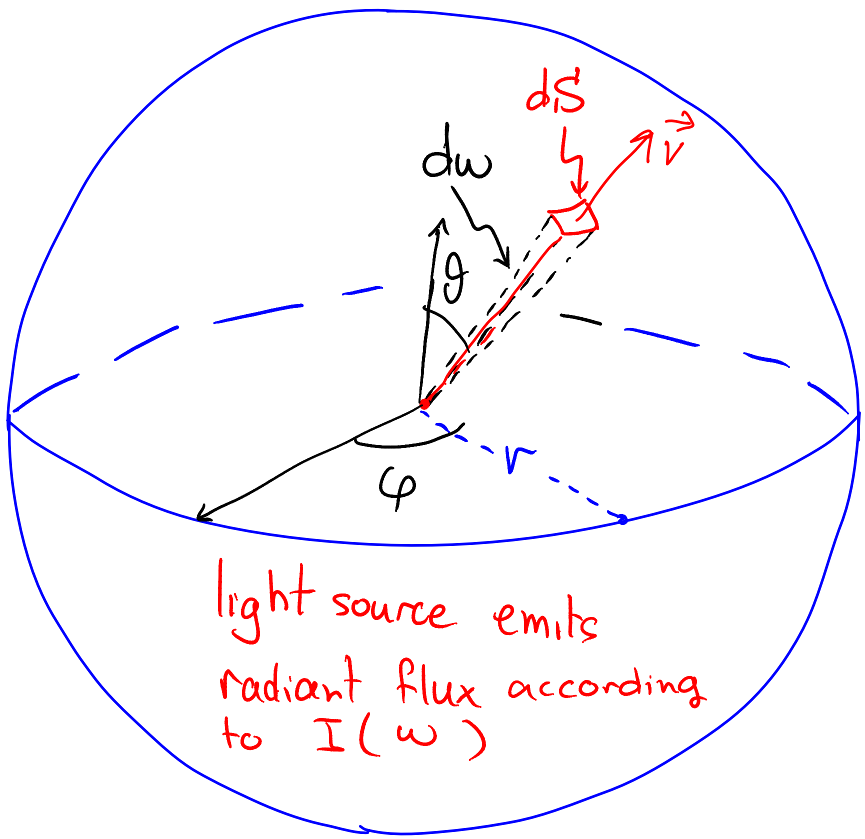
power per unit solid angle

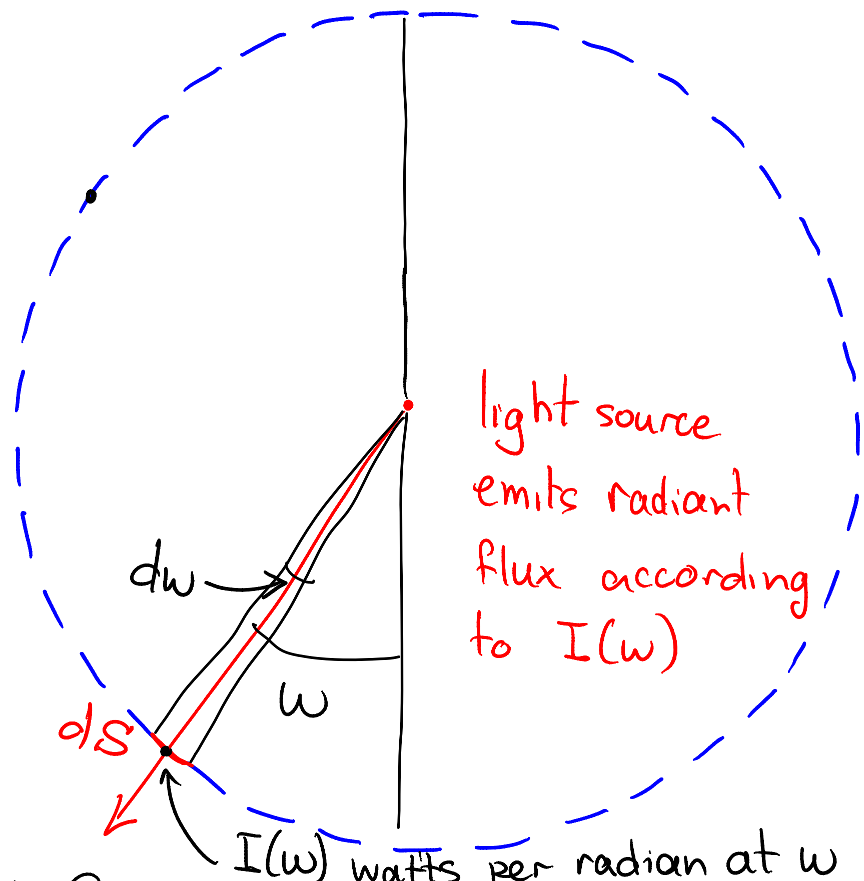
if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

$\text{area}(\Delta\omega) \simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$

when $\Delta\omega \to 0$, $\boxed{d\omega = d\vartheta\, d\varphi \sin\vartheta}$

Radiant Intensity (for 3D, Non-Uniform Src)





differential flux along direction $\vec{v}$:

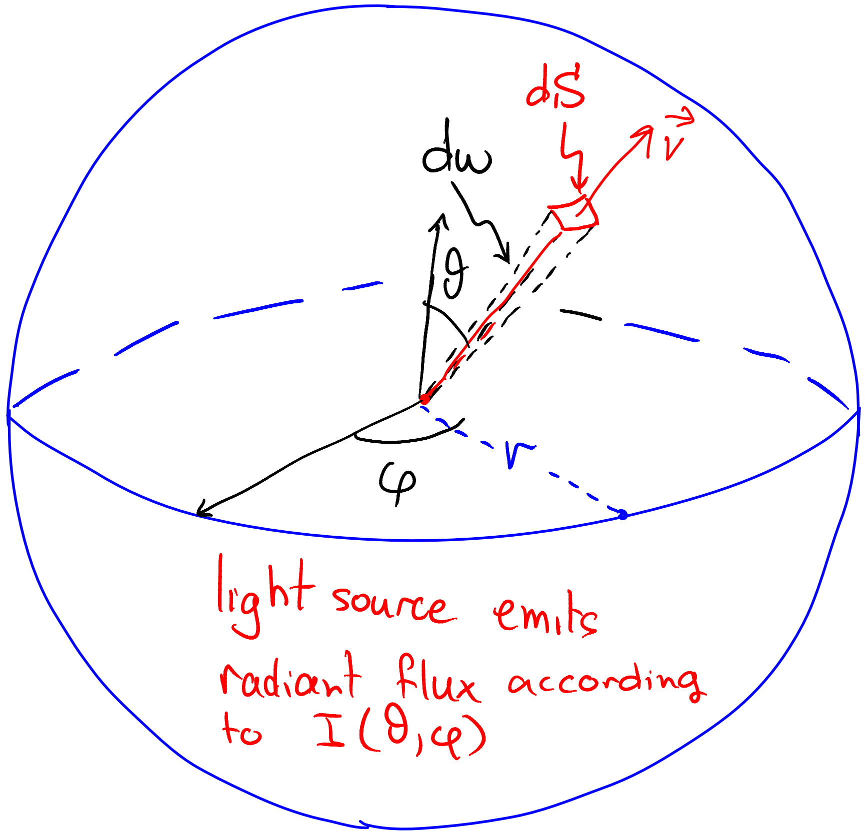
$$d\Phi = d\omega \, I(\omega)$$

Radiant intensity (watts/steradian)

differential flux along direction $\vec{v}$:

$$d\Phi = d\omega \cdot I(\omega)$$

Radiant Intensity (for 3D, Non-Uniform Src)



- $I(\omega)$ is called radiant intensity (flux along specific direction)
- Measured using W/sr
- Can also be written as a function of ϑ,φ:

  $I(\vartheta,\varphi)$

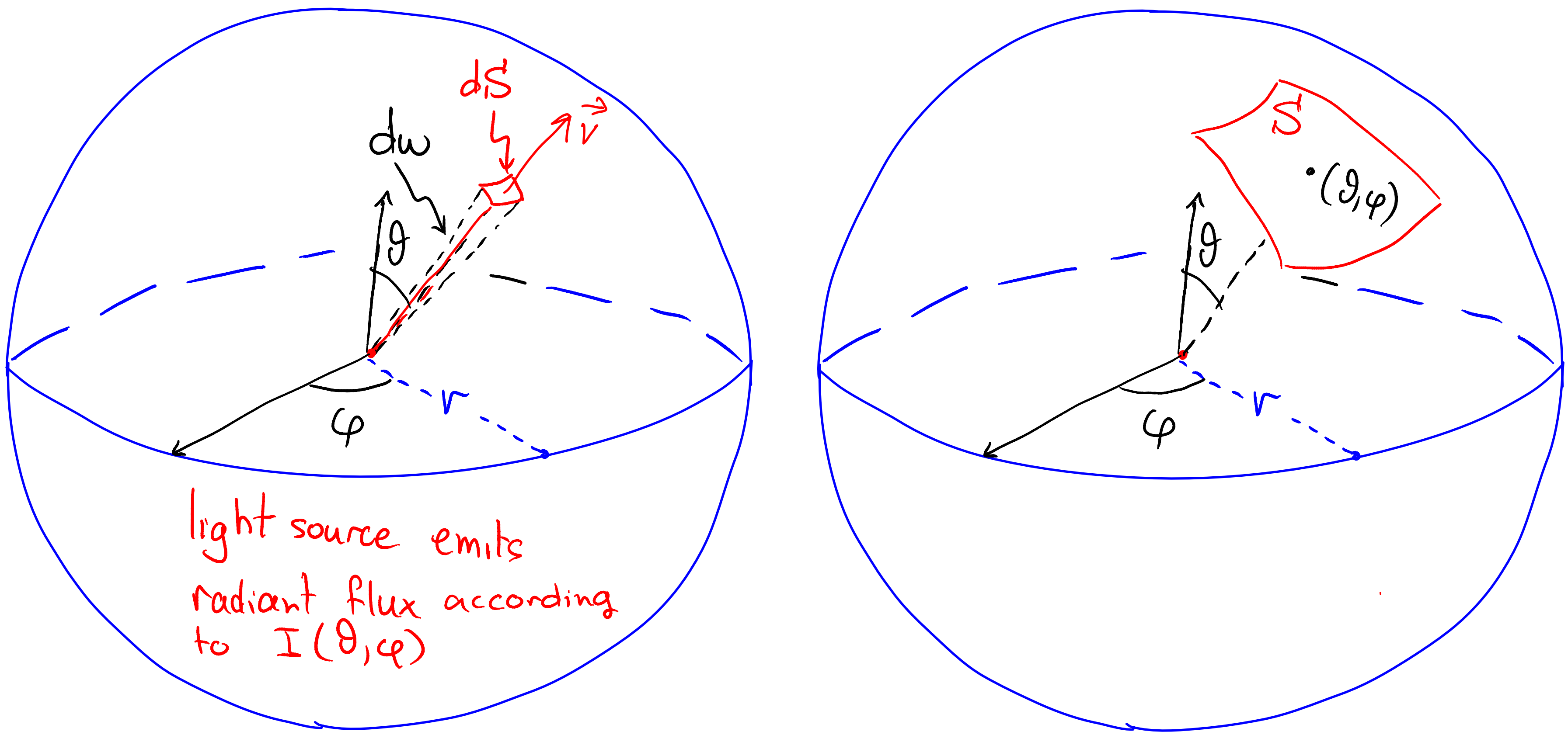
differential flux along direction $\vec{v}$:

$$d\Phi = d\omega\, I(\omega)$$

- differential flux along (ϑ,φ):

$$\boxed{d\Phi = d\vartheta\, d\varphi \sin\vartheta\, I(\vartheta,\varphi)}$$

Flux Through a General Patch (3D, Non-Uniform)



differential flux along direction $\vec{v}$:

$$d\Phi = d\vartheta\, d\varphi \sin\vartheta\, I(\vartheta,\varphi)$$

total flux through region S:

$$\Phi_S = \iint_{(\vartheta,\varphi)\in S} \sin\vartheta\, I(\vartheta,\varphi)\, d\vartheta\, d\varphi$$

