Volumetric Michell Trusses for Parametric Design & Fabrication

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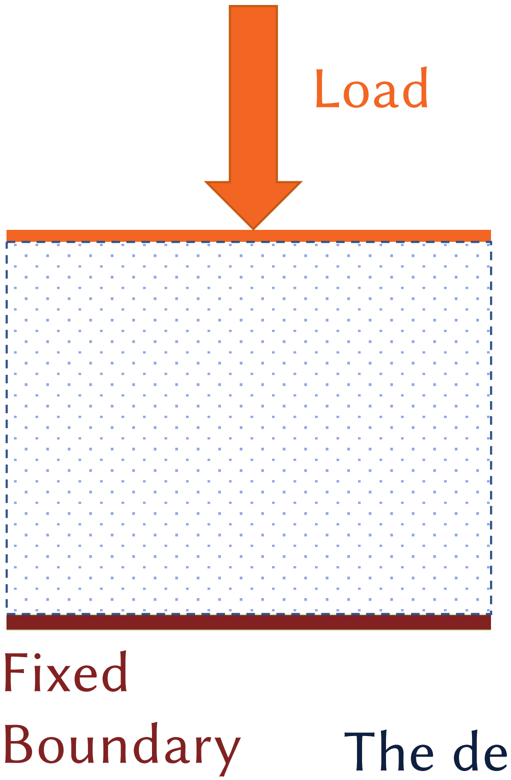
with

Alec Jacobson, Timothy R Langlois, Yijiang Huang, Caitlin Mueller, Wojciech Matusik, Ariel Shamir, Karan Singh, David IW Levin

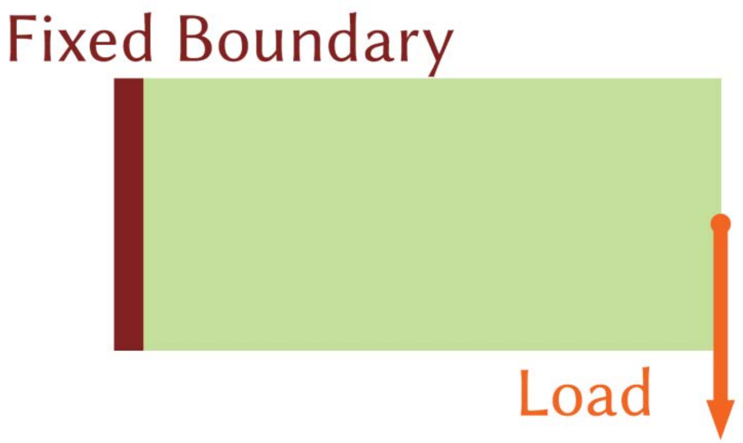
Structural Optimization

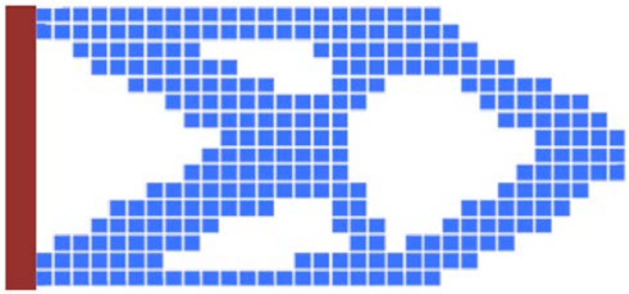


The design of *optimal* load-carrying structures

Engineer image by GraphicMama-team from Pixabay.

Topology Optimization

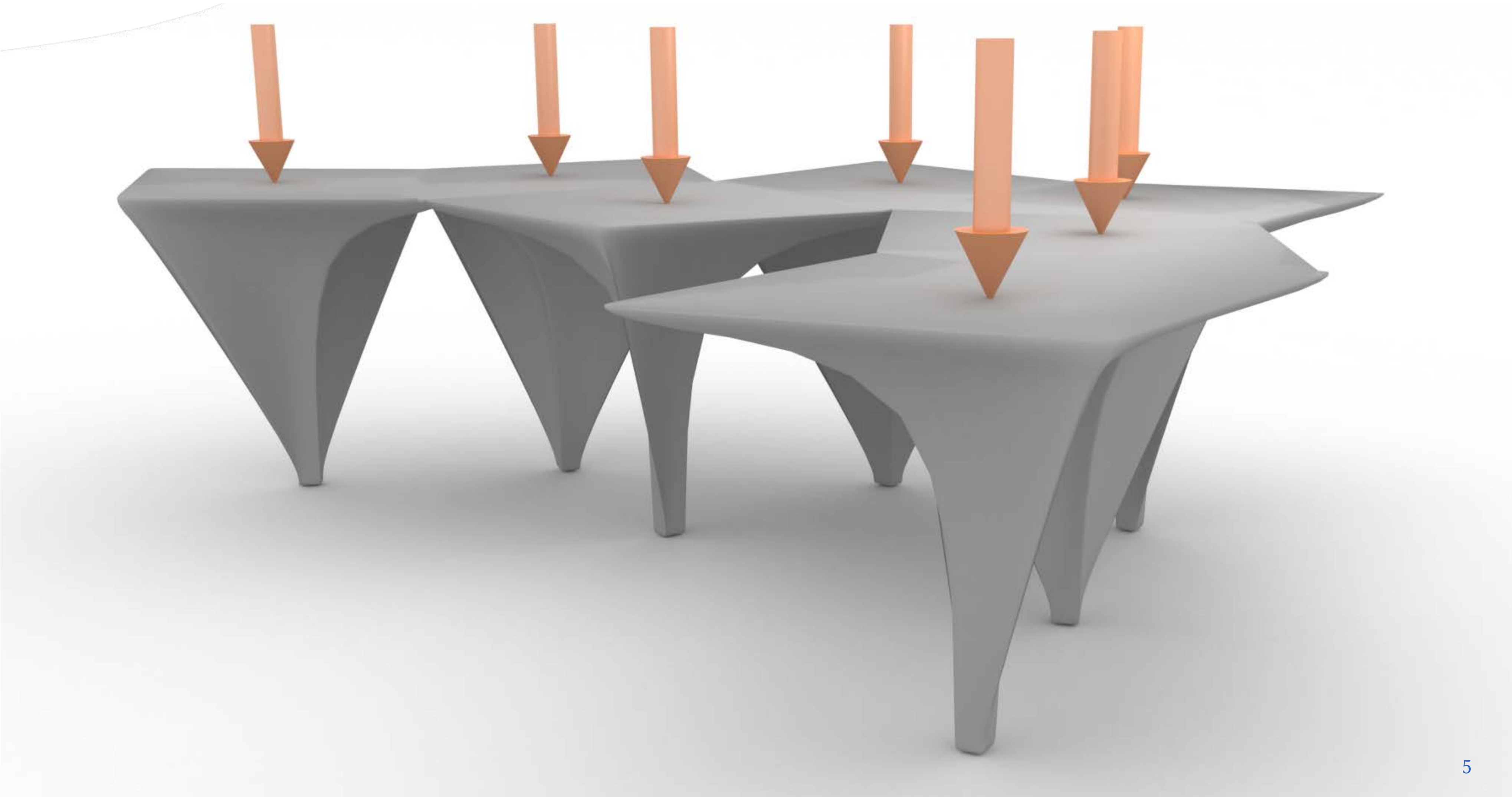


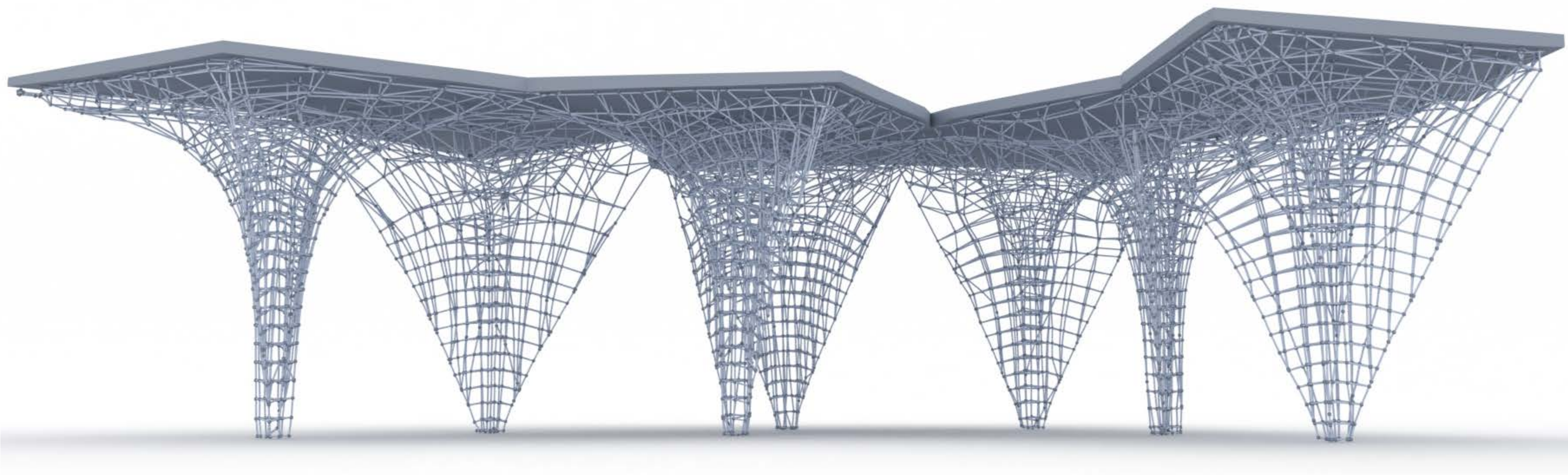


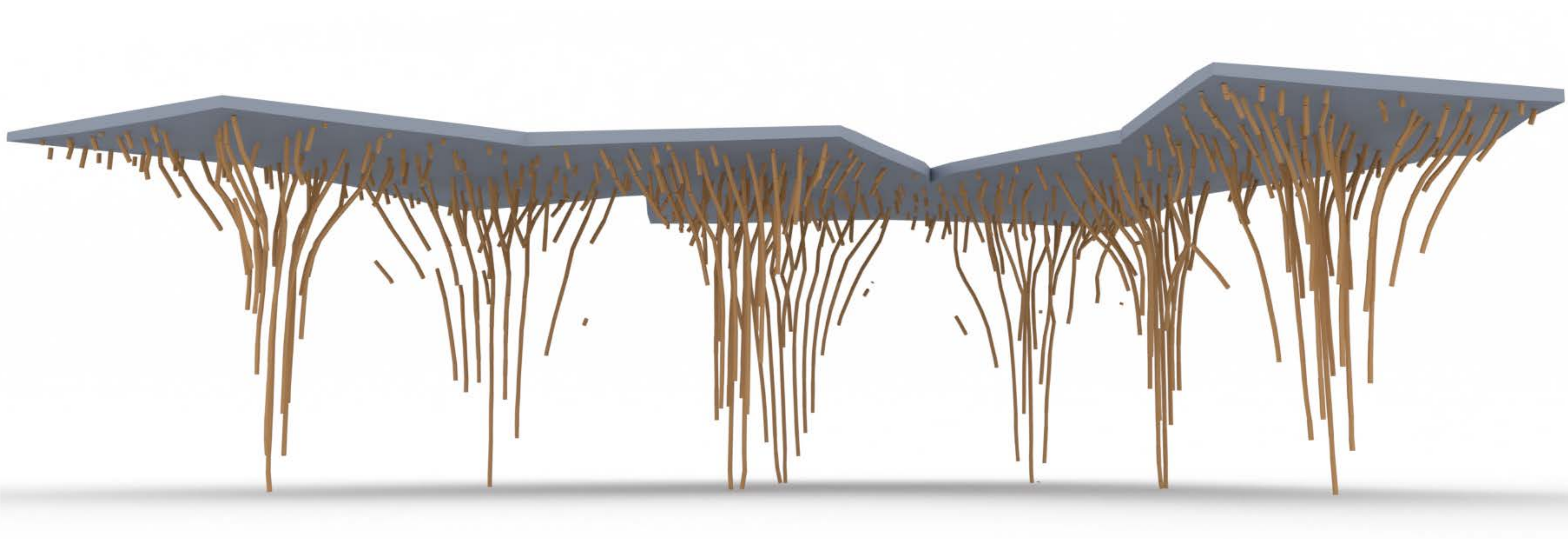
Structural optimization methods that can introduce topological changes

