CSC 270F, Fall 1996 Fundamental Data Structures and Techniques
Brief solutions to Midterm by Mikhail Soutchanski

1 Short Answer - 15 points.

1. Not if the function is discontinuous on [a,b], like $f(x) = \frac{1}{x}$ and $a < 0$, $b > 0$. If function is continuous on the interval [a,b], then according to the Intermediate Value Theorem, if $f(a) * f(b) < 0$, the function has a root and bisection mathod will find one.

2. Let X_0 be a point such that tangent to f(x) at X_0 goes through the point (0,0) (such point X0 exists and it is not a multiple of π). Hence, X_1=0, but because tangent to f(x)=cos(x) at 0 is horizontal, Newton's method will NOT converge if it starts at X_0.

3. Yes. Each undirected edge u–v can be represented in adjacency matrix as two arcs $u \to v$ and $v \to u$; in other words, adjacency matrix for an undirected graph must have diagonal symmetry.

4. No. The general form of the composite trapezoid rule is given in the formula (9.3.25) (see Readings, p.439 of the section 9.3) which in our case ($h = x_2 - x_1 = x_3 - x_2$) amounts to

$$\begin{array}{l} \int_{x_1}^{x_3} f(x)\,dx \;=\; \int_{x_1}^{x_2} f(x)\,dx \;+ \int_{x_2}^{x_3} f(x)\,dx \;= \\ \frac{h}{2}\cdot(f(x_1)+f(x_2)) + \frac{h}{2}\cdot(f(x_2)+f(x_3)) \;= \frac{h}{2}\cdot(f(x_1)+2\cdot f(x_2)+f(x_3)) \end{array}$$

   Similarly, the general form of the composite Simpson rule is given in the formula (9.3.26). In our case, it reduces to

$$\int_{x_1}^{x_3} f(x)\,dx \;= \frac{h}{3}\cdot(f(x_1)+4\cdot f(x_2)+f(x_3))$$

   Obviously, both rules take the same number of operations (additions and multiplications). Hence, Simpson's rule is **not** faster than the Trapezoid rule for our computation.

2 Secant Method - 10 points.

1. At $x_{i-1} = 1$ the function $f(x) = x^2$ has the value 1.
   At $x_i = 4$ the function has the value 16. The secant is the line going through (1,1) and (4,16). This line intersects the axis x at some point x_{i+1} such that $x_{i-1} - x_{i+1} = \delta x$

2. We are given 2 right angled triangles. The bigger one has sides 4-1=3 and 16-1=15, while the smaller one has sides 1 and δx. Because sides of triangles are proportional to each other, we can compute

$$\frac{1}{\delta x} = \frac{15}{3}$$

   Thus, δx=0.2, and consequently, $x_{i+1} = x_i - \delta x$=0.8

3. No. It converges arbitrary closely (if we ignore numerical rounding errors), but never reaches it, since the secant line always passes below (0,0).

3 C Programming - 10 points.

```
void find_path (int s, int d)

{
  int i;

  i = s;
  while (i != d ) {
    printf( "%d\n", i);
    i = next_on_path(s,i,d);
  }
  printf( "%d\n", i);
}
```

4 Dijkstra's Algorithm - 10 points.

Let $\delta(u, v)$ denote the weight of the shortest path from u to v, and let $d[x]$ denote the upper bound on the length of the shortest path from the source vertex s to x (see lecture notes for more details).

We denote by the capital letter S the set of already "settled" vertices of a graph. Dijkstra's Algorithm can be summarized as follows:

- At each step, add to S the vertex x outside S that has the smallest value $d[x]$.
- After adding x, update $d[y]$ for all vertices y outside S that are adjacent to x iff $d[y]$ is greater than the sum of $\delta(s, x)$ and the weight of the edge (x, y).

Solution of the part 1.
Because $d[u]$=6 is the smallest value according to the figure, at the first step we add u to the set of settled vertices: $S \leftarrow S \bigcup u$. Because $delta(s, u) = 6$, weight of the edge (u, v) equals 5 and $d[v] = 15 > 6 + 5$, we set $d[v]$ to 11.

The second step, $S \leftarrow S \bigcup y$, because now $d[y]$ has the smallest value 7. Because $delta(s, y) = 7$, weight of the edge (y, w) equals 1 and $d[w] = 9 > 7 + 1$, we have to adjust $d[w]$ to 8.

The third step is $S \leftarrow S \bigcup w$, because $d[w]$ has the smallest value 8 among the unsettled vertices {w,v,x}. Because $delta(s, w) = 8$ and weight of the edge (w, v) is –4 (negative), the new value of $d[v]$ becomes 4=8–4 (istead of 11 before).

Of course, at the forth step we add v to the set S of settled vertices. At the last step, we add the vertix x to S.

$vertex$	u	y	w	v	x
$\delta(s, vertex)$	6	7	8	4	9

Solution of the part 2.
The path could go out the edge with negative weight as many times as it wants. That is, the shortest path can have an arbitrary negative weight.