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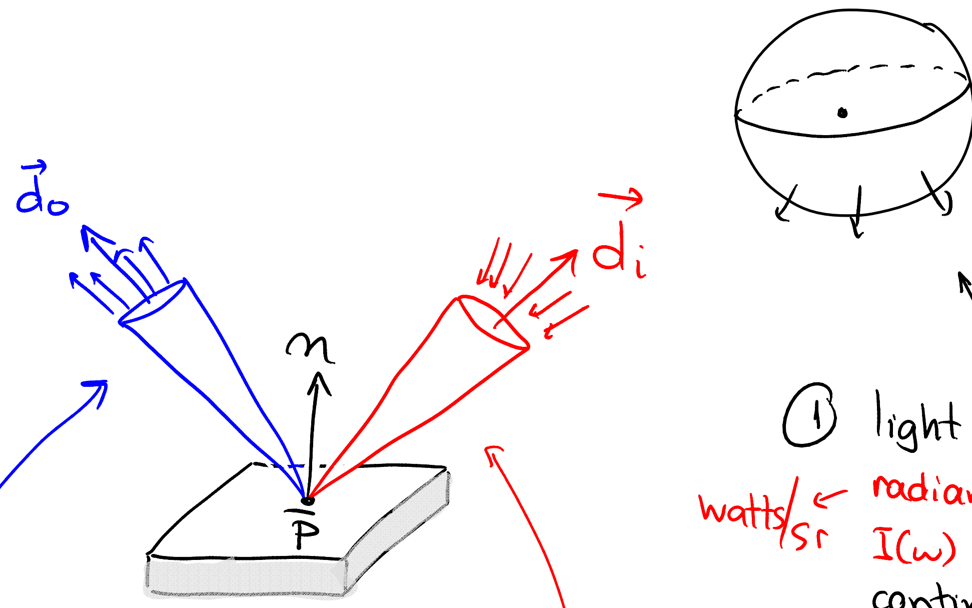
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Light Transport Cycle
- The Bidirectional Reflectance Distribution Function

The Basic "Light Transport" Path



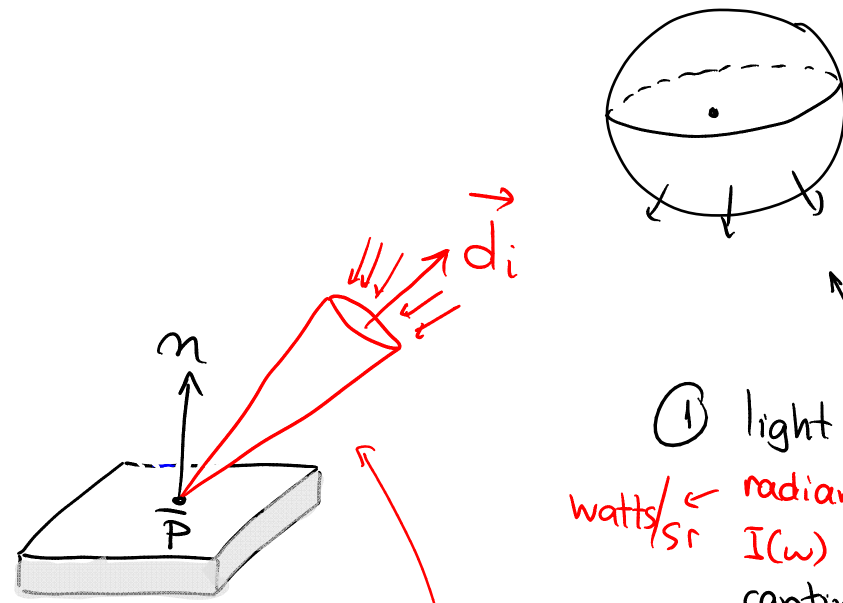
① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

② light converges onto a point from a continuous "cone" of directions

③ light is reflected from a point to a continuous "cone" of directions

The Basic "Light Transport" Path



① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

② light converges onto a point from a continuous "cone" of directions

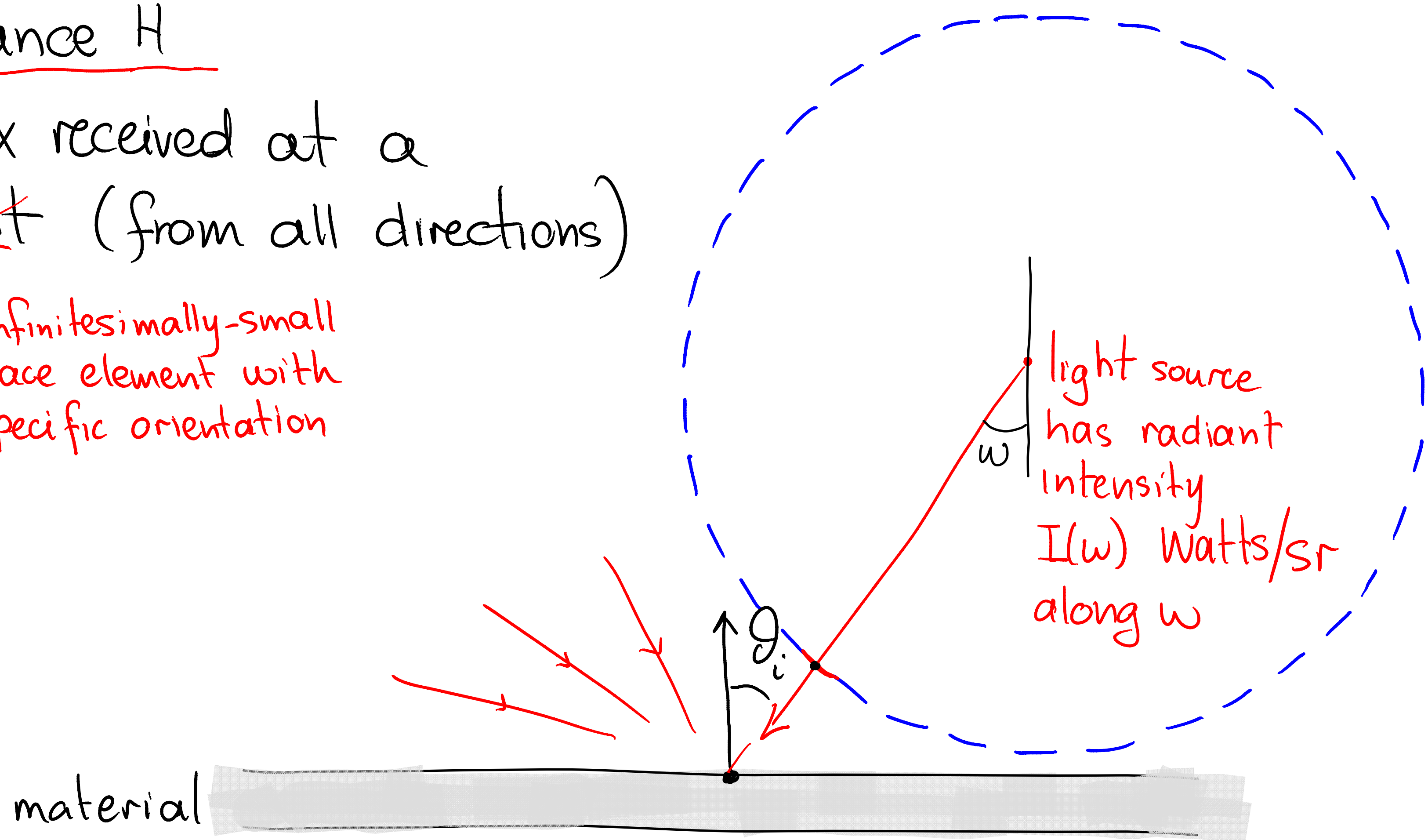
Measuring Incident Light: Irradiance

Irradiance H

Flux received at a ~~point~~ (from all directions)

"

an infinitesimally-small surface element with a specific orientation



Definition of Irradiance (for "small patches")

Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

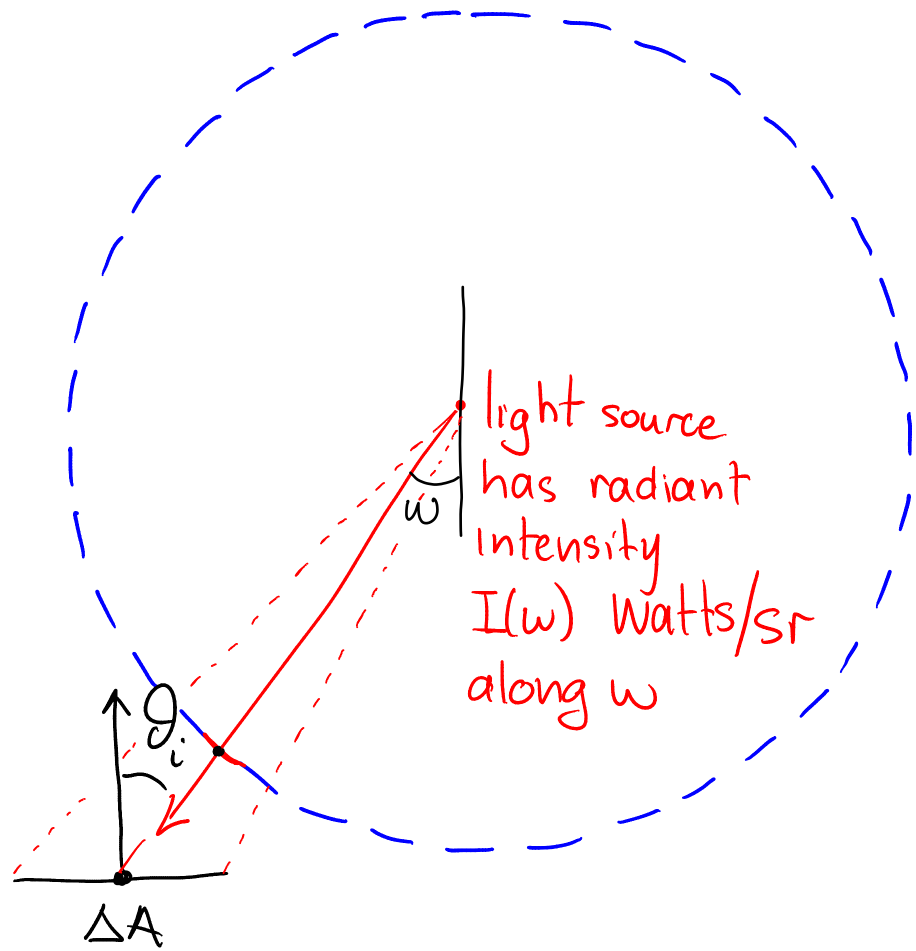
(measured in watts/m^2)

For small patches

$$H = \frac{\Delta \Phi}{\Delta A}$$

total flux received from all directions

area of patch



Definition of Irradiance (for differential areas)

Irradiance H

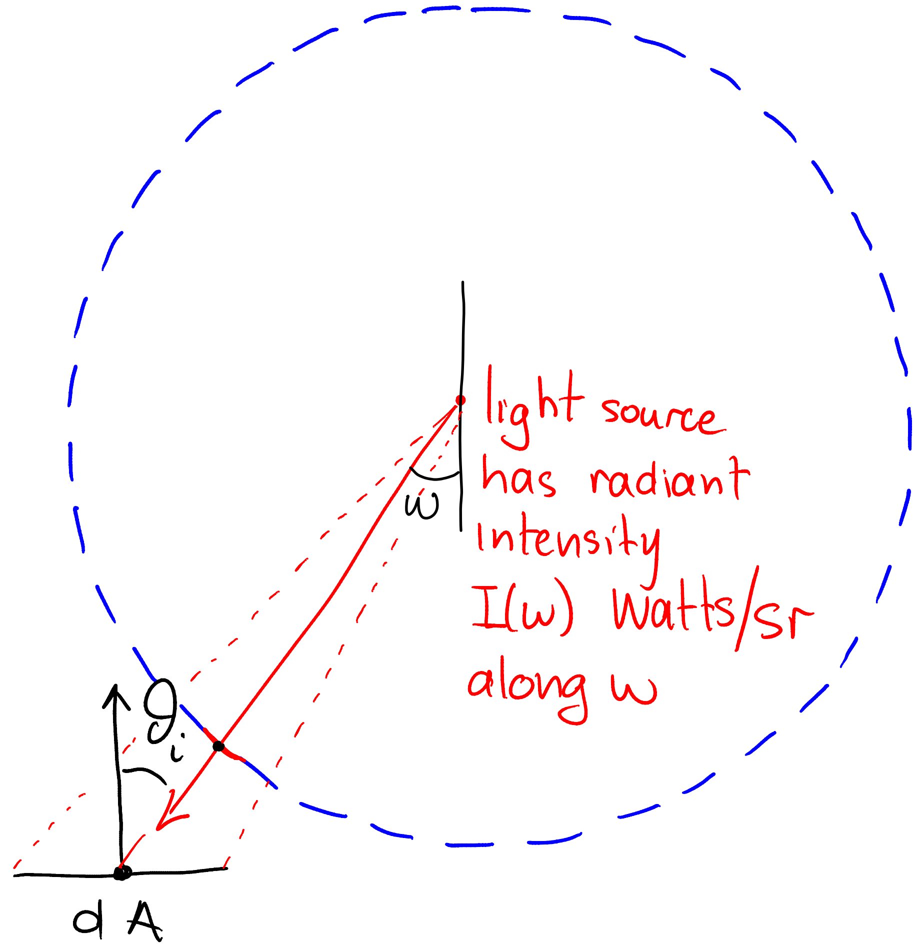
Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)