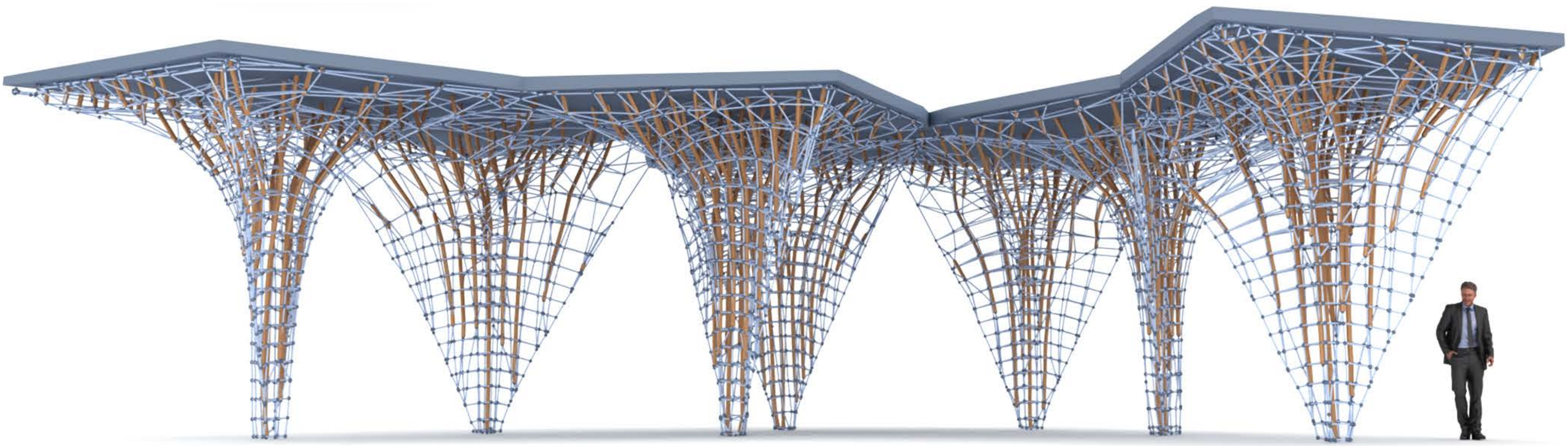
A **user-centric approach** to structural optimization.

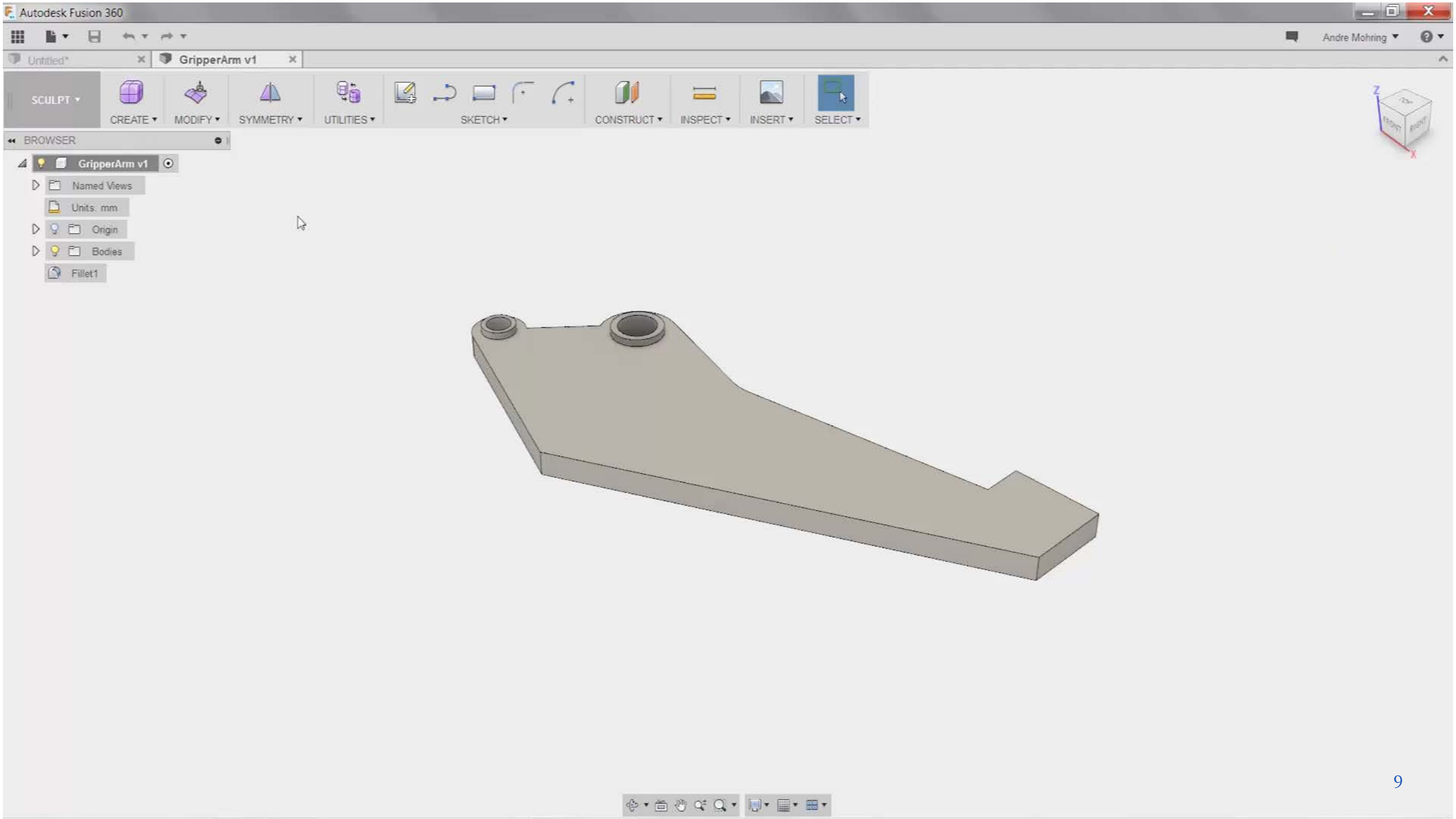
By generating a **parametrized output**, our method generates structures that can be **easily controlled and edited** a posteriori.

