Advance Image Synthesis: Exercises 30.06.2011

Technische Universität Berlin

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1. Properties of Monte Carlo Estimators Develop a *biased* Monte Carlo estimator for integration over the sphere. Estimate or compute the bias.

2. Monte Carlo Estimator and Importance Sampling Implement Monte Carlo integration for the hemisphere $\mathcal{H}^2$ when the integrand is cosine-weighted as in the shading equation,

$$I[f] = \int_{\mathcal{H}^2} f(\omega)\, \cos\theta \, d\omega,$$

for arbitrary $f : \mathcal{H}^2 \to \mathbb{R}$.

a) Implement uniform sampling over the hemisphere $\mathcal{H}^2$ using the formula for uniform sampling over the sphere $\mathcal{S}^2$ that we derived in class.

b) Derive the formula for importance sampling the hemisphere according to the cosine distribution. Have a look at the Stanford lecture notes[1] if you need inspiration. Implement the formula.

c) Implement stratified sampling for the hemisphere.

d) Visualize the samples on the sphere to ensure the distributions are meaningful.

e) Compare the convergence behaviour when the function to be integrated is a spherical harmonic.[2] Does the behaviour change for different orders of spherical harmonics?

3. Bonus: Quadrature on the Sphere Compare the Monte Carlo techniques in Question 2 to a suitable quadrature rule on the sphere (hint: you should consider the space $\mathcal{H}_{\leq L}$ of all spherical harmonics up to band L).

References

Hanrahan, P., E. Veach, and D. Zorin. *Mathematical Models for Computer Graphics Lecture Notes*. 1997. URL: http://www-graphics.stanford.edu/courses/cs448-97-fall/notes.html.

[1] Hanrahan, Veach, and Zorin, *Mathematical Models for Computer Graphics Lecture Notes*, Lecture 8, Exercise 2.4; also at http://www.dgp.toronto.edu/people/lessig/teaching/spring2011/ais/data/stanford_math_for_cg.pdf

[2] A Matlab function that evaluates an arbitrary spherical harmonic is available here: http://www.dgp.toronto.edu/people/lessig/teaching/spring2011/ais/data/shEval.m.