For infinitesimal patches

$$H = \lim_{\Delta A \to 0} \frac{\Delta \Phi}{\Delta A} = \frac{d\Phi}{dA}$$



Computing Irradiance: Normal Incidence

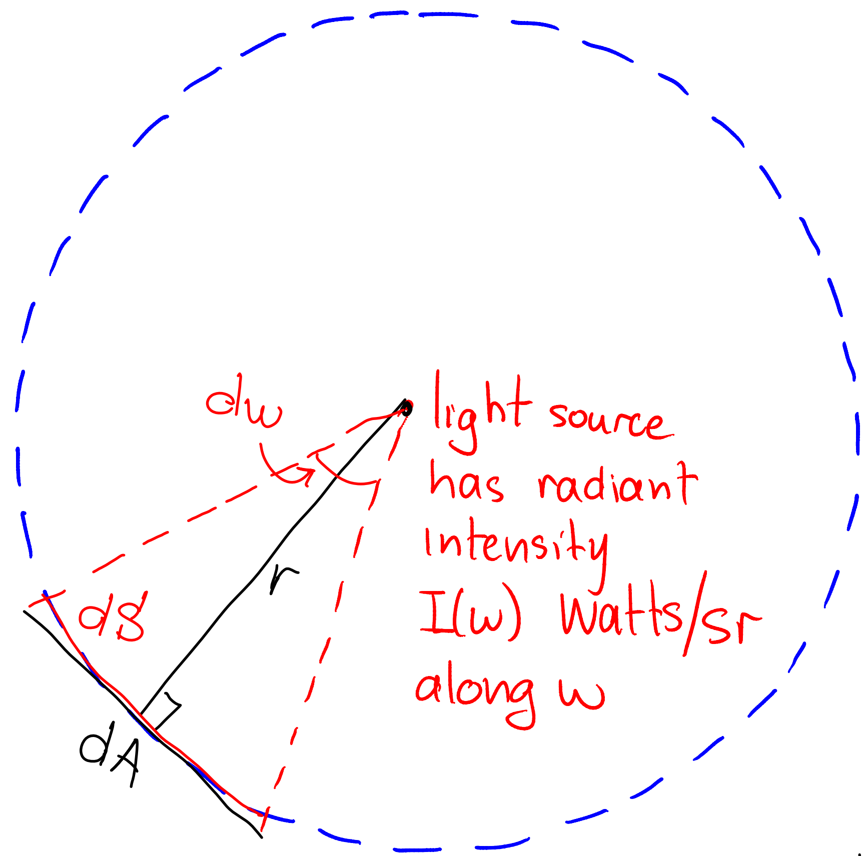
Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)

Example 1: Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it



- let $d\Phi$ be the flux through dA
- $d\Phi$ = flux through $dS = d\omega \cdot I(\omega) = \frac{dS}{r^2} \cdot I(\omega)$
- for infinitesimal patches $dA \simeq dS \Rightarrow d\Phi = \frac{dA}{r^2} I(\omega)$

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}}$$

Computing Irradiance: Normal Incidence

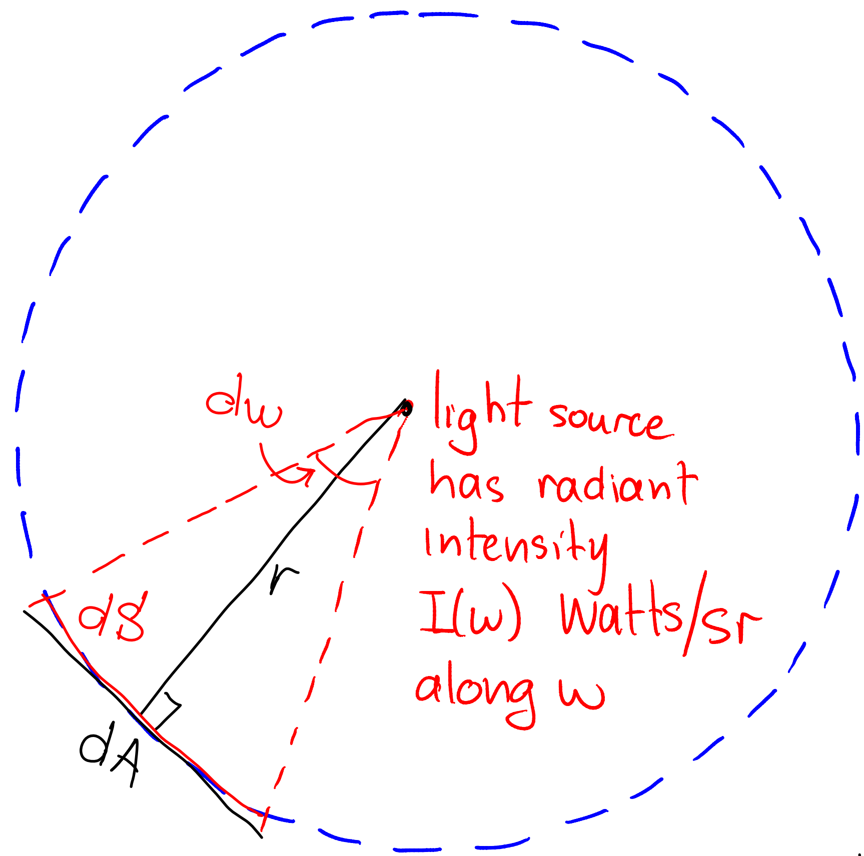
- Irradiance H

  Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)

Example 1: Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it



$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}}$$

⇒ Irradiance decreases quadratically with distance ("squared-distance fall-off")

⇒ the farther the patch is, the less light it gets

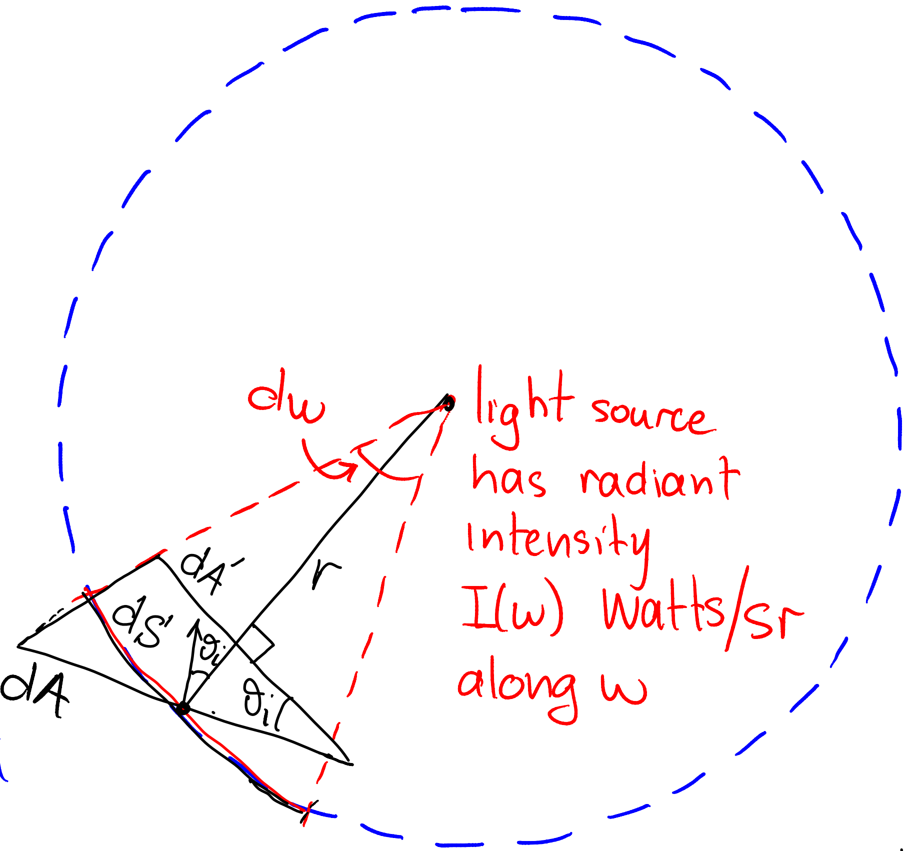
Computing Irradiance: Tilted Patches

- Irradiance H

  Flux received per unit area:

  $$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

  (measured in Watts/m^2)



Example 2: Calculate the irradiance at a planar patch dA at angle θ with source and distance r away from it

- define dA′ to be the patch at distance r that faces the light source

$$\boxed{H' = \frac{d\Phi}{dA'} = \frac{I(\omega)}{r^2}}$$

- for infinitesimal patches, $dA' \simeq dA \cos\theta \Rightarrow$

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}\cos\theta_i}$$

Computing Irradiance: Foreshortening

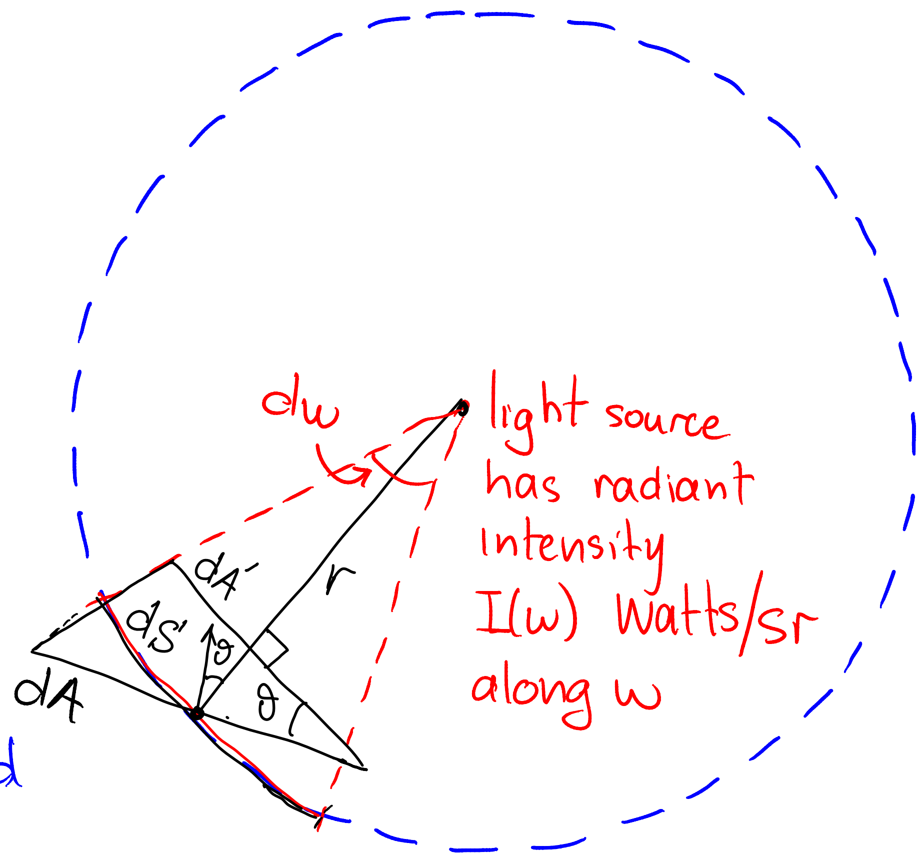
- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)

Example 2: Calculate the irradiance at a planar patch dA at angle ϑ with source and distance r away from it



The foreshortening effect: patches tilted relative to the source receive less light per unit area

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}\cos\vartheta}$$

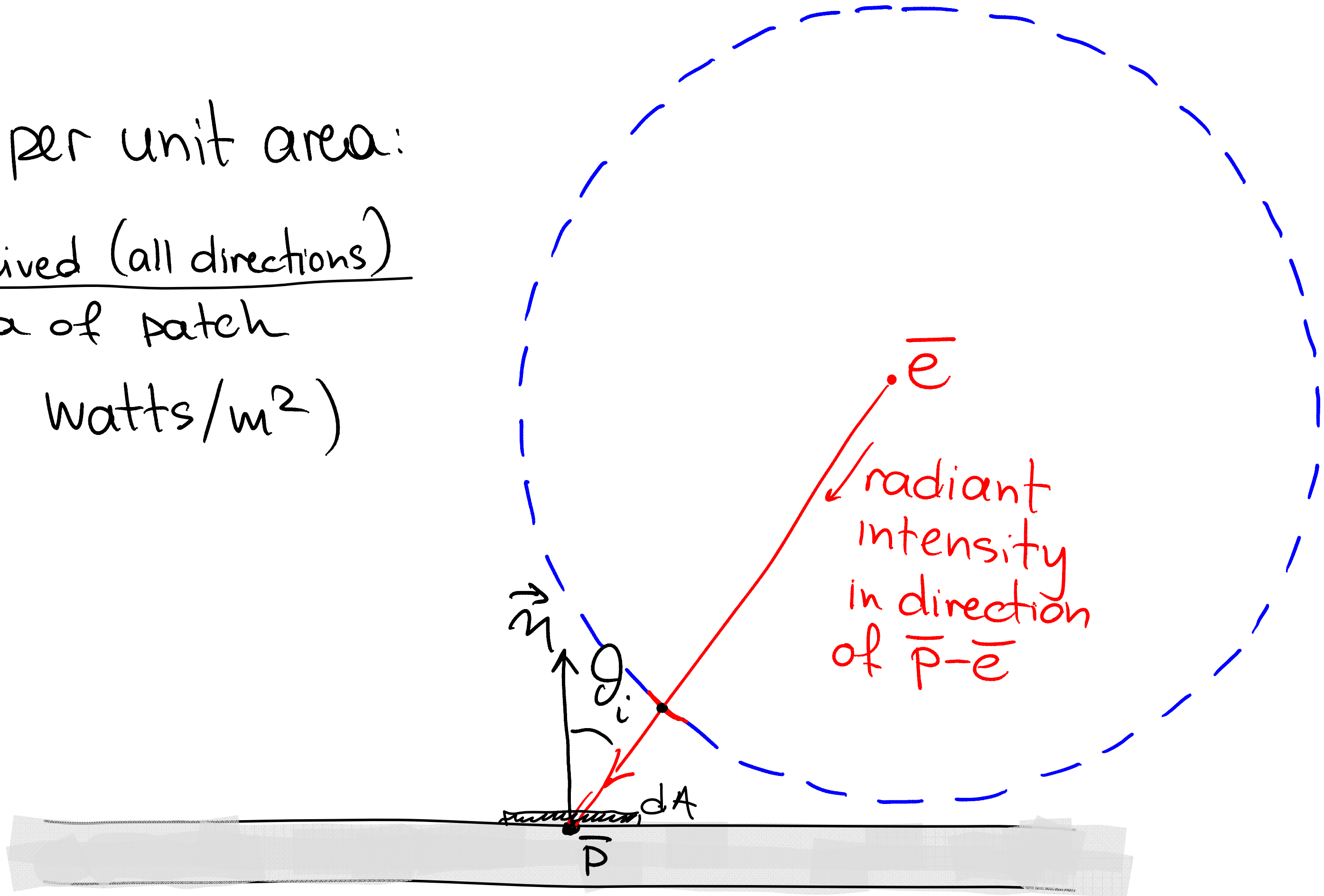
Example: Irradiance due to Point Light Source