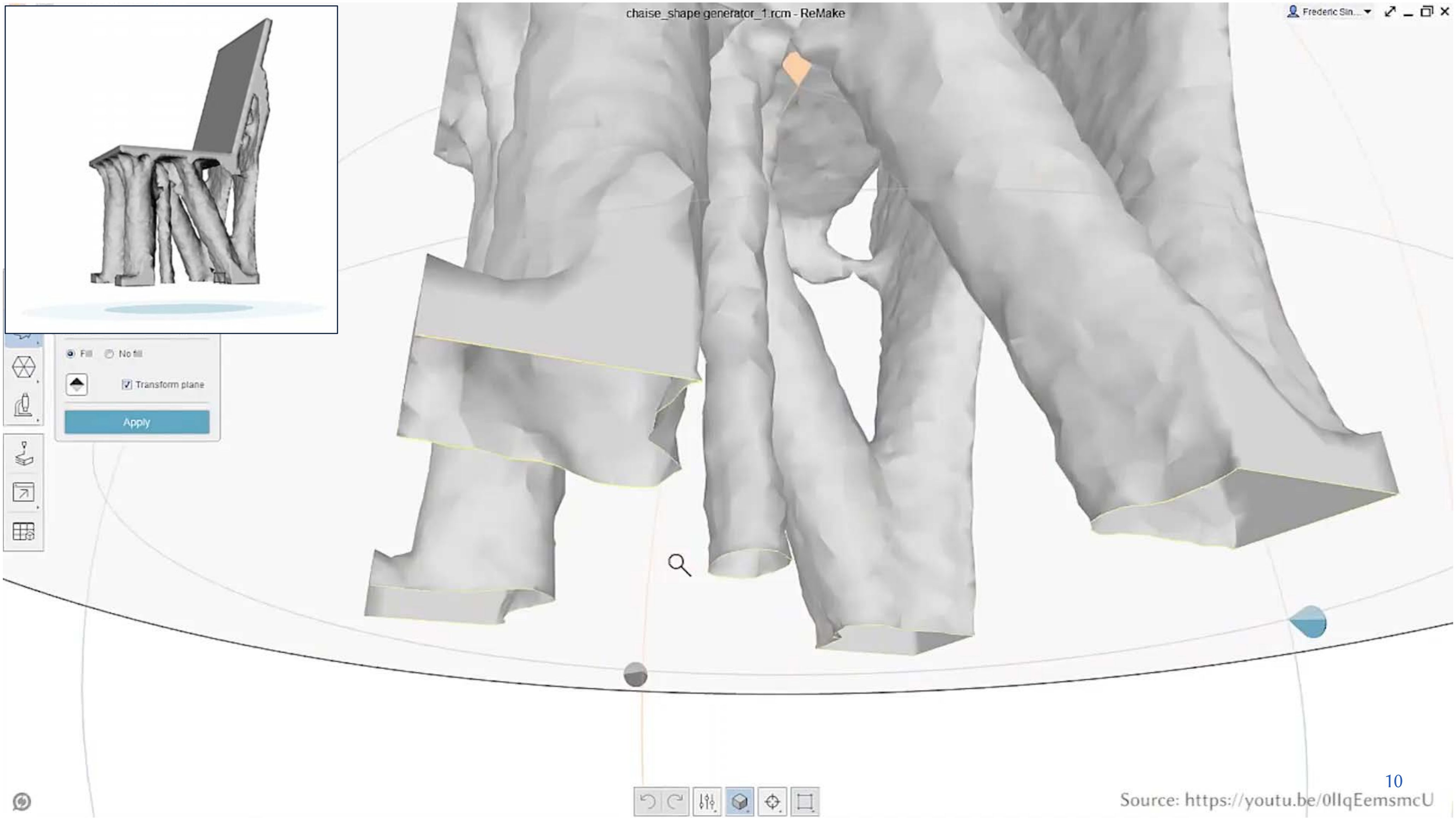




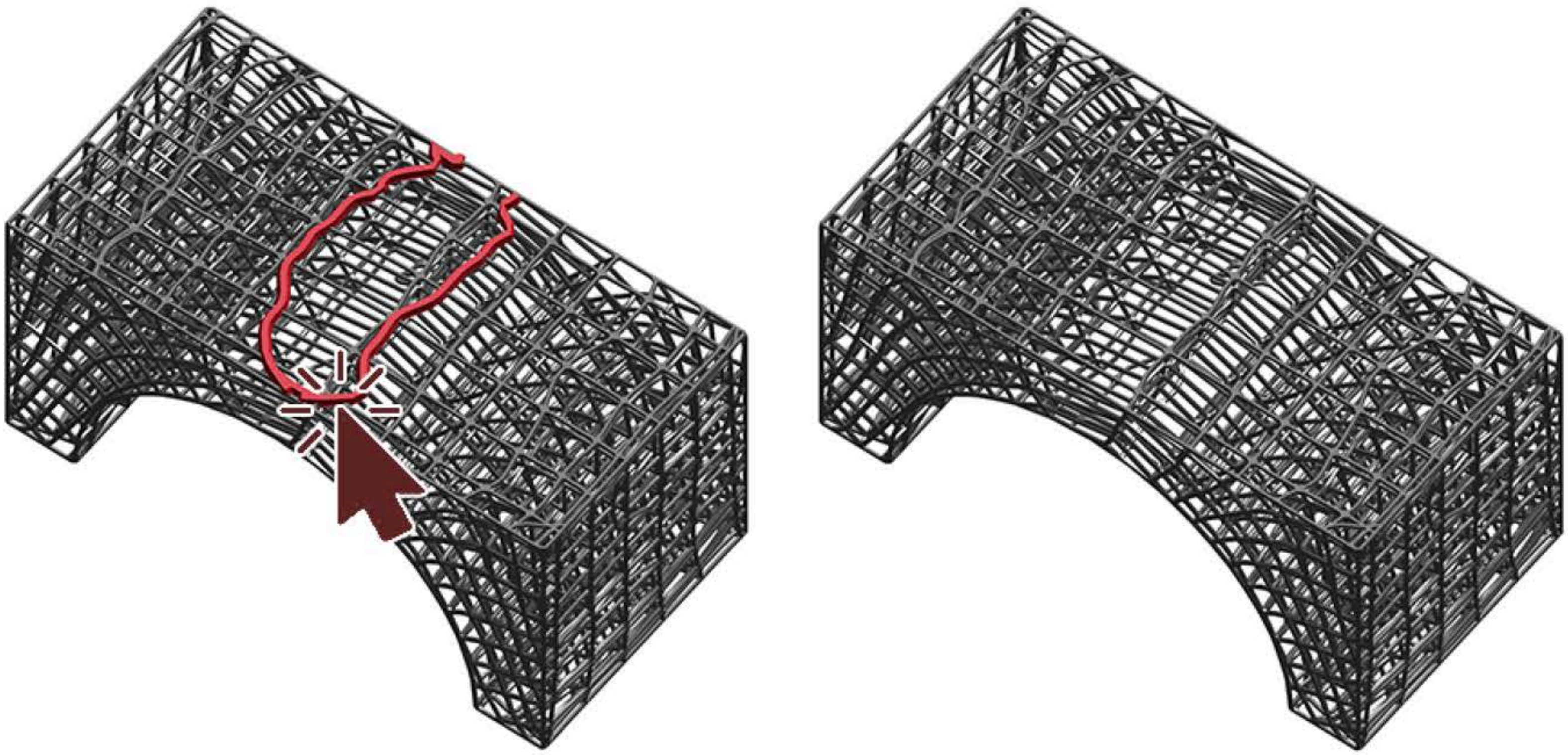


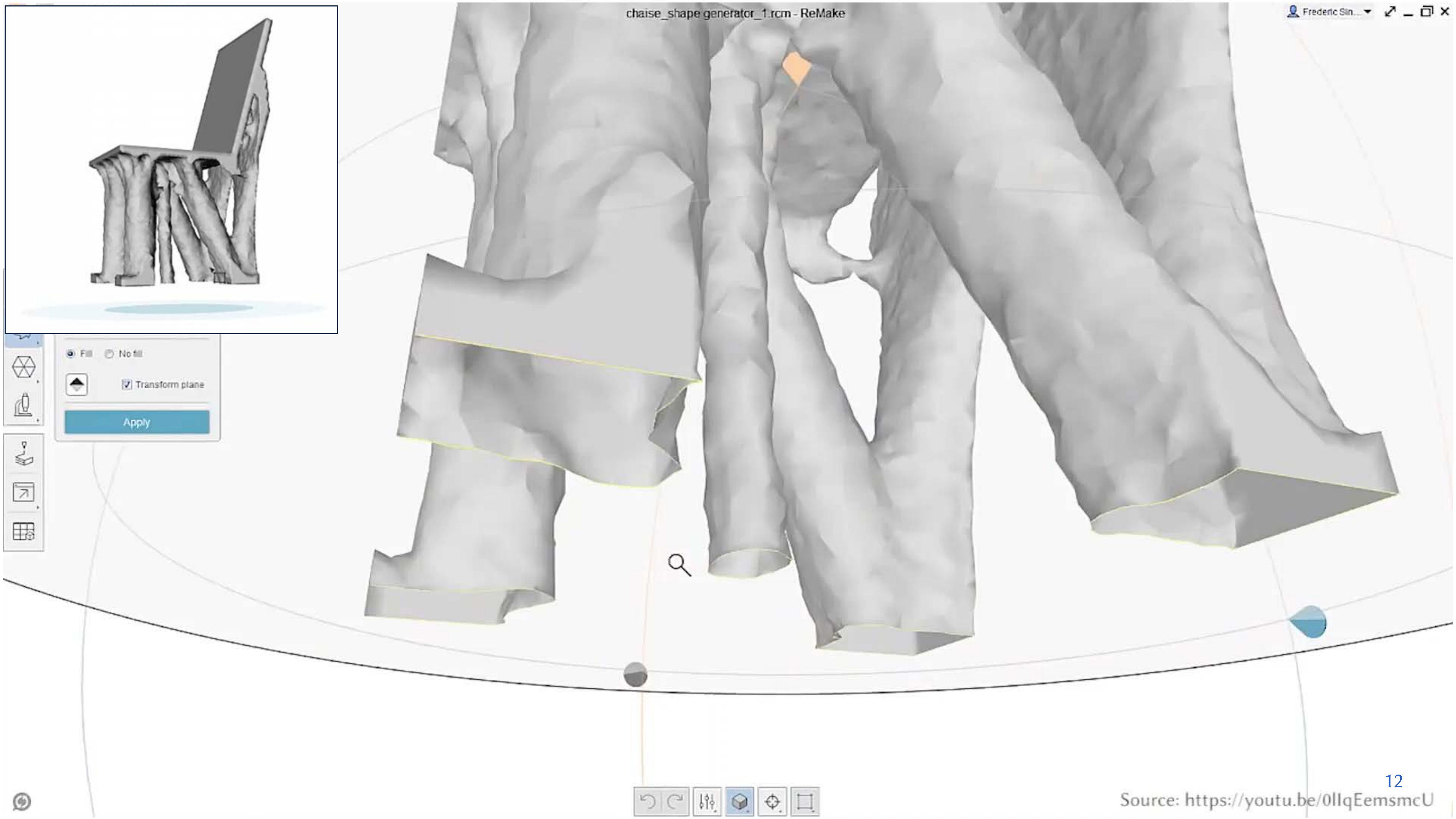



Autodesk Fusion 360
Andre Mohring
Untitled*
GripperArm v1
SCULPT
CREATE
MODIFY
SYMMETRY
UTILITIES
SKETCH
CONSTRUCT
INSPECT
INSERT
SELECT
BROWSER
GripperArm v1
Named Views
Units: mm
Origin
Bodies
Fillet1