Irradiance:

Flux received per unit area:

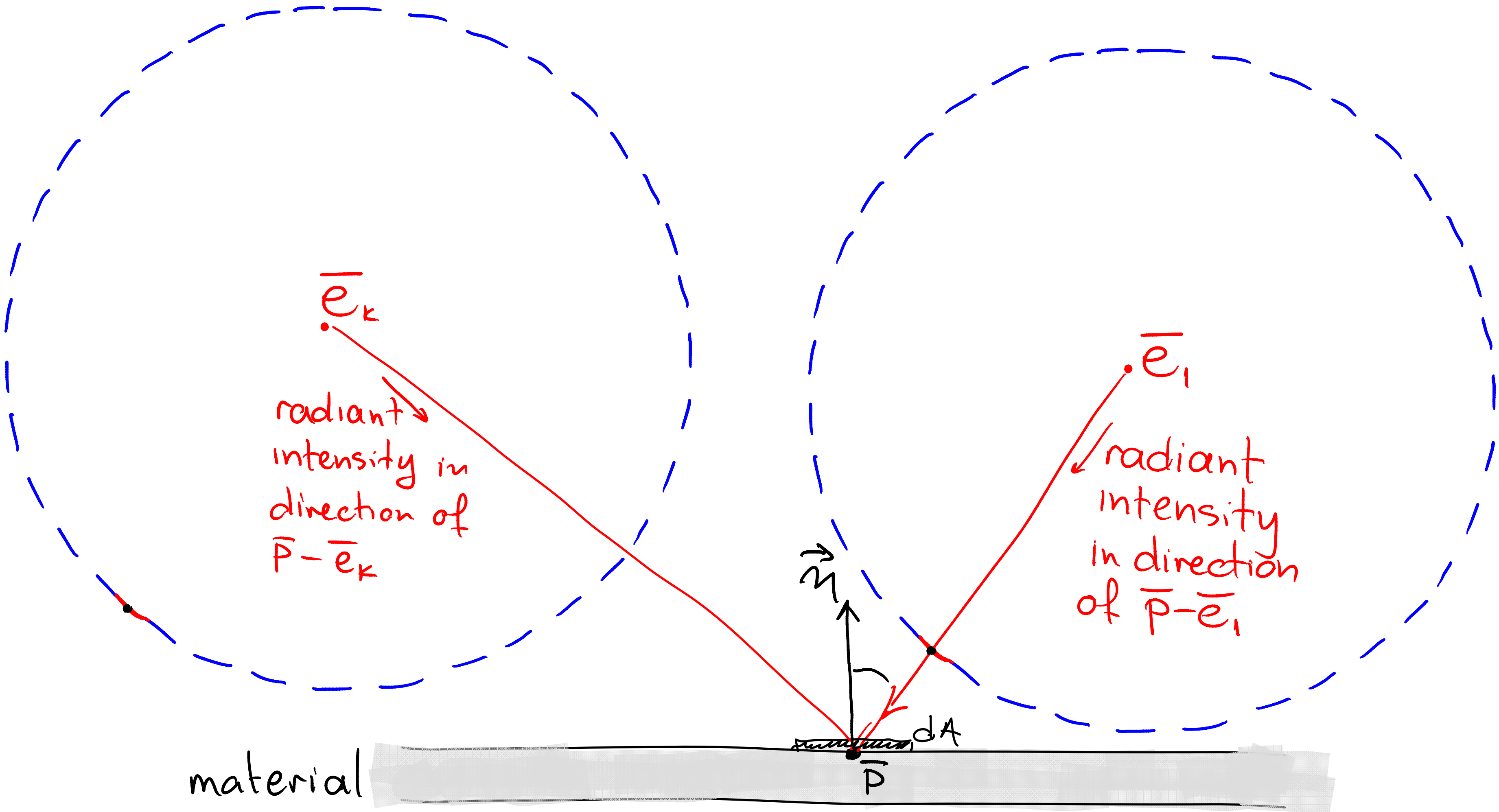
$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)



Irradiance at $\bar{p}$ (one light): $H(\bar{p}) = \dfrac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}\,\vec{n}\cdot\dfrac{(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|}$

Example: Irradiance due to Multiple Sources



Irradiance at $\bar{p}$ (K lights): $H(\bar{p}) = \sum_{k=1}^{K} \frac{I(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|^2} \vec{n} \cdot \frac{(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|}$

Topic 13:

Radiometry

- The big picture
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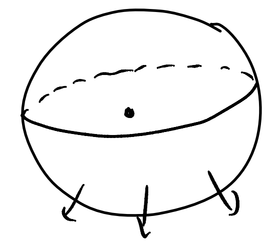
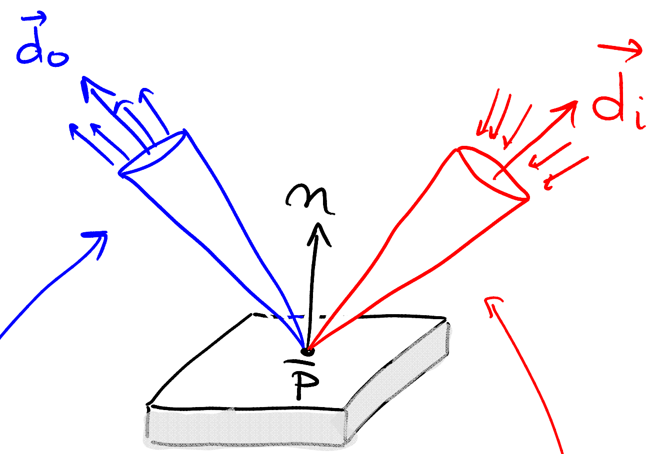
- The Bidirectional Reflectance Distribution Function

The Basic “Light Transport” Path





① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuoum of) directions

watts/m^2

③ light is reflected from a point to a continuous "cone" of directions

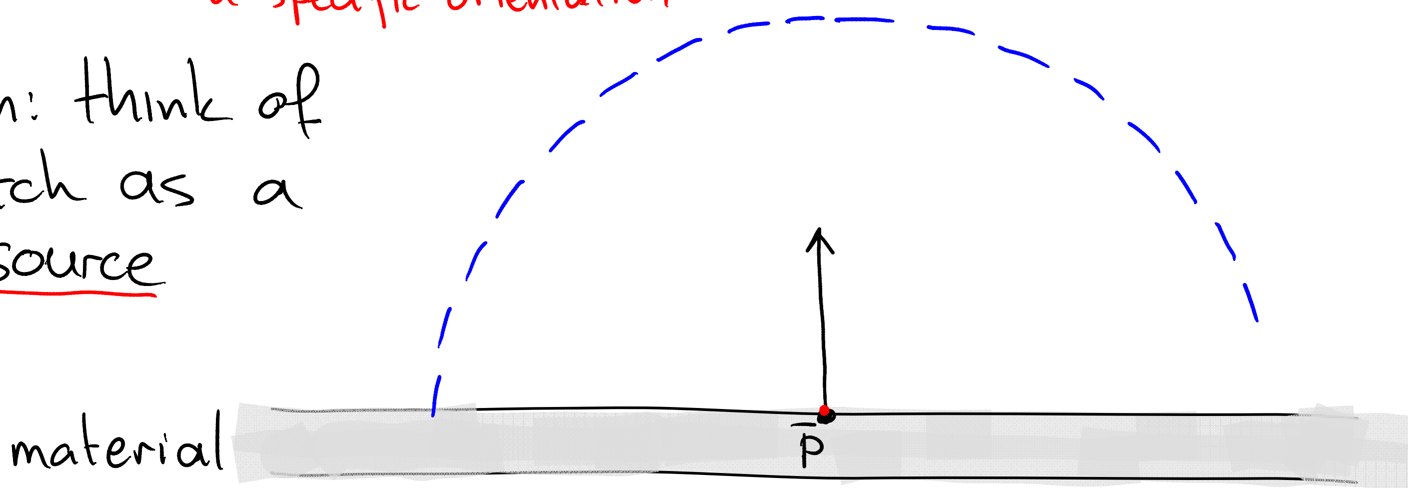
Measuring Outgoing Light: Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by a ~~surface point~~

an infinitesimally-small surface element with a specific orientation

Intuition: think of the patch as a light source



Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

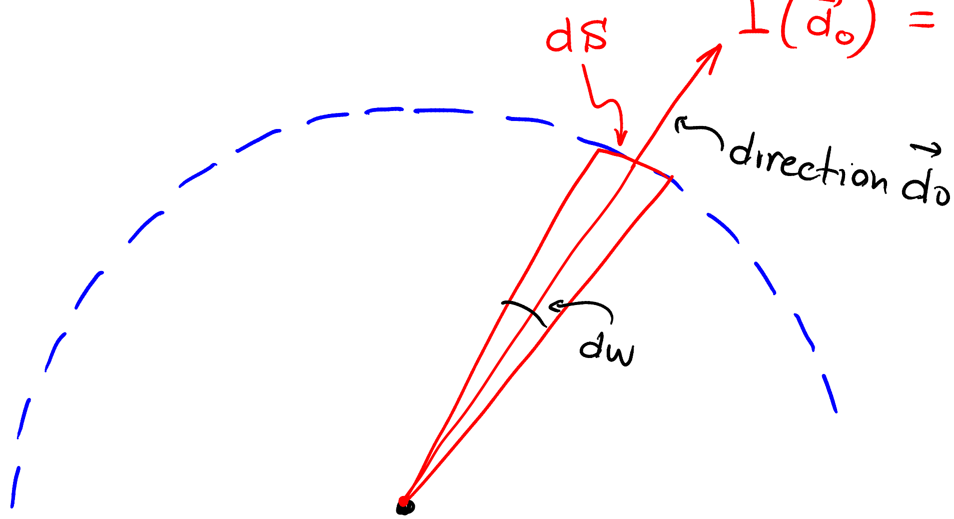
Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source

flux through dS

$$I(\vec{d}_o) = \frac{d\Phi}{d\omega}$$

solid angle of dS



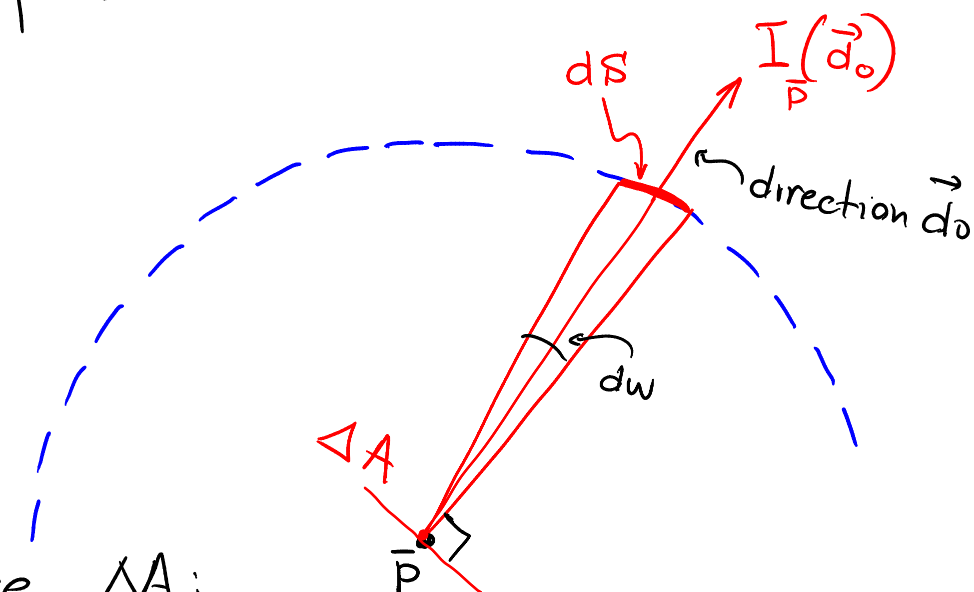
When the light source is a point its emission is quantified using its radiant intensity $I(\vec{d}_o)$

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

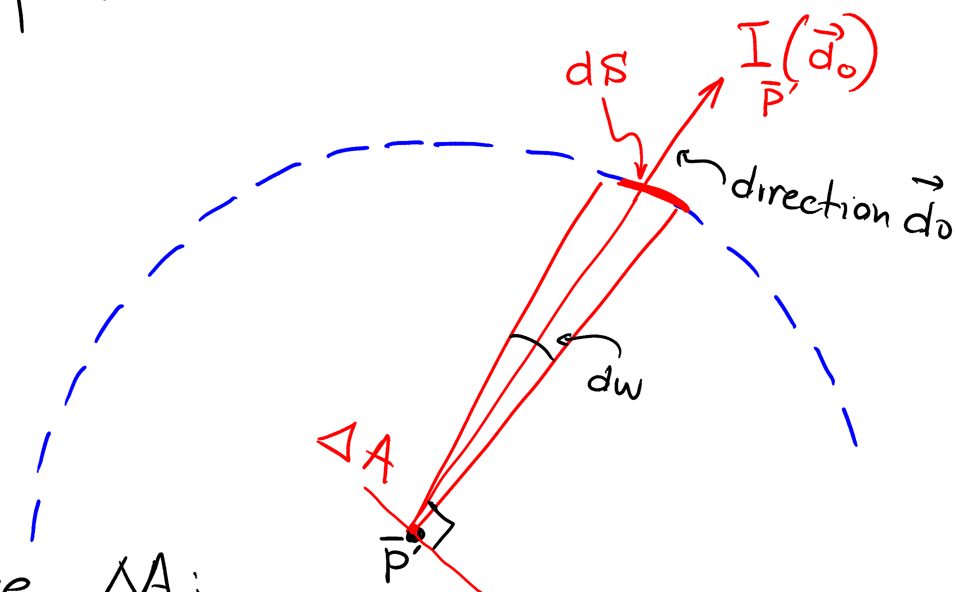
- Measure total radiant intensity ΔI through dS due to ΔA

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



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Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

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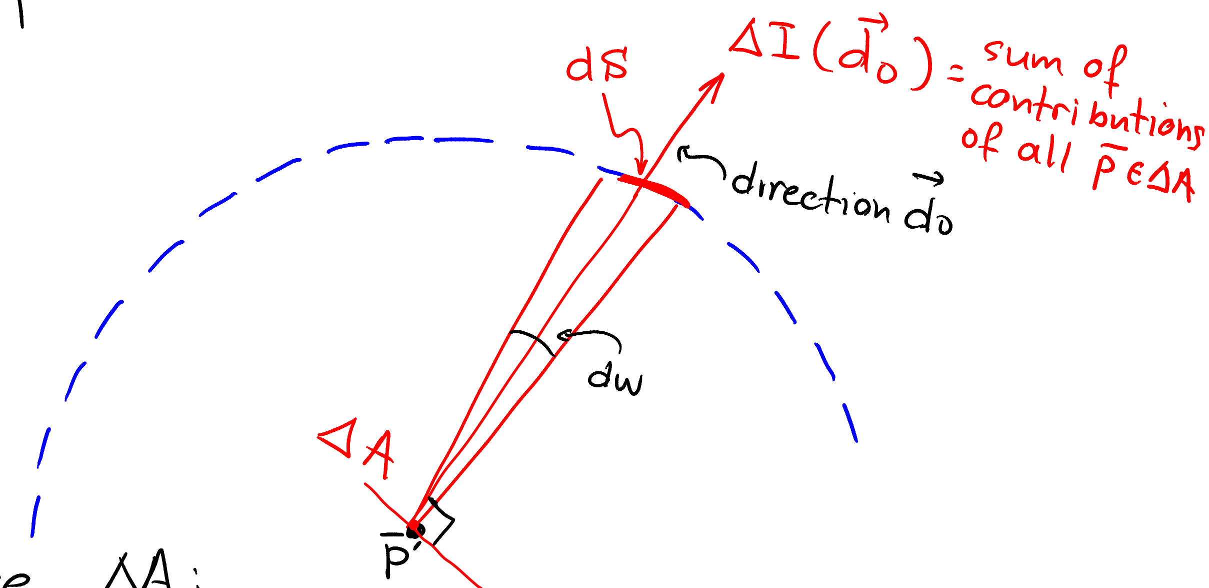


For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch

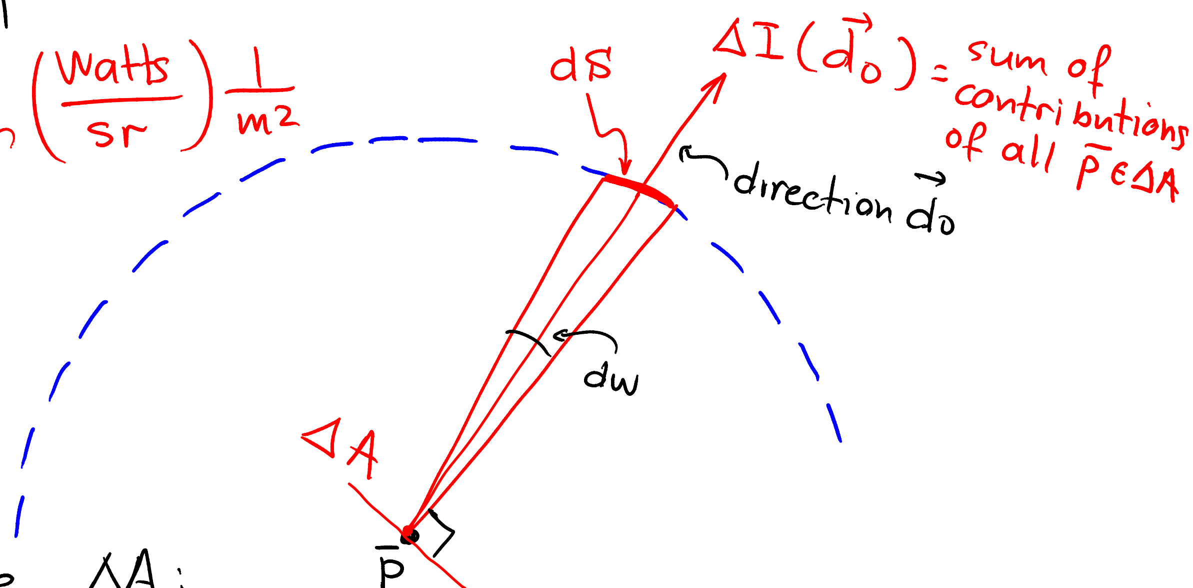
Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \lim_{\Delta A \to 0} \frac{\Delta I(\vec{d}_o)}{\Delta A}$$

$\left(\frac{\text{Watts}}{sr}\right)\frac{1}{m^2}$



For a patch source ΔA:

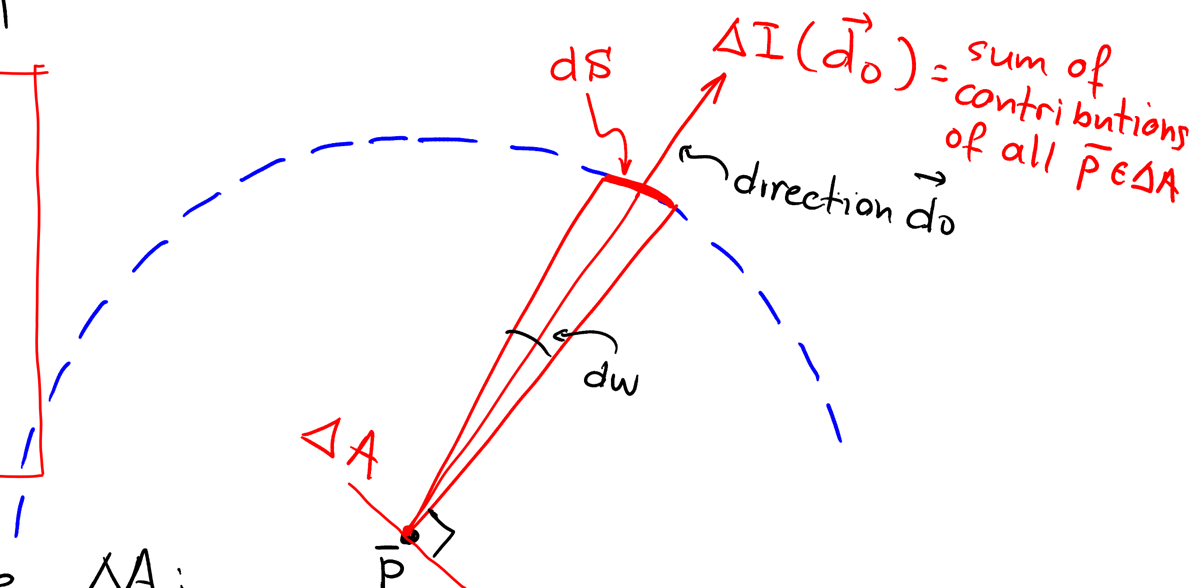
- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch (and take limit)

Definition of Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$\boxed{L(\bar{p}, \vec{d}_o) = \frac{dI}{dA} = \frac{d}{dA}\left(\frac{d\phi}{dw}\right) = \frac{d^2\phi}{dA\,dw} \quad (\text{in Watts}/sr \cdot m^2)}$$



For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch

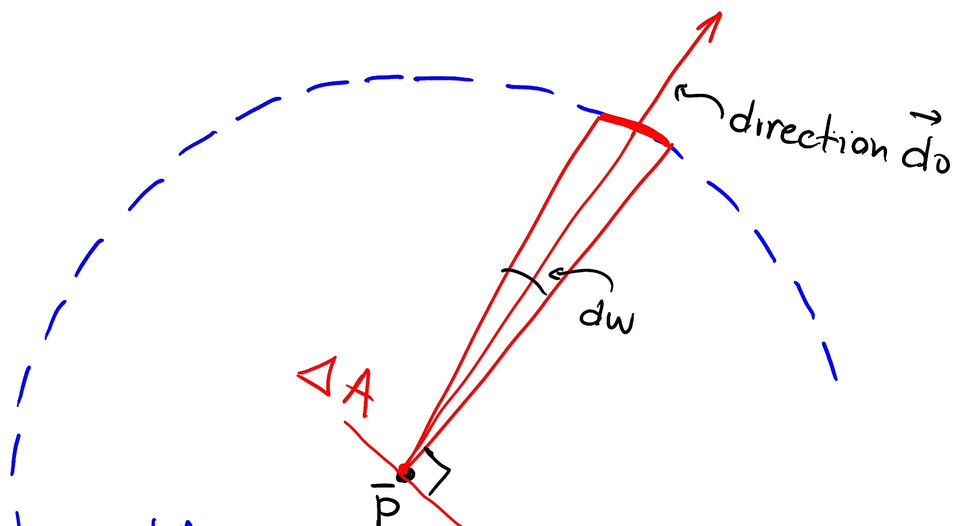
Definition of Radiance: Normal Exitance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \frac{dI}{dA}$$

$$= \frac{d}{dA}\left(\frac{d\Phi}{d\omega}\right) = \frac{d^2\Phi}{dA\, d\omega}$$



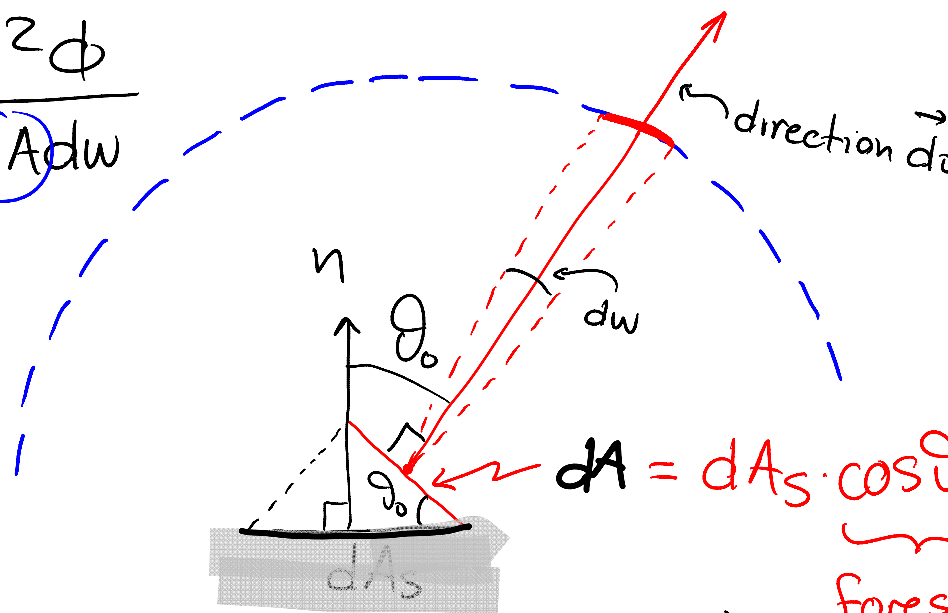
Attention: Division by dA assumes that patch is perpendicular to emission direction $\vec{d}_o$