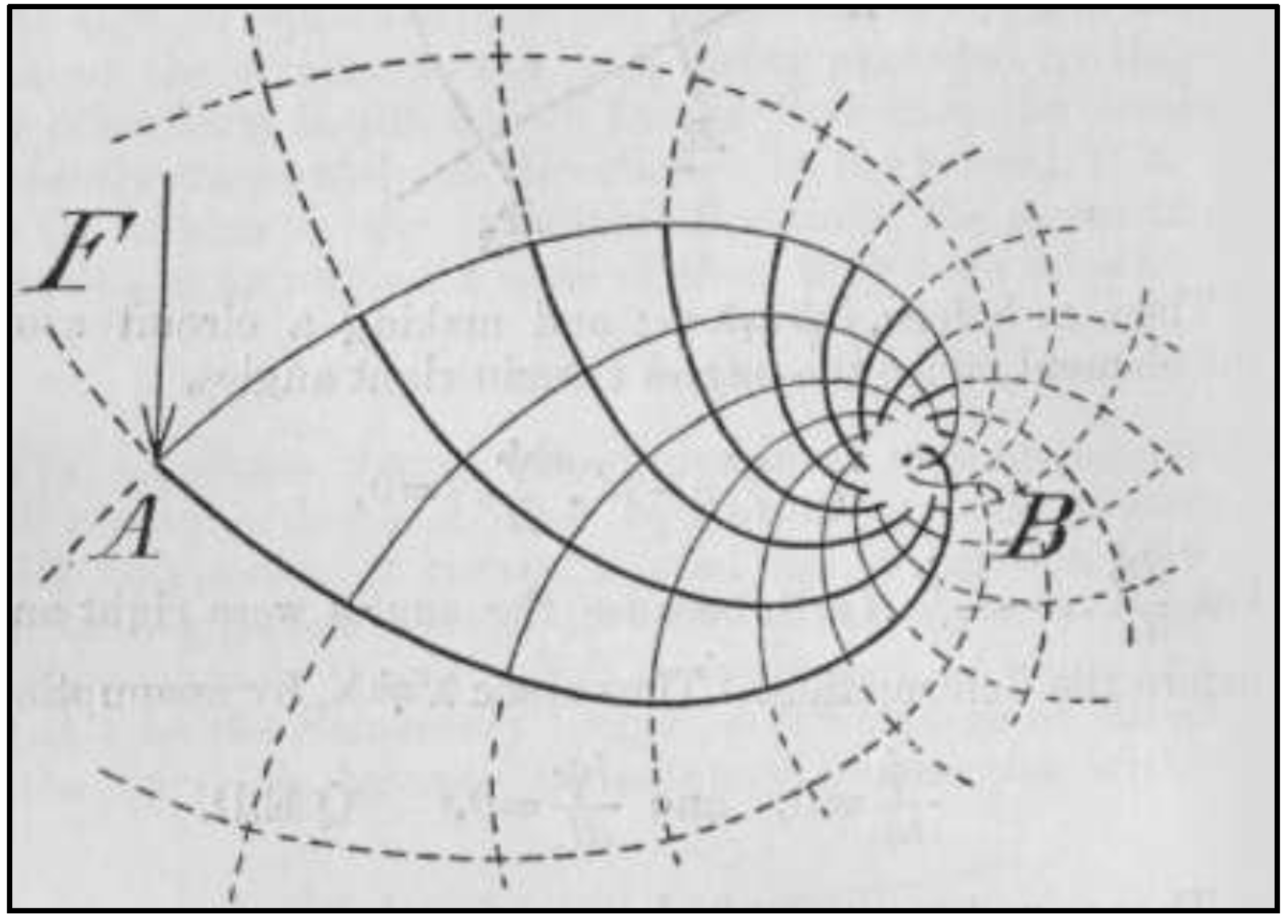
Source: https://youtu.be/0lIqEemsmcU



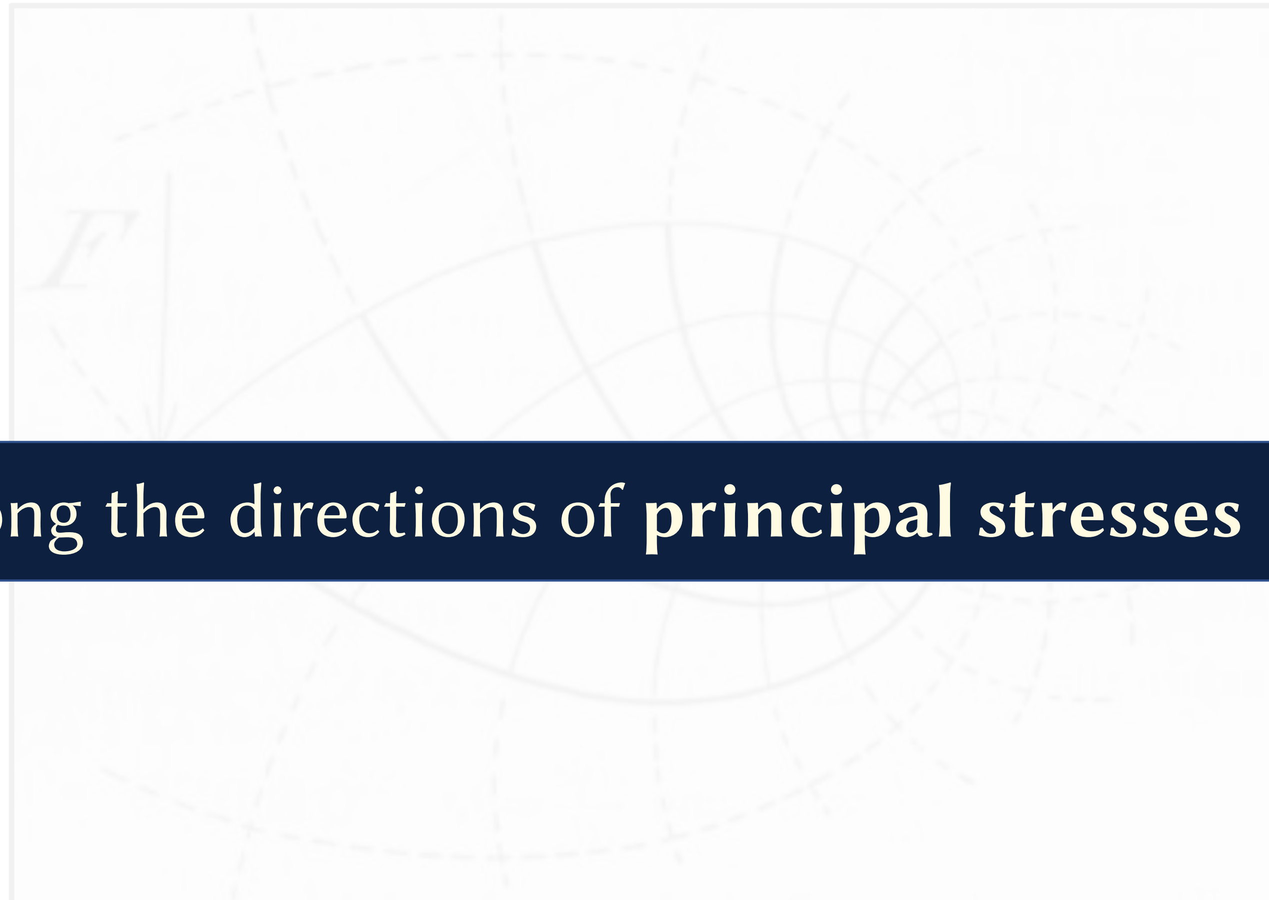


Source: https://youtu.be/0lIqEemsmcU

Background

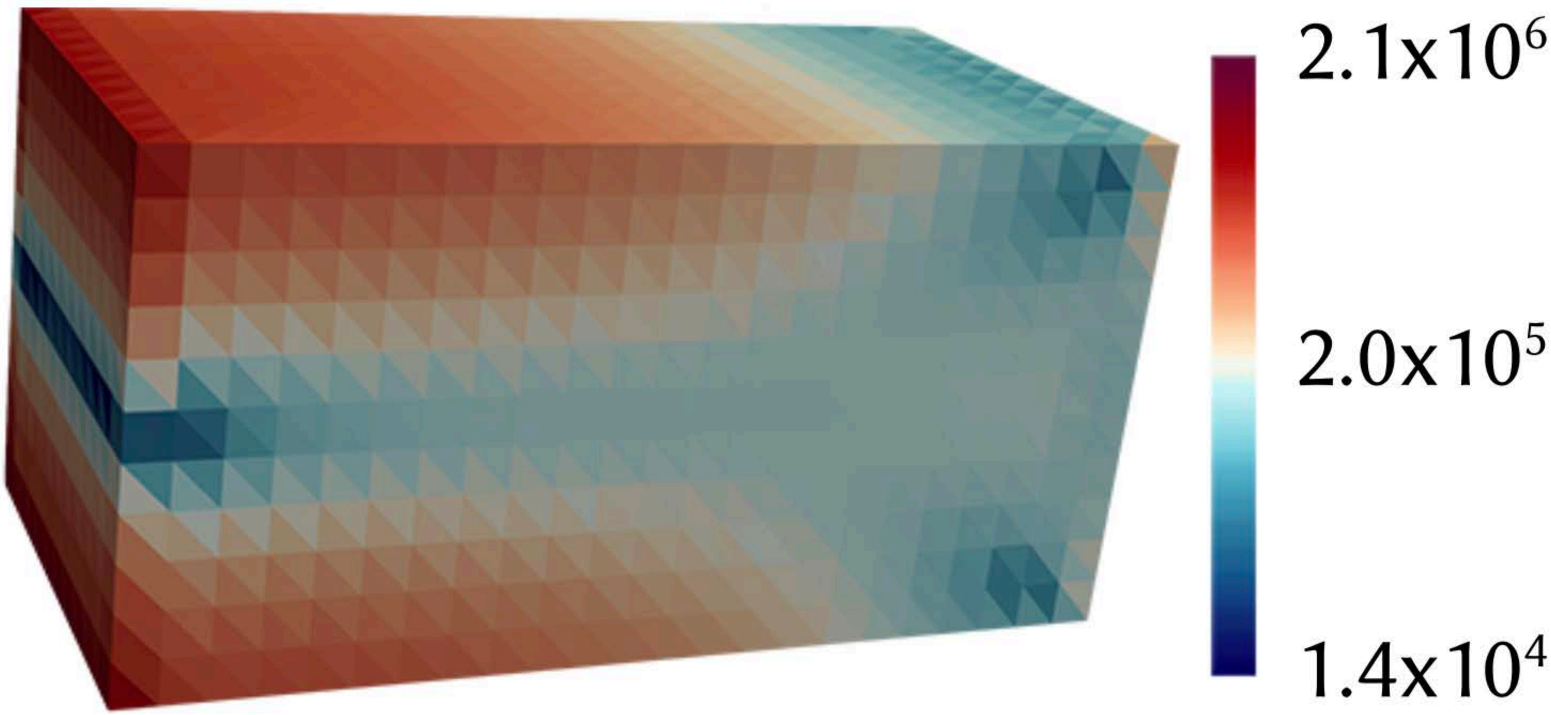


[A.G.M. **Michell**. 1904. The limits of economy of material in frame-structures.]



[A.G.M. **Michell**. 1904. The limits of economy of material in frame-structures.]

Position material along the directions of principal stresses

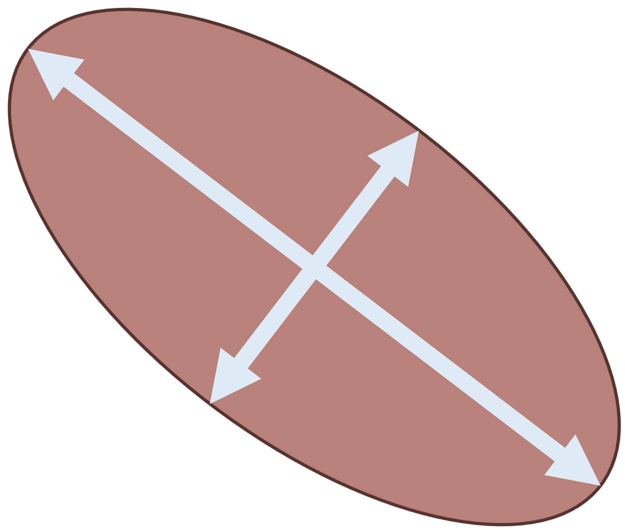


Stress magnitude visualization

$$\sigma \equiv d \times d \text{ symmetric matrix}$$

Stress

$d = 2$ in 2D, $d = 3$ in 3D



Stress (Symmetric matrix)

$$\sigma = Q\widetilde{\Lambda}Q^T$$

Orientation
(rotation matrix)

Scaling (diagonal matrix)

Eigendecomposition of a stress matrix

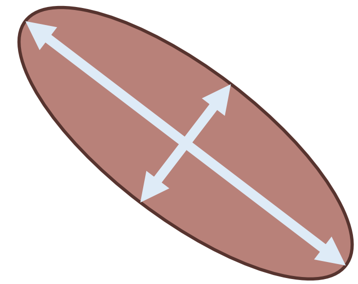
Stress (Symmetric matrix)

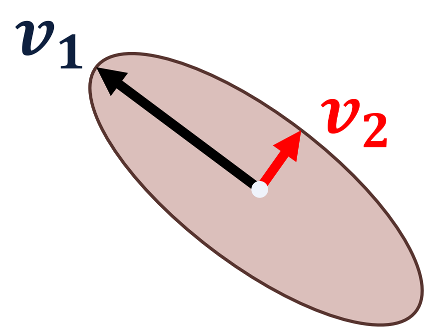
$$\sigma = Q\tilde{\Lambda}Q^T$$

Orientation
(rotation matrix)

Scaling (diagonal matrix)