Definition of Radiance for a Tilted Patch

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \frac{dI}{dA} = \frac{d^2\phi}{dA\, d\omega}$$



$$dA = \underbrace{dA_s \cdot \cos\vartheta_o}_{\text{foreshortening term}}$$

dA: patch orthogonal to $\vec{d}_o$

dAs: tilted surface patch

Definition of Radiance for a Tilted Patch

Surface Radiance for a tilted patch

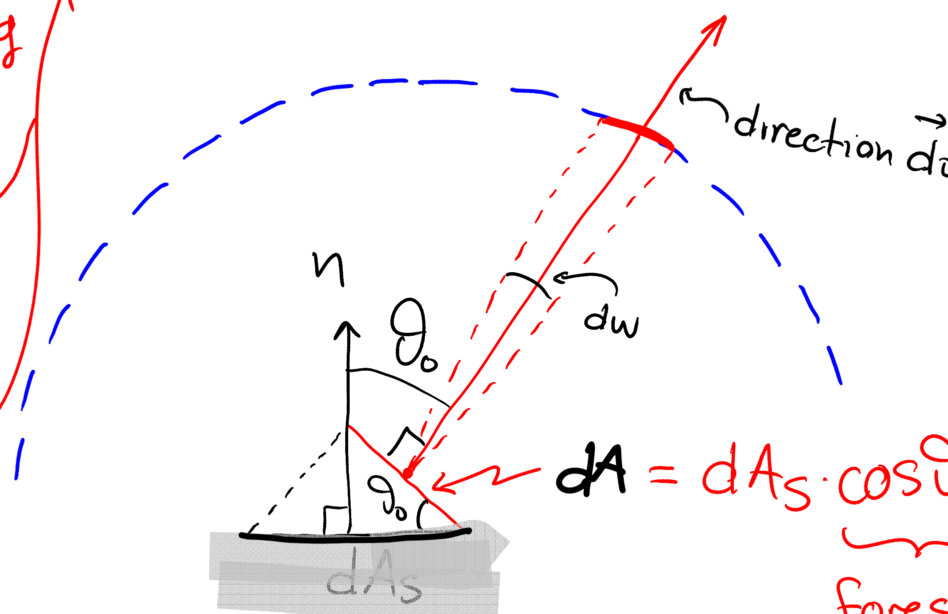
$$\frac{d^2\phi}{dA_s\,d\omega}(\bar{p},\vec{d}_o) = \cos\vartheta_o \cdot L(\bar{p},\vec{d}_o)$$
$$= (\vec{m}\cdot\vec{d}_o)\,L(\bar{p},\vec{d}_o)$$

combining (*), (**)

So we have:

$$(*)\quad L(\bar{p},\vec{d}_o) = \frac{d^2\phi}{dA\,d\omega}$$

$$(**)\quad dA_s = \frac{1}{\cos\vartheta_o}\cdot dA$$



$$dA = dA_s \cdot \underbrace{\cos\vartheta_o}_{\text{foreshortening term}}$$

dA: patch orthogonal to $\vec{d}_o$

dAs: tilted surface patch

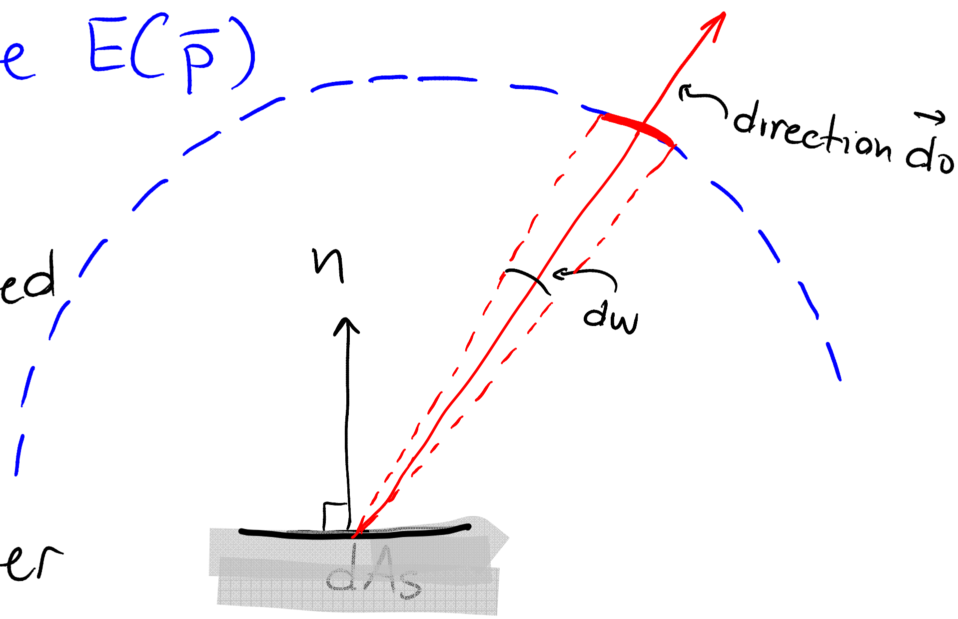
Measuring All Outgoing Light: Radiant Exitance

Surface Radiance for a tilted patch

$$\frac{d^2\phi}{dA_s\,d\omega}(\bar{p},\vec{d}_o) = (\vec{n}\cdot\vec{d}_o)\,L(\bar{p},\vec{d}_o)$$

Radiant exitance $E(\bar{p})$
(aka Radiocity)

Total flux emitted from dA_s in <u>all</u> directions



⇒ an integral over directions:

$$E(\bar{p}) = \int_{\vec{d}_o} (\vec{n}\cdot\vec{d}_o)\,L(\bar{p},\vec{d}_o)\,d(\vec{d}_o)$$

Topic 13:

Radiometry

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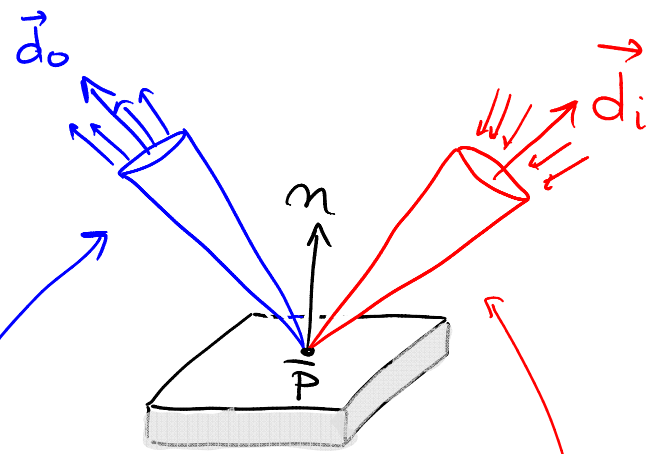
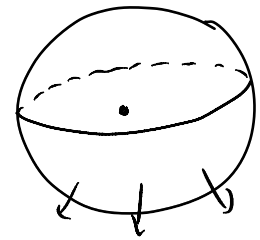
The Basic "Light Transport" Path





① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

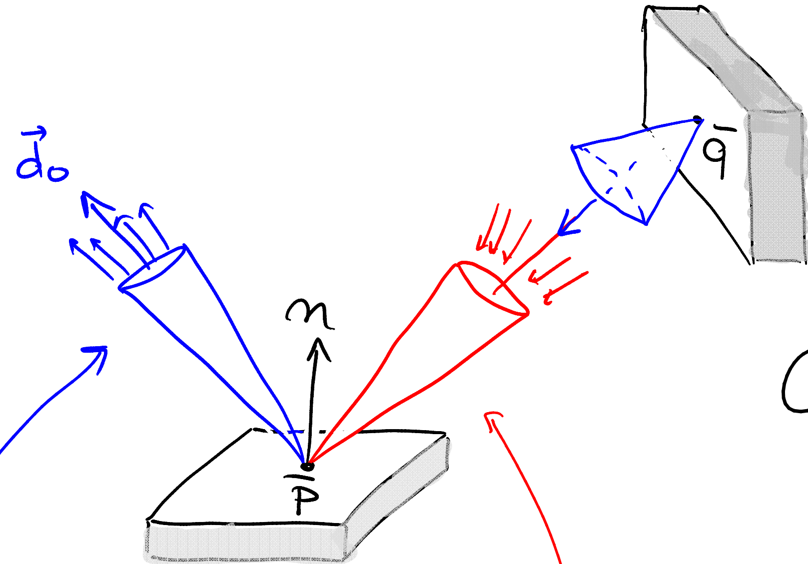
② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

watts/m^2

③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/(sr·m^2)

Light Transport Between Patches



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/(sr·m²)

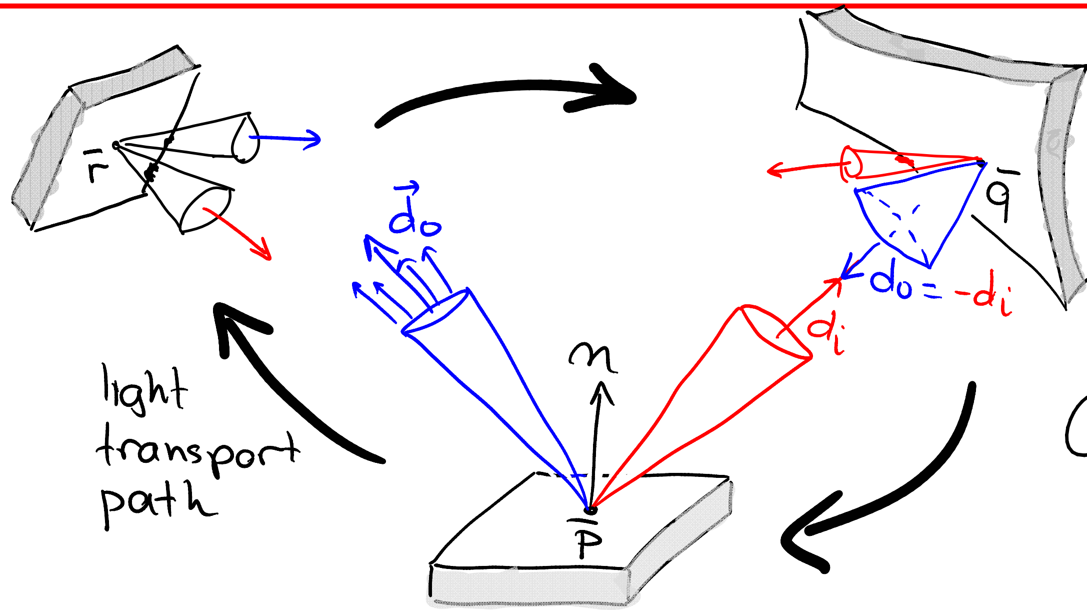
③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Now that we have defined radiance we can think of every surface point as a light source!

The General Light Transport Cycle



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/m²

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Watts/(sr·m²)

③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Now that we have defined radiance we can think of every surface point as a light source!

One Step Along Path: Directional Integration

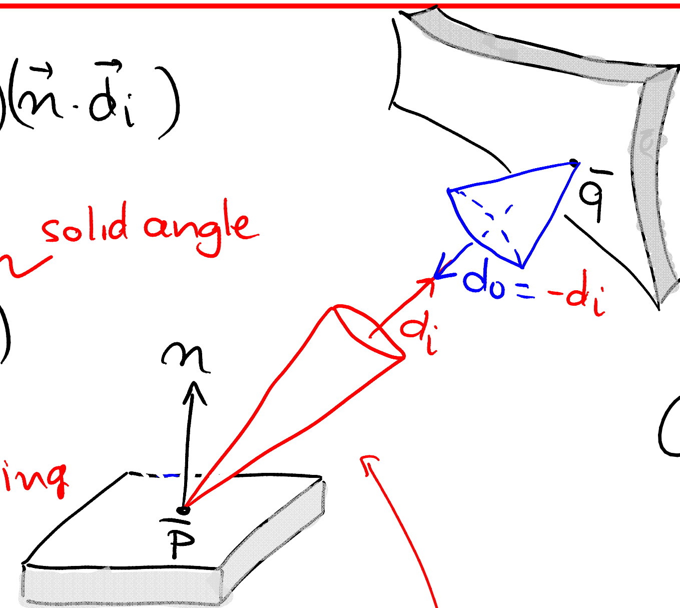
$$H(\bar{p}) = \int_{\text{all directions}} (\text{radiance along } -d_i)(\vec{n}\cdot\vec{d}_i)$$

$$= \int_{\vec{d}_i} L(\bar{q}, -d_i)(\vec{n}\cdot\vec{d}_i)\, d(d_i)$$

solid angle

radiance travelling to $\bar{p}$ from $\bar{q}$

foreshortening in case patch at $\bar{p}$ is slanted



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/m^2

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

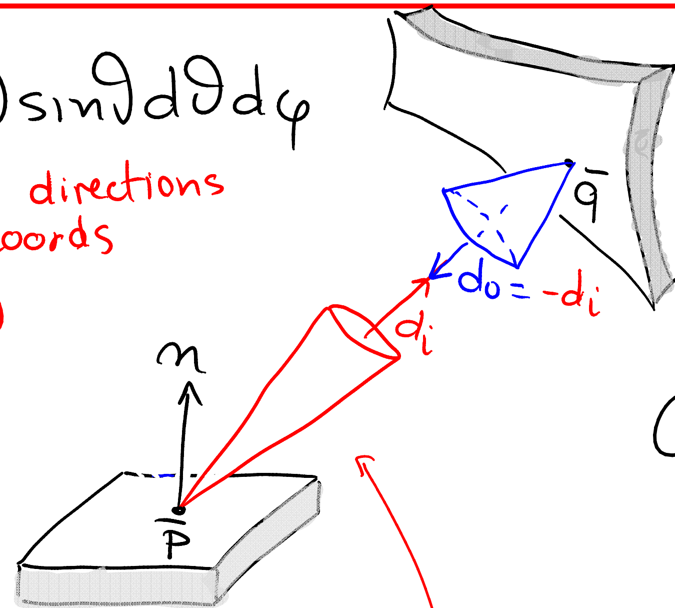
Now that we have defined radiance we can think of every surface point as a light source!

One Step Along Path: Directional Integration

$$H(\bar{p}) = \int_{\varphi}\int_{\vartheta} L(\bar{p}, -d_i)(\vec{n} \cdot \vec{d}_i)\sin\vartheta \, d\vartheta \, d\varphi$$

if we express directions in spherical coords

$(\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/m^2

② Irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Now that we have defined radiance we can think of every surface point as a light source!

Topic 13:

Radiometry

- The big picture
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- Measuring light leaving a patch: Radiance
- The Light Transport Cycle
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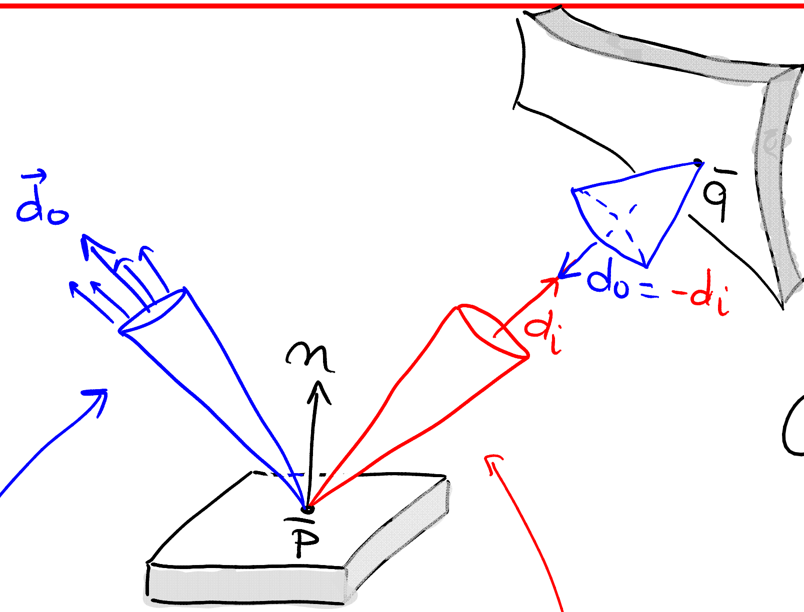
General Light Transport Cycle: Closing the Loop



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/m^2

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Watts/$(sr \cdot m^2)$

③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

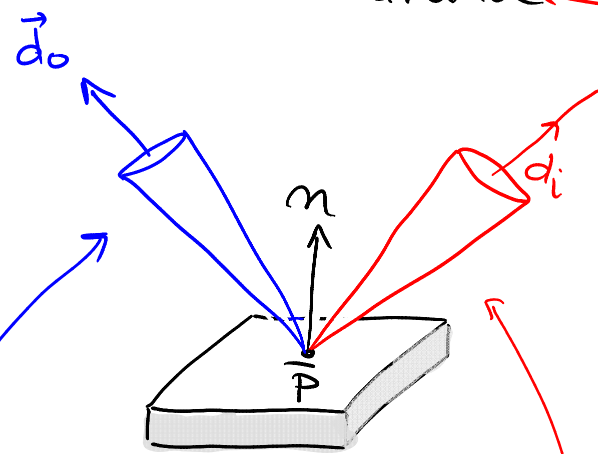
Q: How is incident light "transformed" to outgoing light?

Definition: The BRDF of a Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Watts/(sr·m²)

③ radiance $L(\bar{p}, \vec{d}_o)$
emitted from infinitesimal
patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

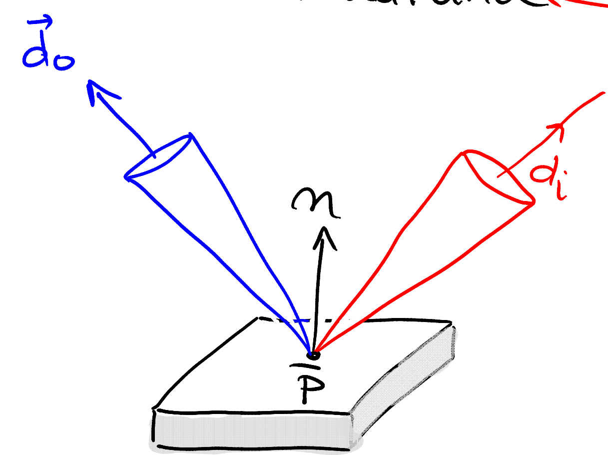
② irradiance $H(\bar{p})$ at an
infinitesimal patch due to
light received along one
or more (or a continuum of)
directions

Definition: The BRDF of a Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



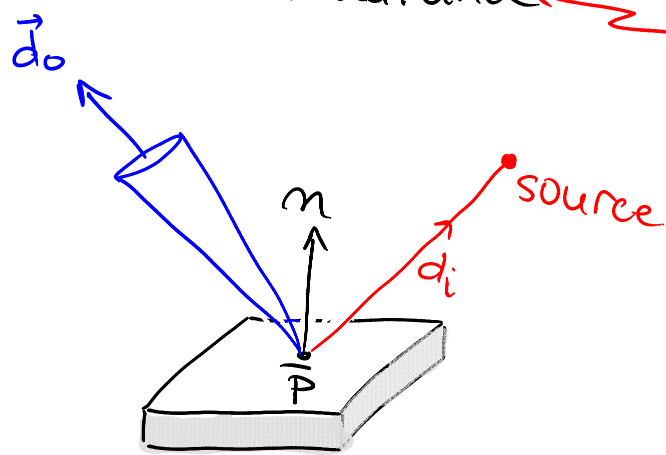
Intuition: The BRDF tells us how bright $\bar{p}$ will appear if viewed along $\vec{d}_o$ when it receives light from a small cone of directions along $\vec{d}_i$

Definition: The BRDF of a Point

BRDF: $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

in direction $\vec{d}_o$

due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Simpler: Suppose we only have a point light source

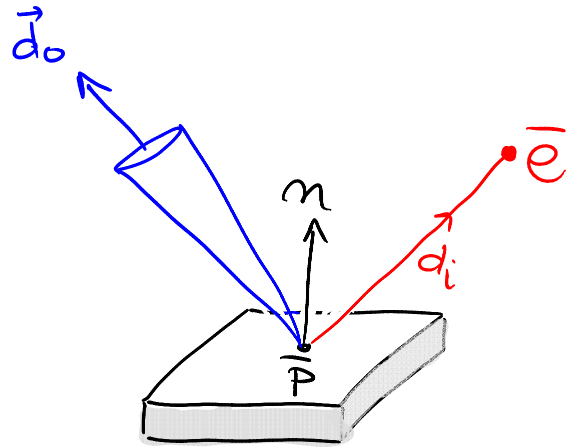
Intuition: The BRDF tells us how bright $\bar{p}$ will appear if viewed along $\vec{d}_o$ and the source is along $\vec{d}_i$

Radiance Due to a Point Light Source

BRDF: $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around



Example #1: Source is at $\bar{e}$

Radiant intensity is $I(\bar{p}-\bar{e})$

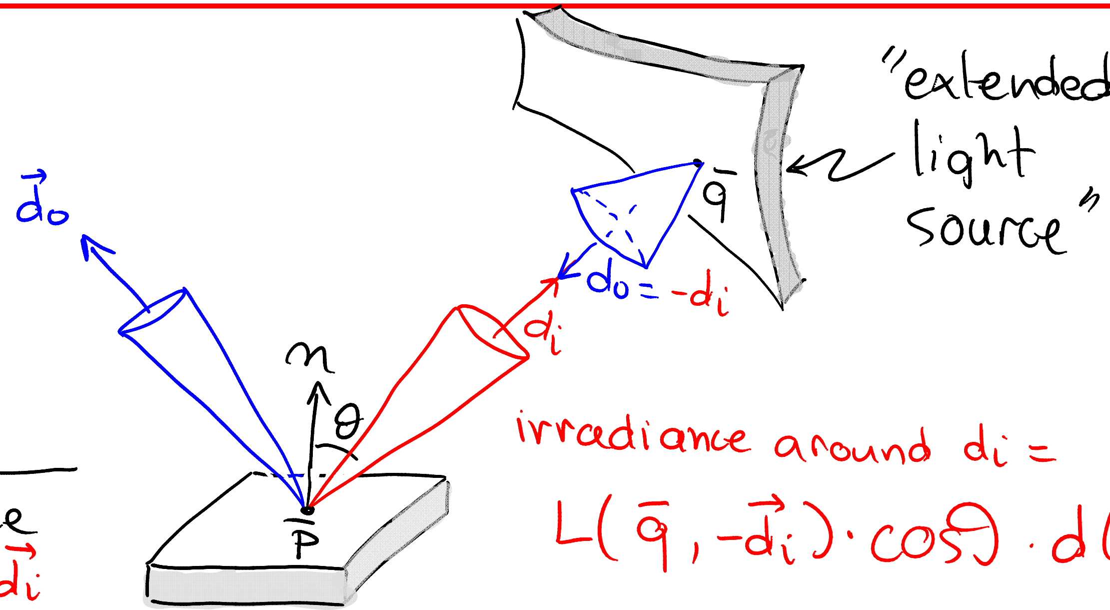
BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

Q: What is the radiance along $\vec{d}_o$?

$\boxed{\text{Reminder: } H(\bar{p}) = \dfrac{I(\omega)}{r^2}\cos\theta}$

Ans: $L(\bar{p}, \vec{d}_o) = \rho(\vec{d}_i, \vec{d}_o) H(\bar{p}) = \rho(\vec{d}_i, \vec{d}_o) \dfrac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2} \cos\theta$

Radiance Due to an Extended Source



along $\vec{d}_o$

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

around $\vec{d}_i$

irradiance around d_i =

$$L(\bar{q}, -\vec{d}_i) \cdot \cos\theta \cdot d(\vec{d}_i)$$

Example #2: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

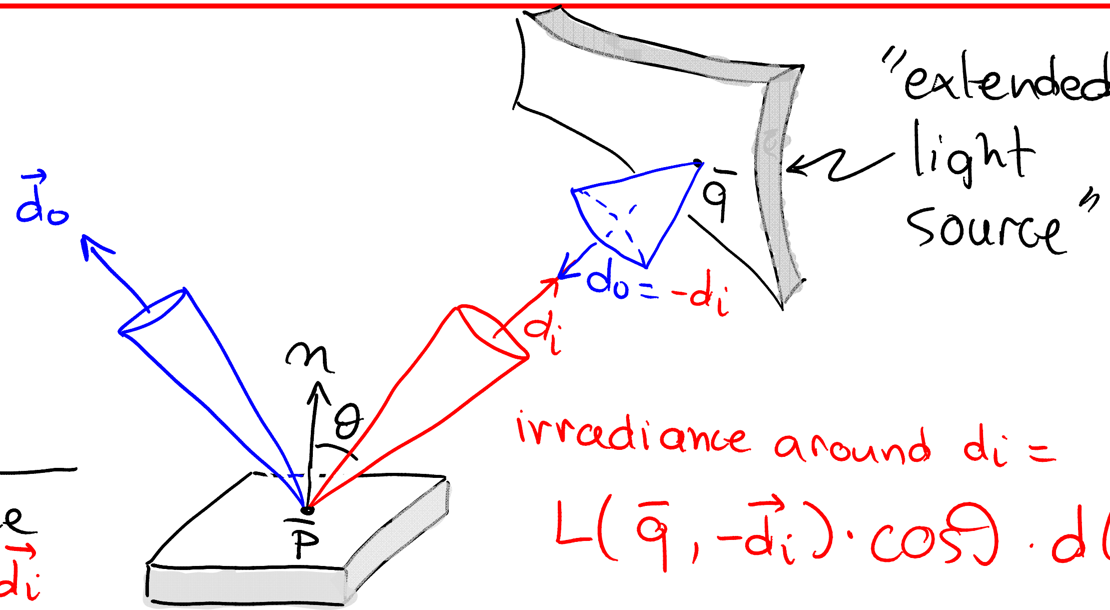
Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \int_{\vec{d}_i} \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot (\text{irradiance around } \vec{d}_i) \, d(\vec{d}_i)$$

$$\text{irradiance around } \vec{d}_i = L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)$$

Radiance Due to an Extended Source

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\overbrace{\text{radiance}}^{\text{along } \vec{d}_o}}{\underbrace{\text{irradiance}}_{\text{around } \vec{d}_i}}$$



irradiance around d_i =
$L(\bar{q}, -\vec{d}_i) \cdot \cos\theta \cdot d(\vec{d}_i)$

Example #2: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

Q: What is the radiance along $\vec{d}_o$?