Principal stress directions

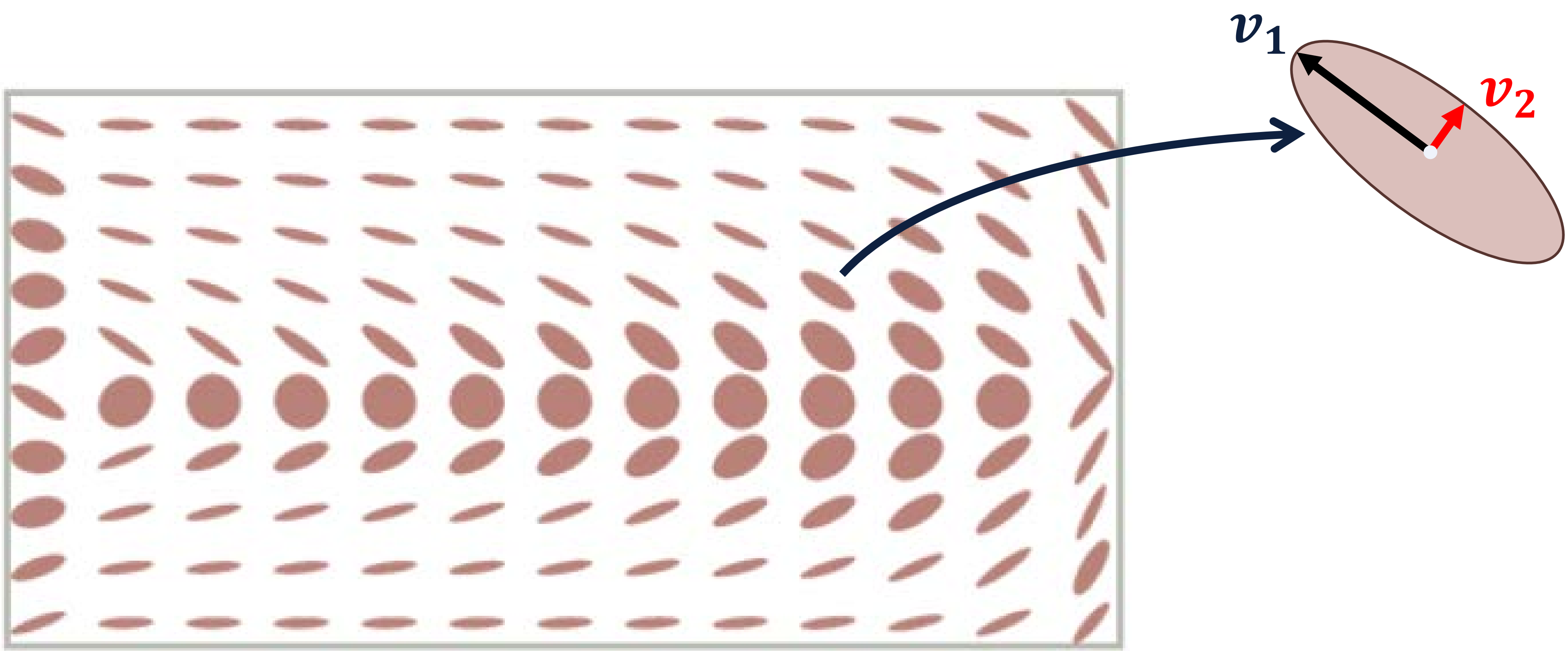


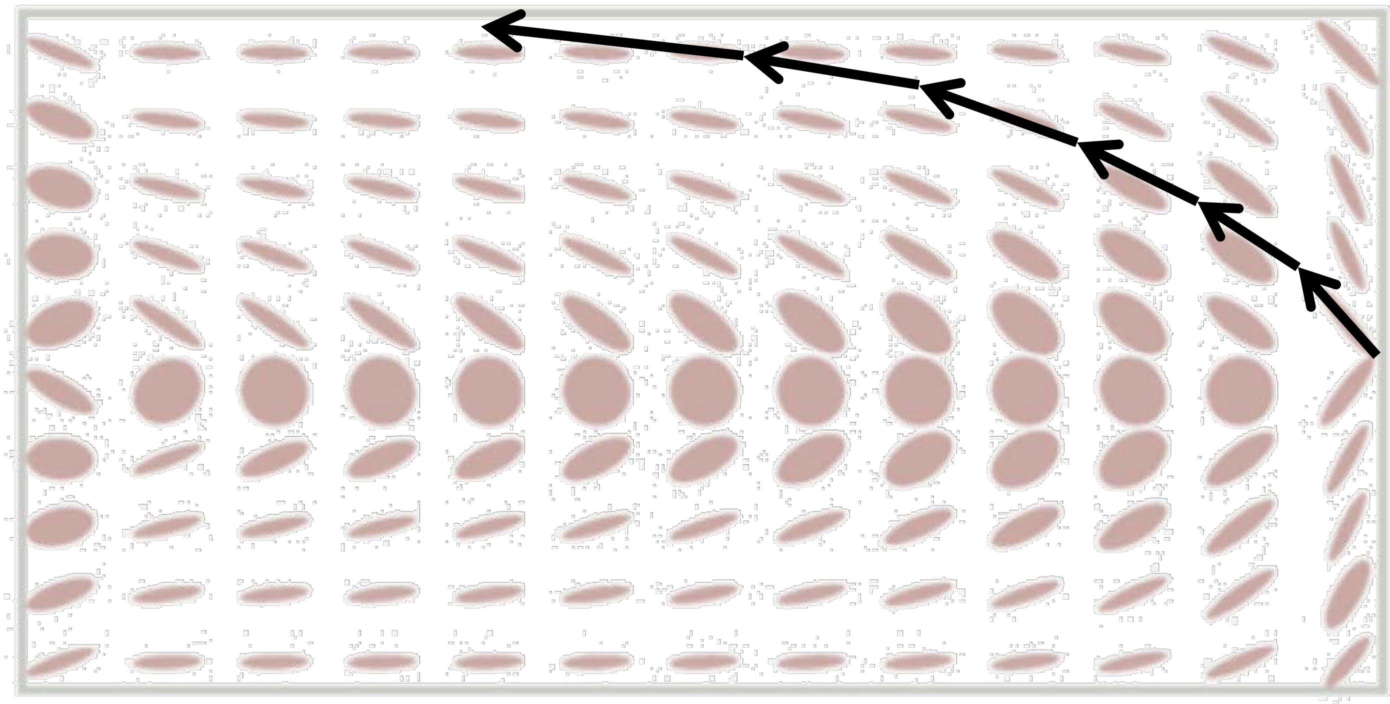


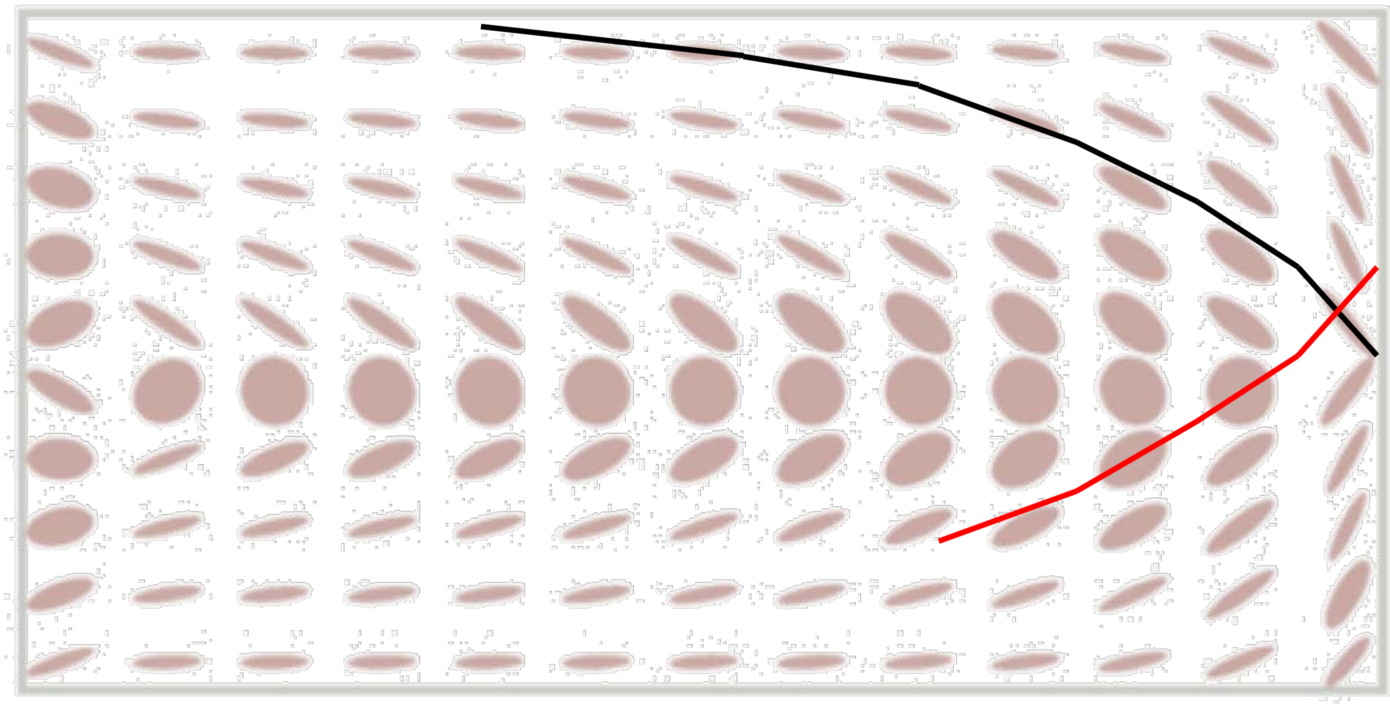
Computed eigenvectors

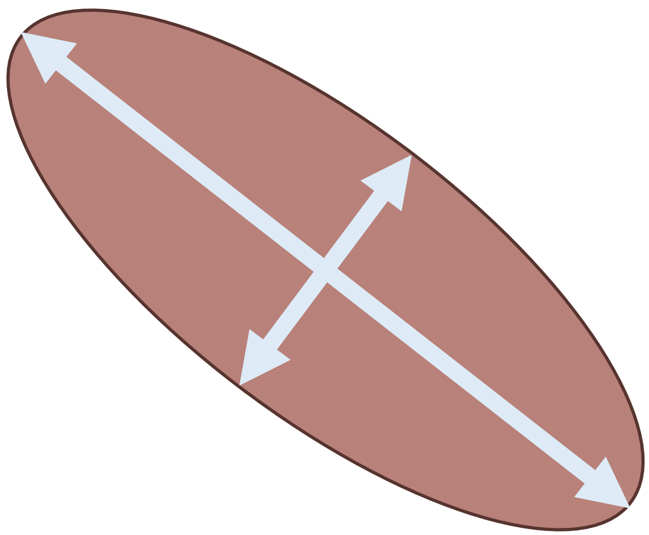
Eigendecomposition of a stress matrix

Computed eigenvectors

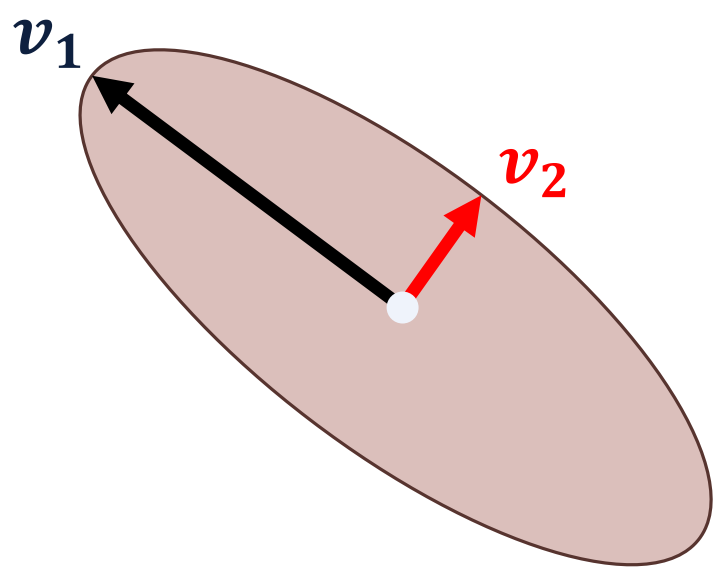




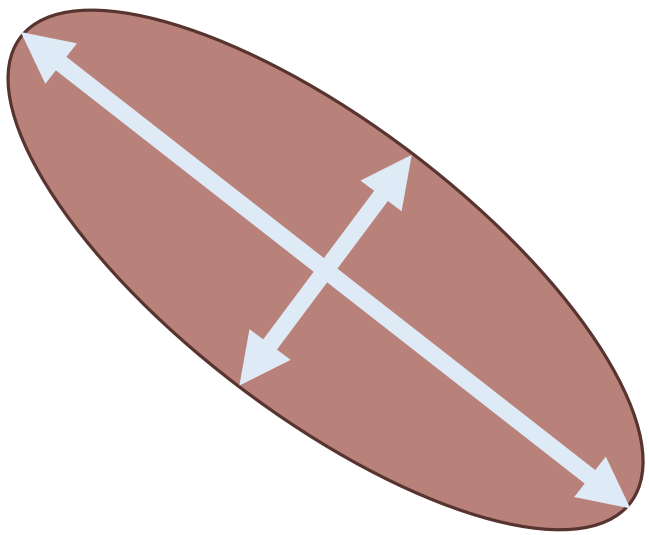




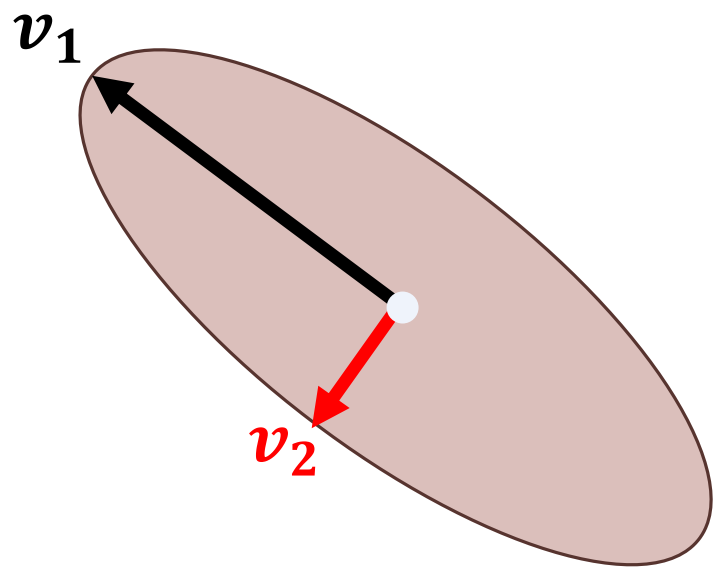
Cross-field symmetry



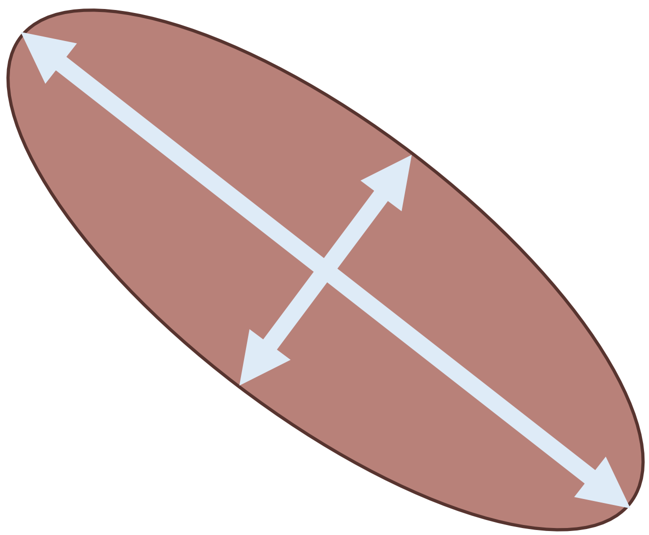
Computed eigenvectors



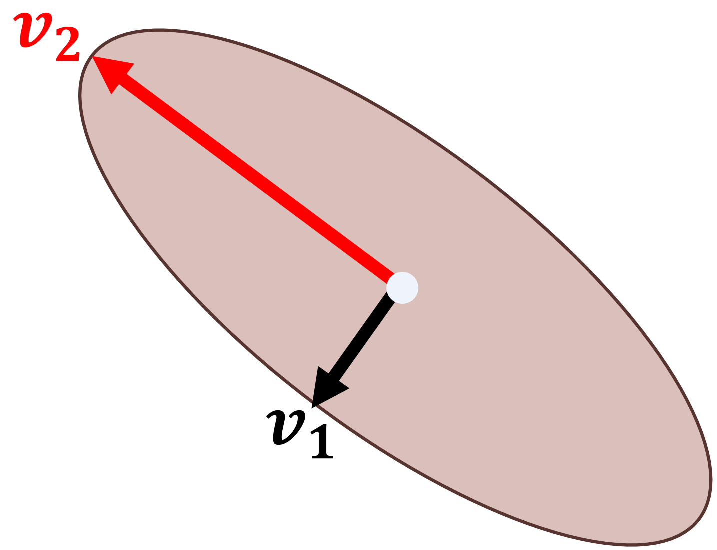
Cross-field symmetry



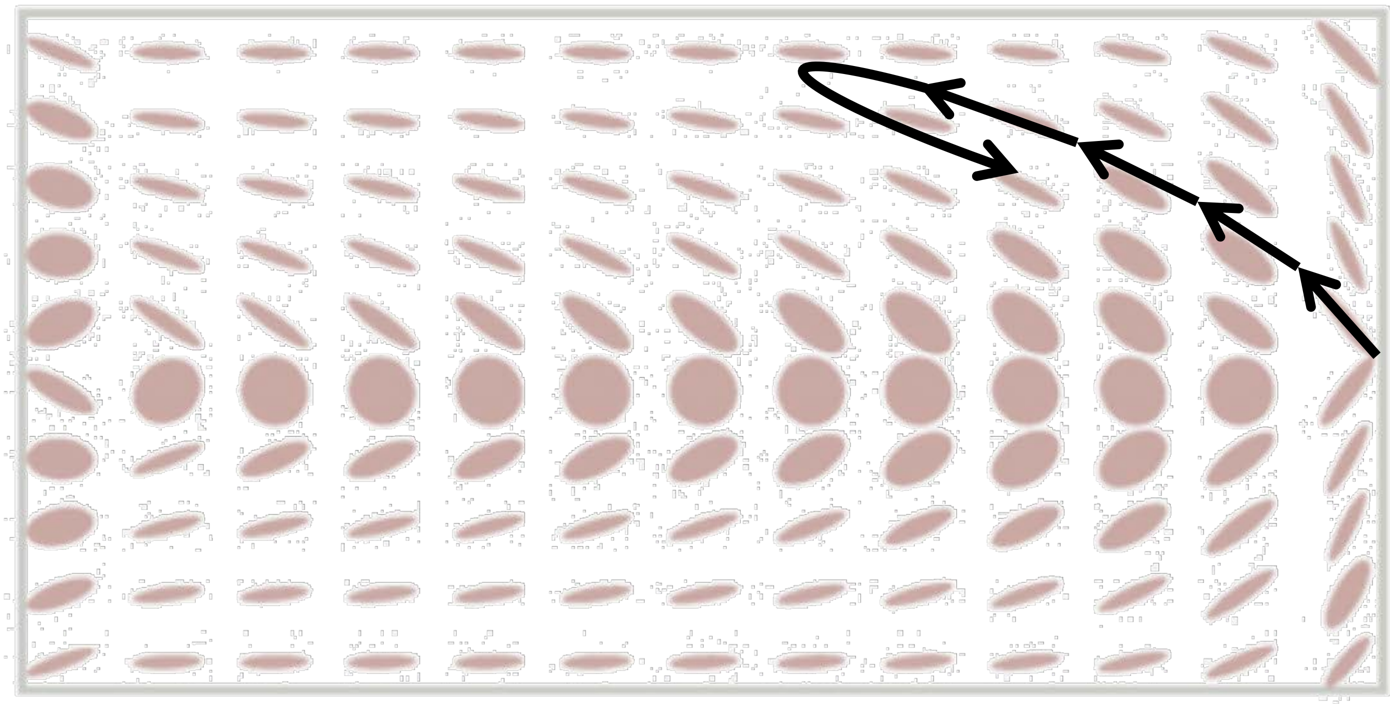
Computed eigenvectors

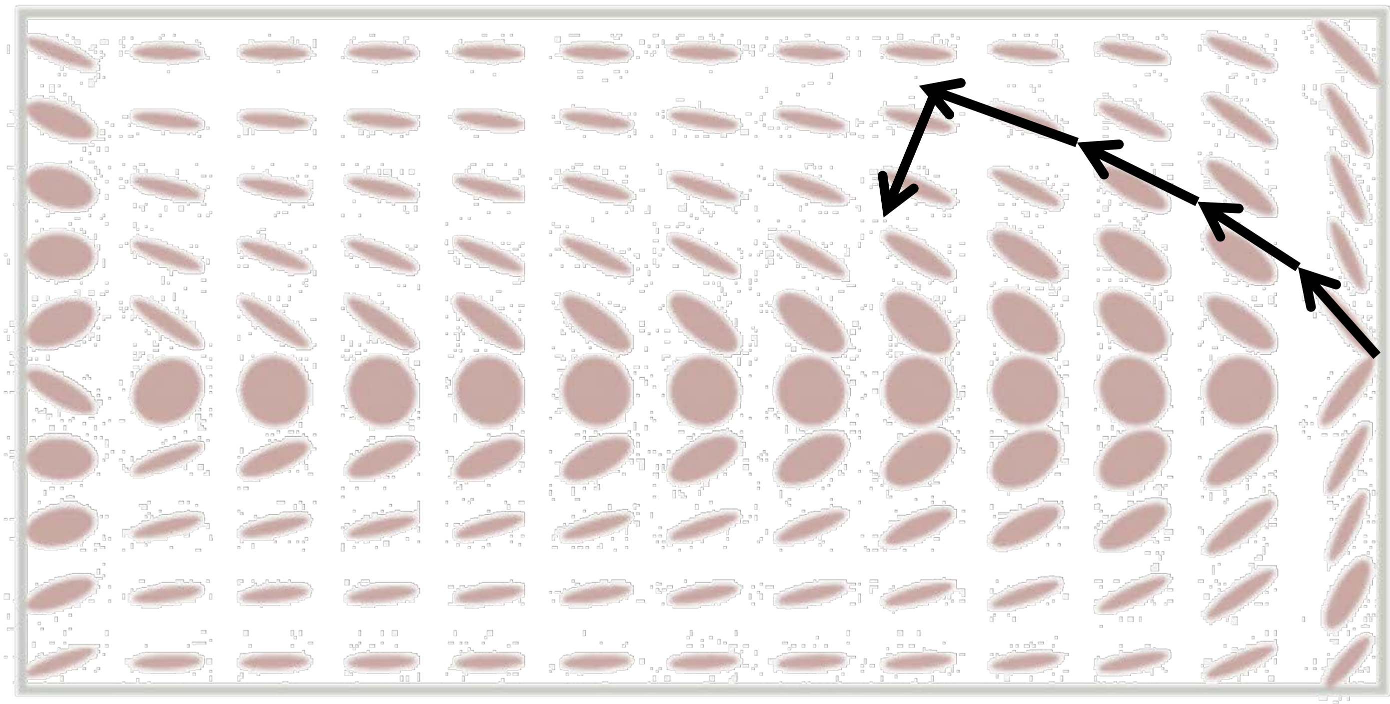


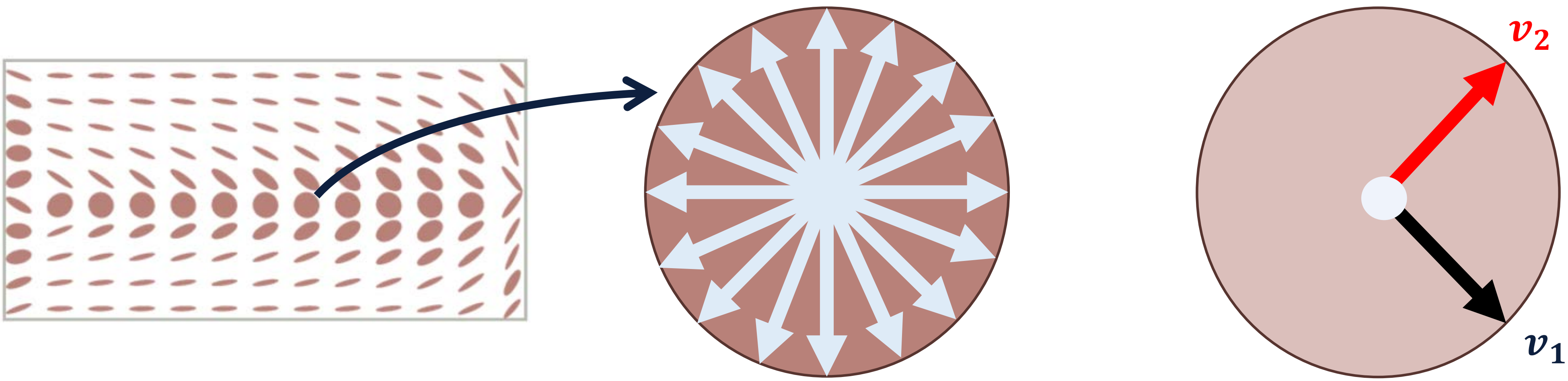
Cross-field symmetry



Computed eigenvectors







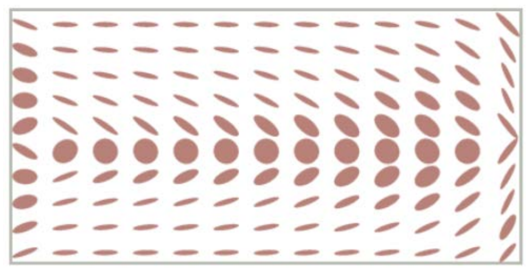
Isotropic stress tensor
(tensor field singularity)