Using spherical coords (θ, φ) for $\vec{d}_i$:

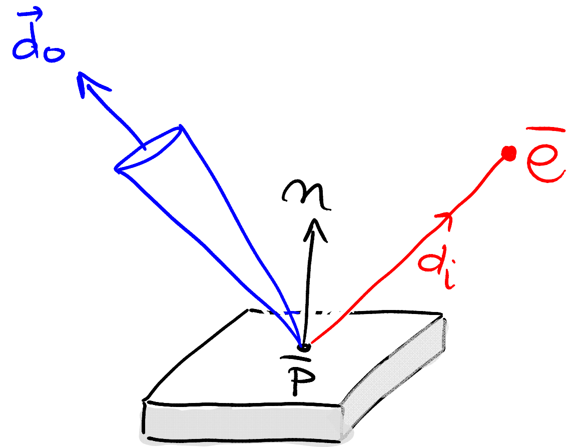
Ans: $$L(\bar{p}, \vec{d}_o) = \int_\theta \int_\varphi \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot \big(\underbrace{\text{irradiance around } \vec{d}_i}_{L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)}\big) \sin\theta \, d\theta \, d\varphi$$

The BRDF of a Diffuse Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around



Example #3: What is the BRDF of a diffuse surface point?

For diffuse points:

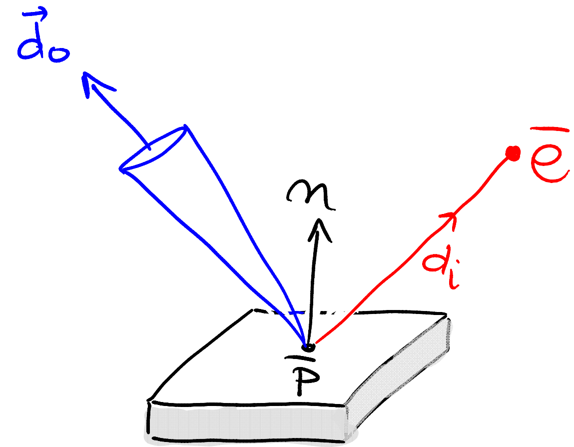
- brightness independent of $\vec{d}_o$
- brightness depends only on total incident flux (i.e. irradiance) not illumination dir

The BRDF of a Diffuse Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Example # 3: What is the BRDF of a diffuse surface point?

For diffuse points:

radiance = constant fraction of irradiance

$\Rightarrow \quad \rho(\vec{d}_i, \vec{d}_o) = \text{constant}$

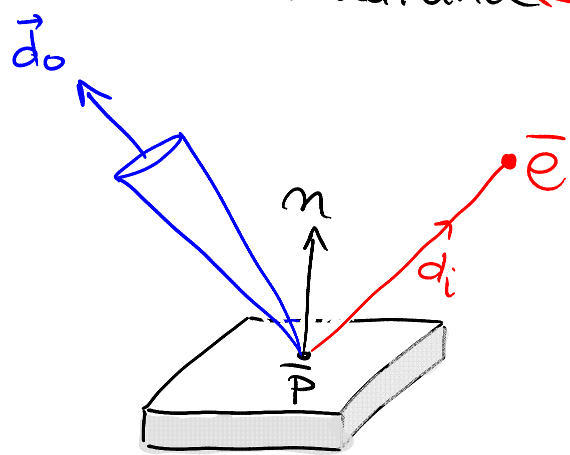
↑ what is it equal to?

The BRDF of a Diffuse Point

BRDF: $\rho_{\bar{p}}(\vec{d_i}, \vec{d_o}) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance — in direction $\vec{d_o}$

irradiance — due to flux arriving from an infinitesimal solid angle around $\bar{e}$



Example #3: What is the BRDF of a diffuse surface point?

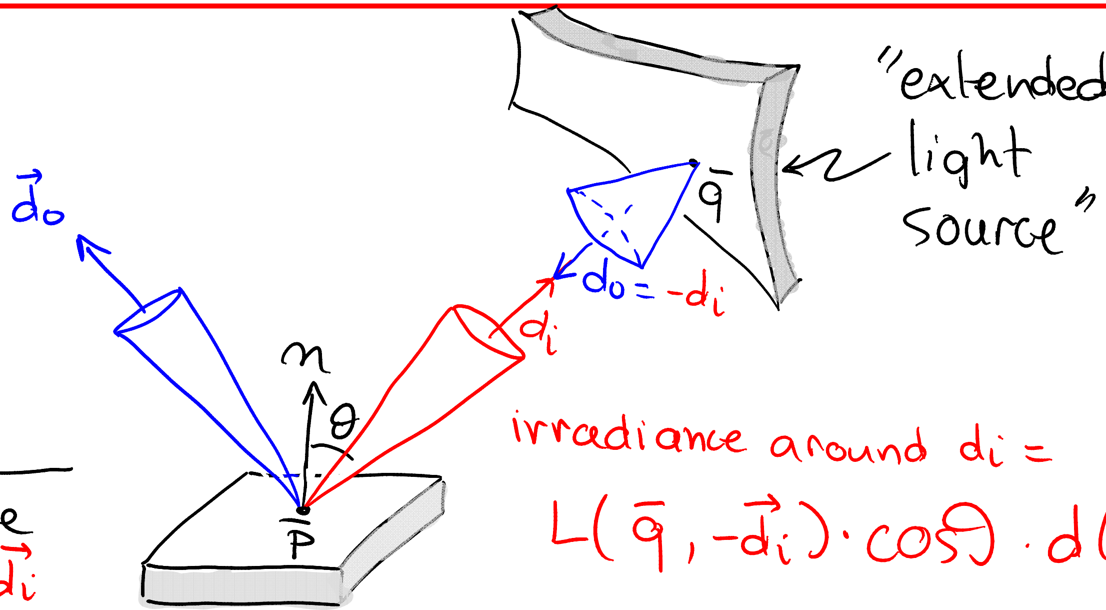
For diffuse points:

conservation of energy

total light coming in (irradiance) = total light going out (radiant exitance)

can show that $\rho = \frac{1}{\pi}$ (see Leonid's slides)

Radiance of a Diffuse Point Due to Extended Src



along $\vec{d}_o$

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

around $\vec{d}_i$

irradiance around d_i = $L(\bar{q}, -\vec{d}_i) \cdot \cos\theta \cdot d(\vec{d}_i)$

Example #4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

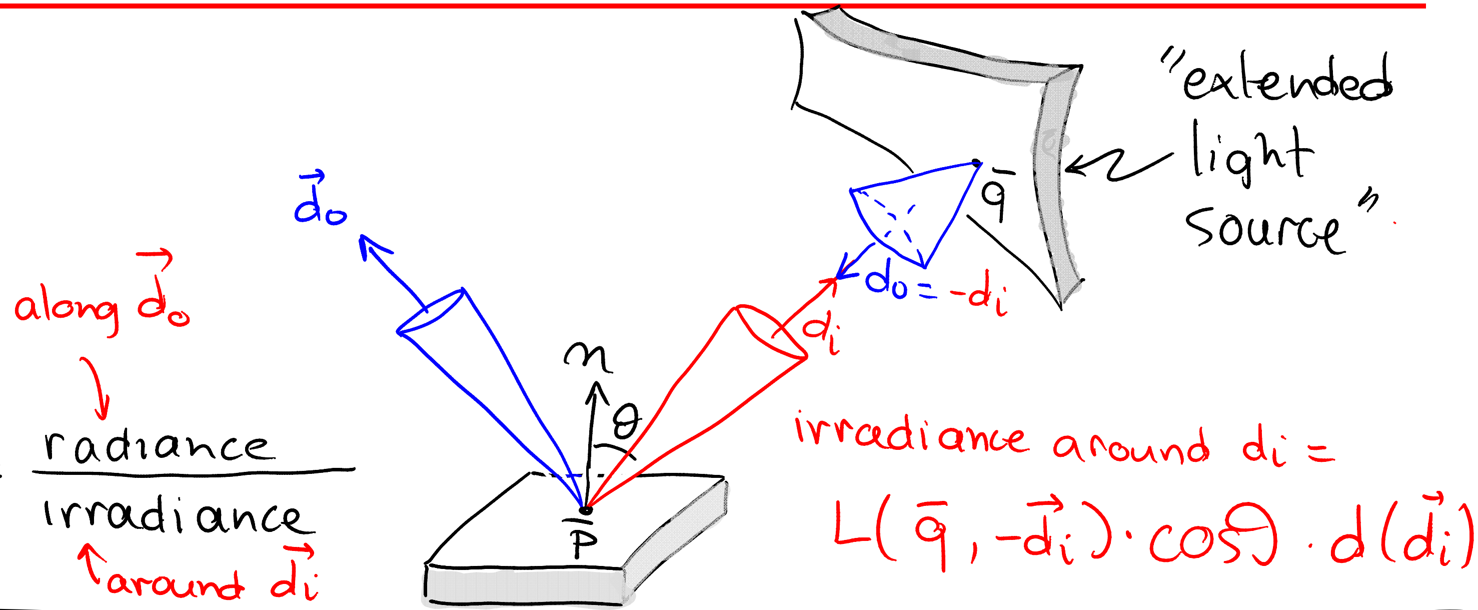
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \int_{\vec{d}_i} \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot (\text{irradiance around } \vec{d}_i)\, d(\vec{d}_i)$$

$$\text{irradiance around } \vec{d}_i = L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)$$

Radiance of a Diffuse Point Due to Extended Src



$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

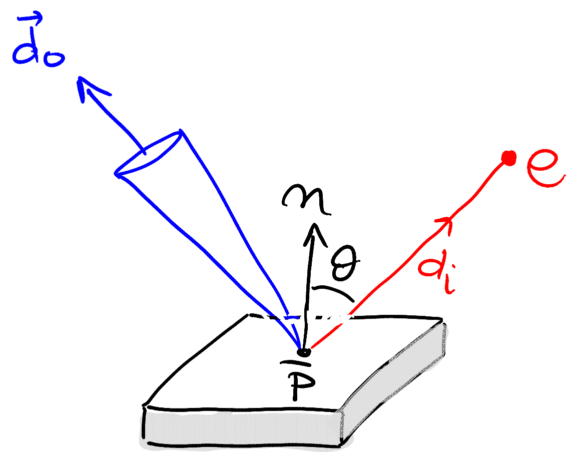
Example #4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \frac{1}{\pi} \int_{\vec{d}_i} L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)\, d(\vec{d}_i)$$

Radiance of Diffuse Point due to Point Light Src



Example # 5 : Point light source at distance r

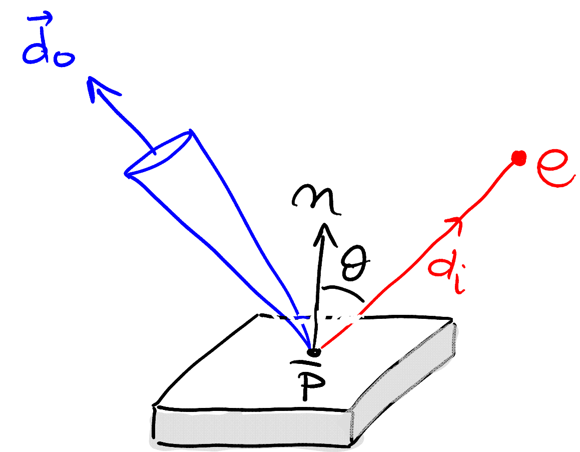
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \frac{1}{\pi} \cancel{\int_{\vec{d}_i}} L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)\, \cancel{d(\vec{d}_i)}$$ only one direction

$$\uparrow \frac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}$$

Radiance of Diffuse Point due to Point Light Src



Example # 5 : Point light source at distance r
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $L(\bar{p}, \vec{d}_o) = \frac{1}{\pi} \cdot \frac{I(\bar{p}-\bar{e})}{\cancel{\|\bar{p}-\bar{e}\|^2}} \cdot (\vec{n} \cdot \vec{d}_i) = \frac{1}{\pi} I(\bar{p}-\bar{e})(\vec{n} \cdot \vec{d}_i)$

can be ignored if light very far away

"Radiometrically-Correct" Ray Tracing

Basic loop:

for each pixel $\bar{q}$

① cast ray r through $\bar{q}$

② find 1st intersection of $\bar{q}$ with scene (i.e. point $\bar{p}$)

③ estimate amount of light reaching $\bar{p}$

④ estimate radiance $L(\bar{q}, \bar{p}-\bar{q})$ from $\bar{p}$ to $\bar{q}$

Implemented by

- spawning a large set of rays at each step
- directional integration to compute $H(\bar{p})$