Method

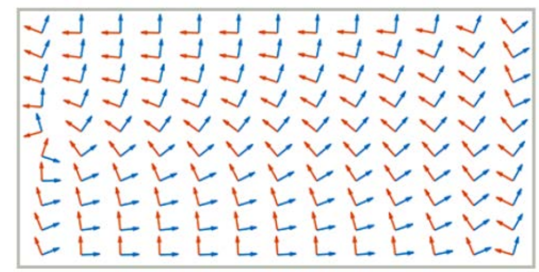
Algorithm: Overview

Step 0: Problem Specification

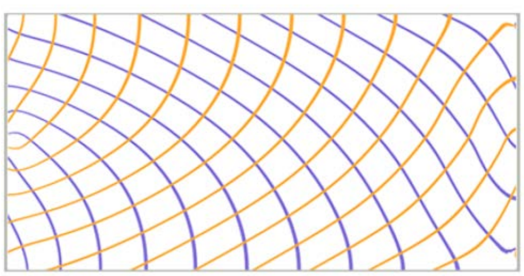
Step 1: Stress-field



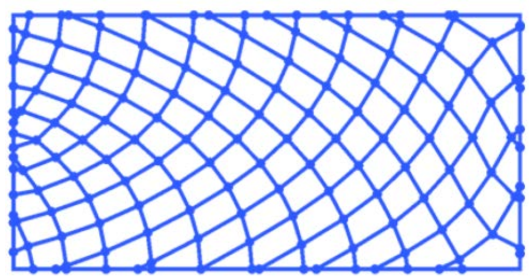
Step 2: Stress-aligned frame-field



Step 3: Texture parametrization



Step 4: Truss layout

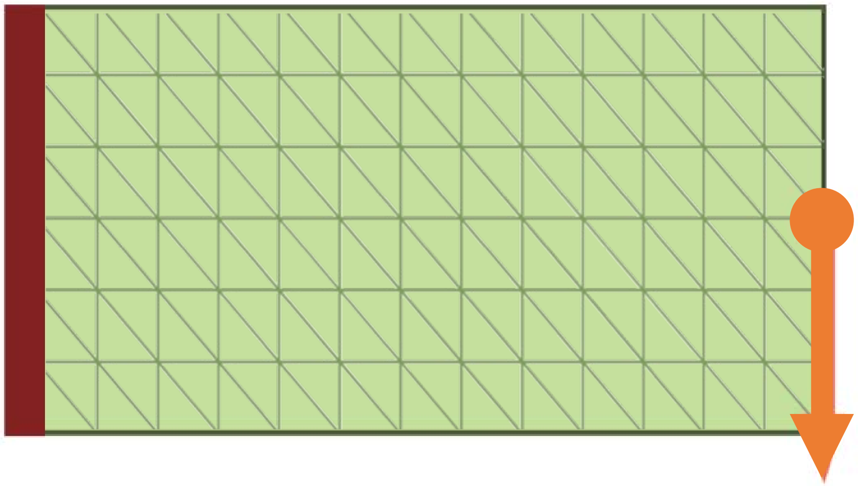


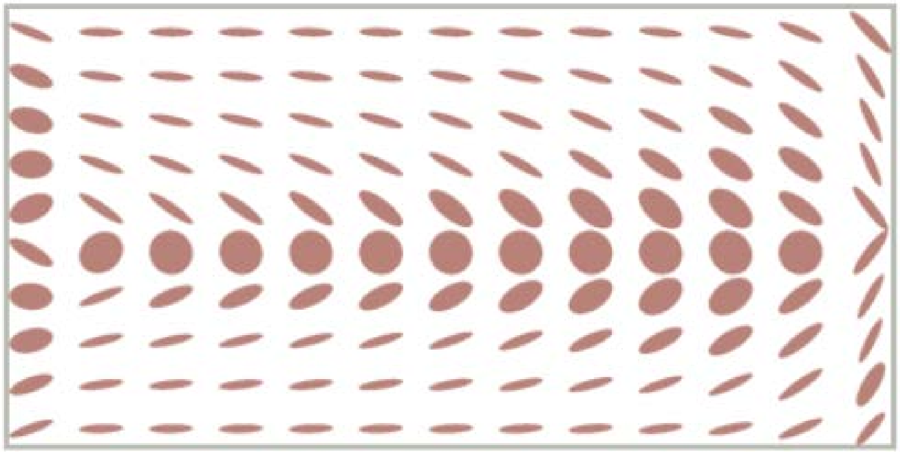
Finally: Editing & fabrication

Later…

1. Stress Field Computation

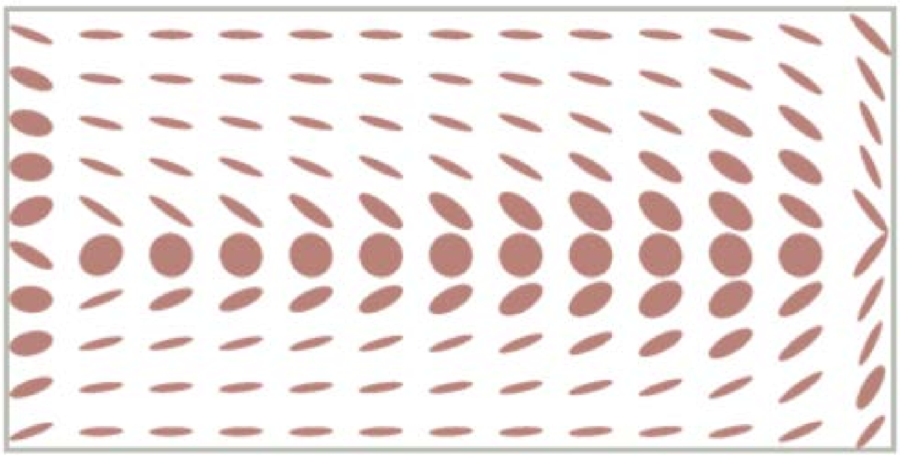
Problem description → Stress field

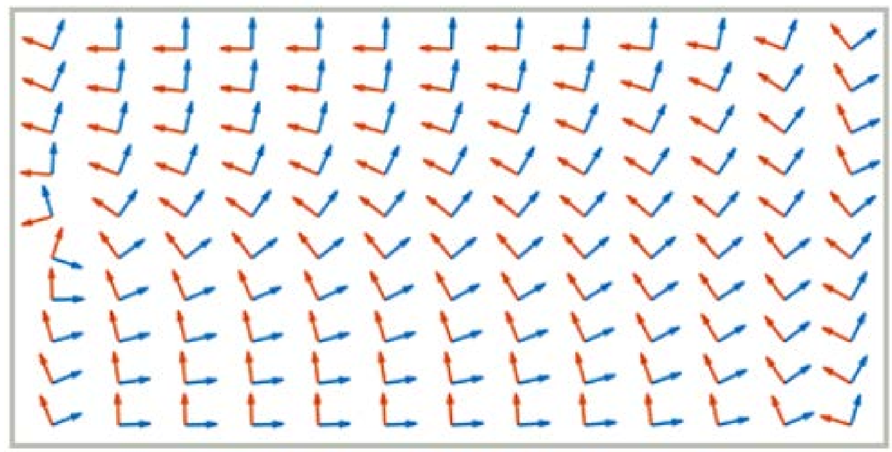




2. Stress-Aligned Frame-Field Generation

Stress field → Smooth frame field





$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3$ are unit vectors

$$R = \begin{bmatrix} | & | & | \\ \boldsymbol{r}_1 & \boldsymbol{r}_2 & \boldsymbol{r}_3 \\ | & | & | \end{bmatrix}_{d \times d}$$

Frame
($d \times d$ rotation matrix)

Stress (Symmetric matrix)

Positive-definite matrix

$$\sigma = Q\widetilde{\Lambda}Q^T \qquad \sigma_+ = Q\Lambda Q^T$$

Orientation
(rotation matrix)

Scaling (diagonal matrix)

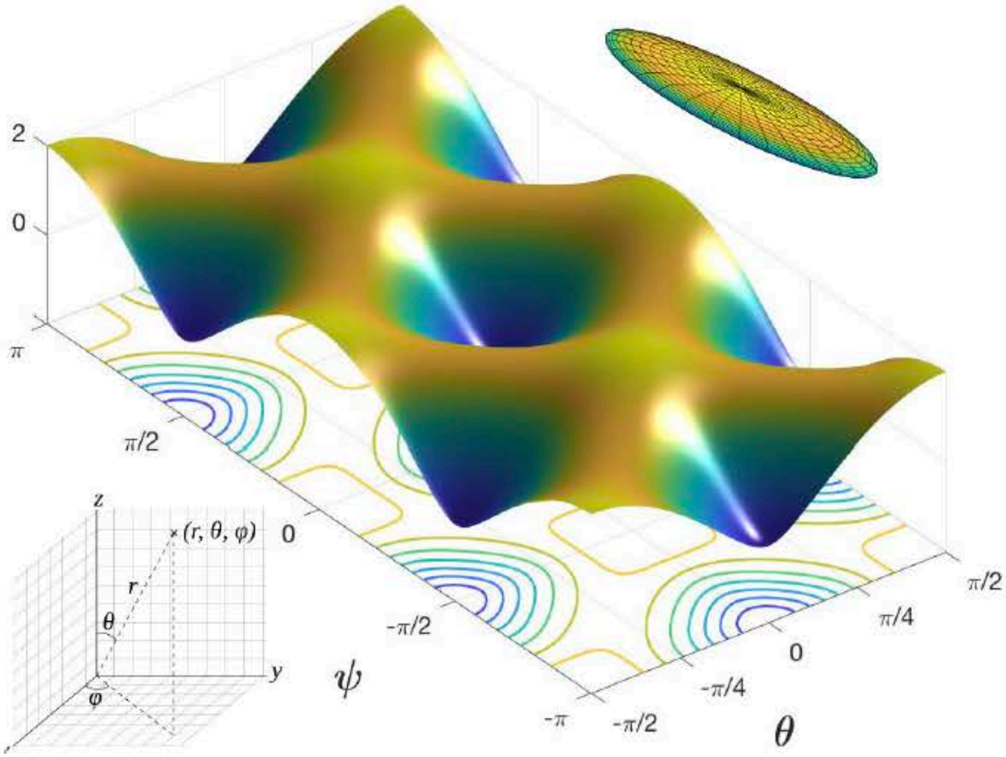
$$R = \begin{bmatrix} | & | & | \\ \boldsymbol{r_1} & \boldsymbol{r_2} & \boldsymbol{r_3} \\ | & | & | \end{bmatrix}_{d \times d}$$

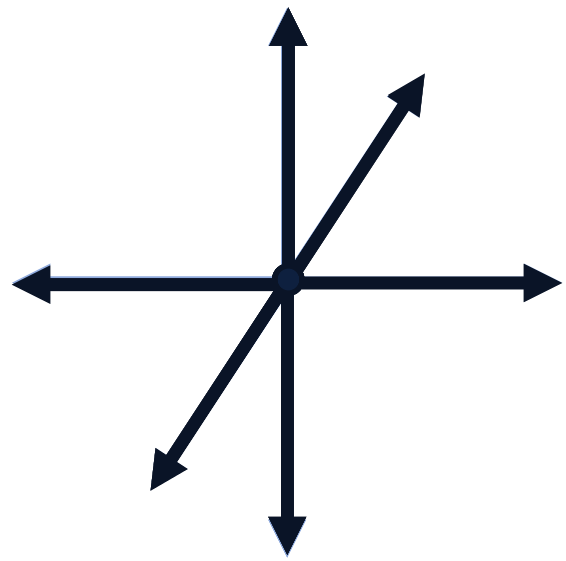
$\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}$ are unit vectors

Frame

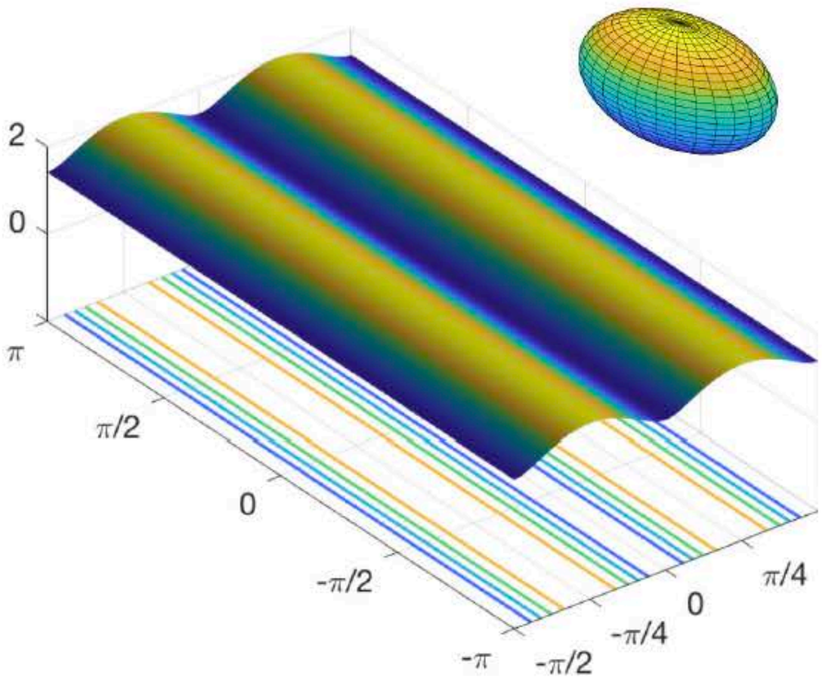
($d \times d$ rotation matrix)

$$E_{align}(R) = \boxed{(\boldsymbol{r_1}^T \sigma_+ \boldsymbol{r_1})}^{1/2} + (\boldsymbol{r_2}^T \sigma_+ \boldsymbol{r_2})^{1/2} + (\boldsymbol{r_3}^T \sigma_+ \boldsymbol{r_3})^{1/2}$$





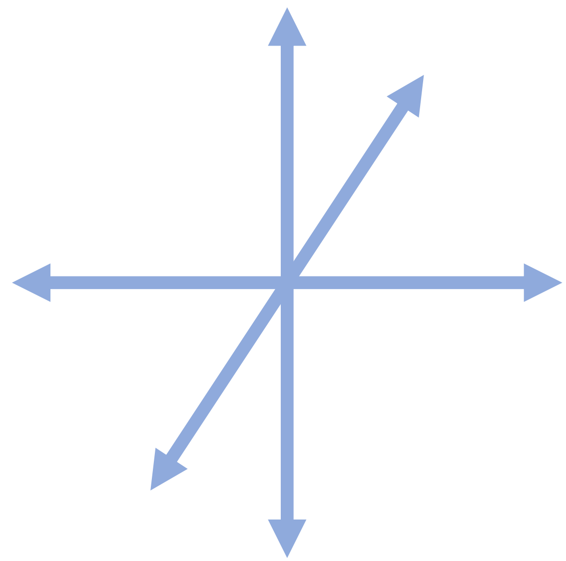
$$\lambda_1 \neq \lambda_2 \neq \lambda_3$$

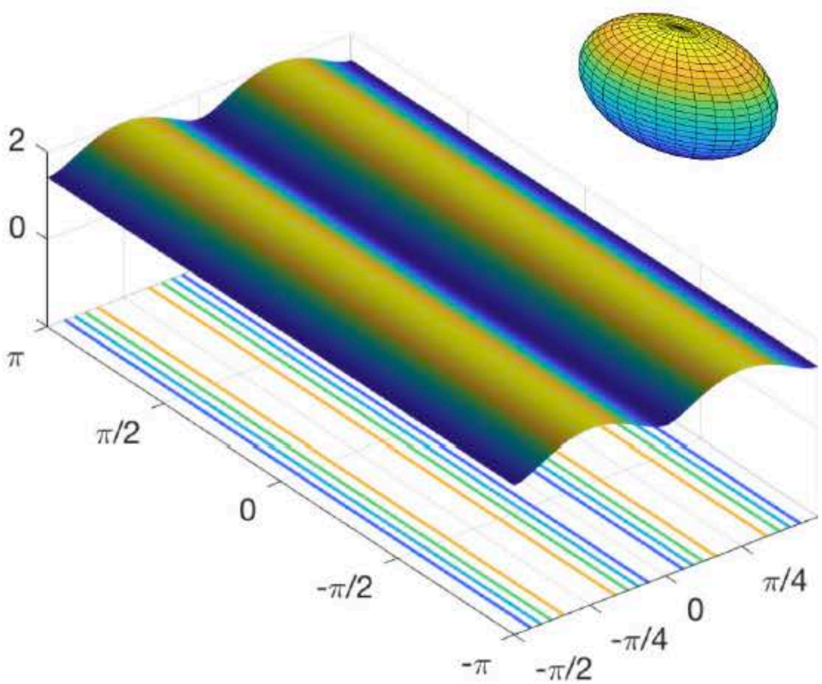


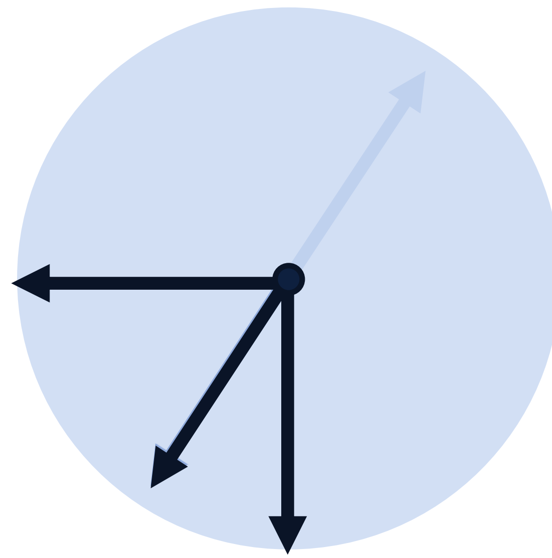
$$\lambda_1 = \lambda_2 \neq \lambda_3$$

or

$$\lambda_1 \neq \lambda_2 = \lambda_3$$







$$\lambda_1 = \lambda_2 \neq \lambda_3$$

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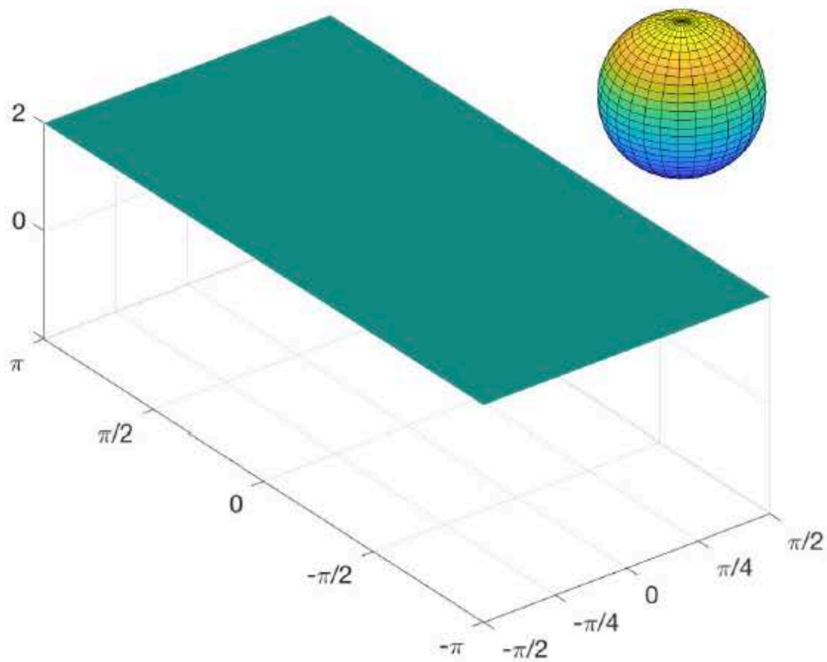
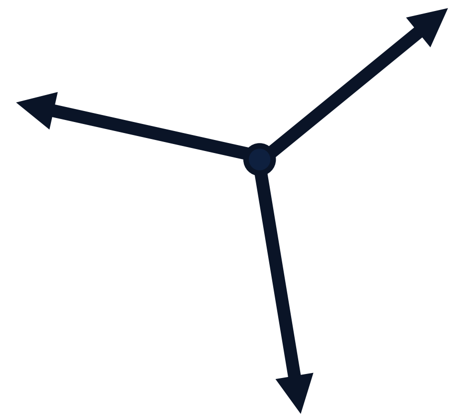




$$\lambda_1 = \lambda_2 = \lambda_3$$

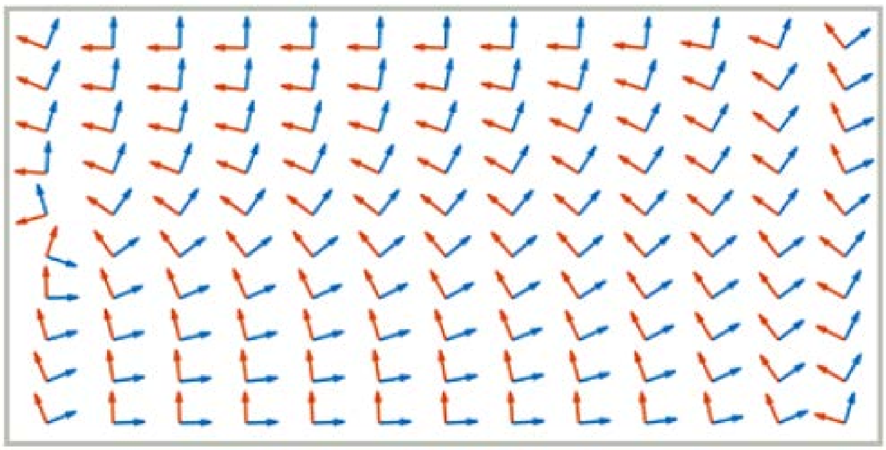
Summed over all tets

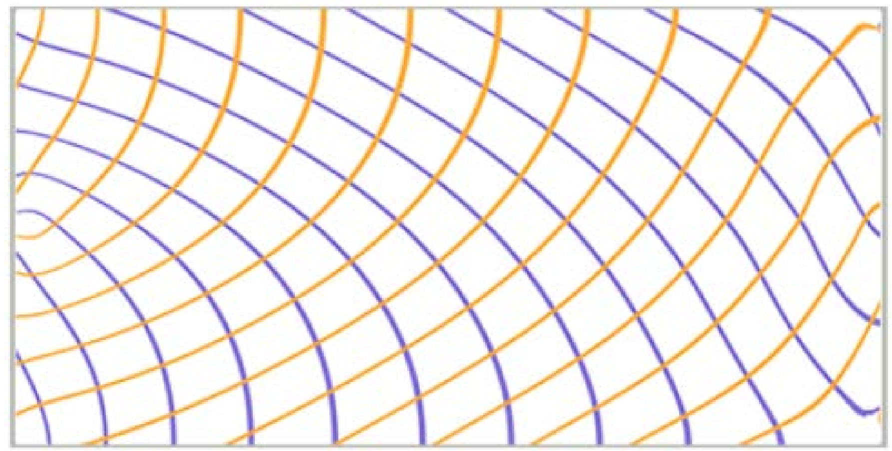
$$E = \sum_{i=1}^{|T|} E_{\mathrm{align}}^{i} + \alpha E_{\mathrm{smooth}}$$

Laplacian-based smoothness cost

3. Texture Parametrization

Frame field → Frame-aligned parametrization

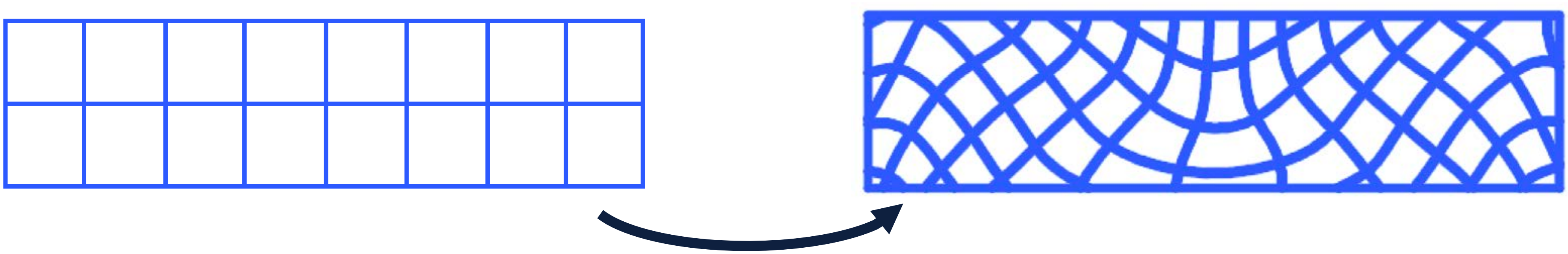




$$\phi : M \to \mathbb{R}^{d}$$

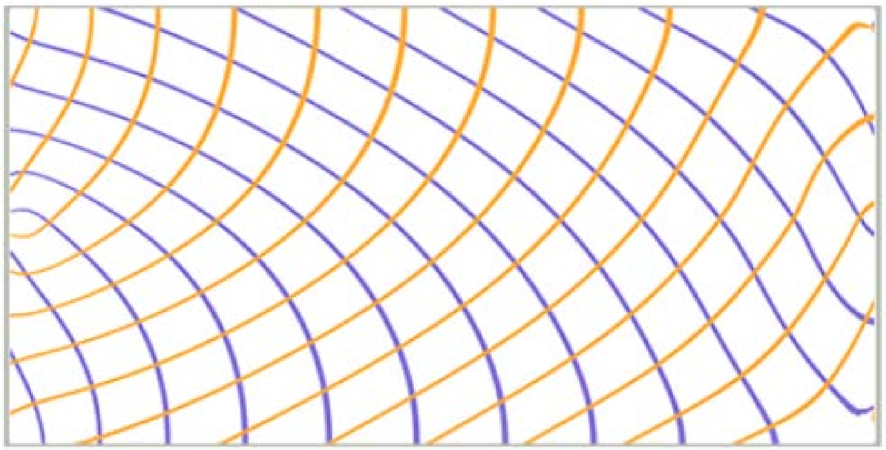
$\mathbb{R}^{d}$-valued parametrization

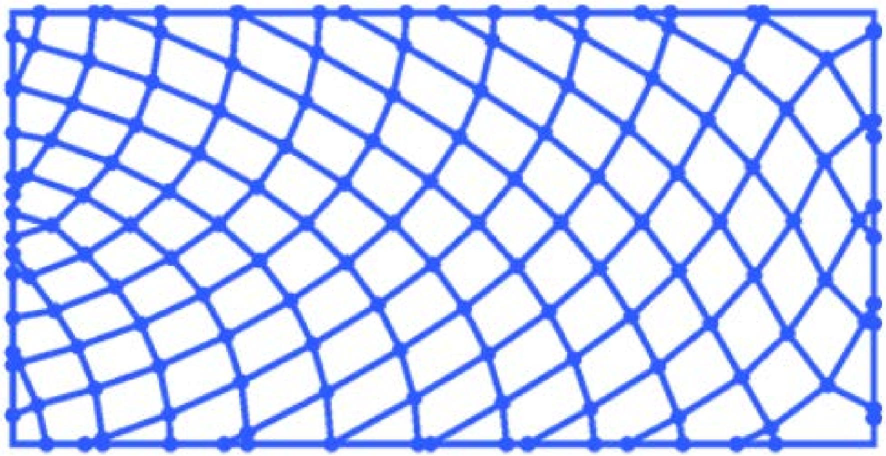
$$\boxed{\nabla \phi = R}$$



4. Truss Layout Extraction

Global parametrization → Truss layout



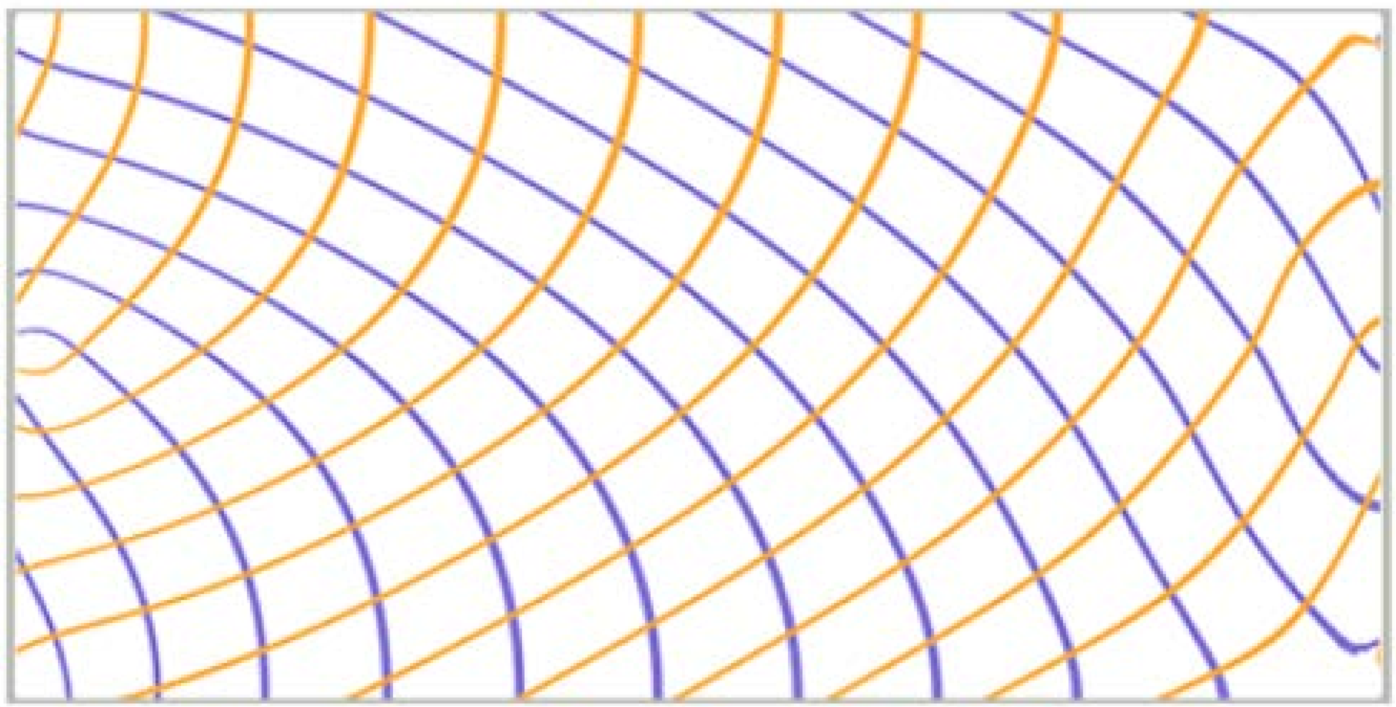


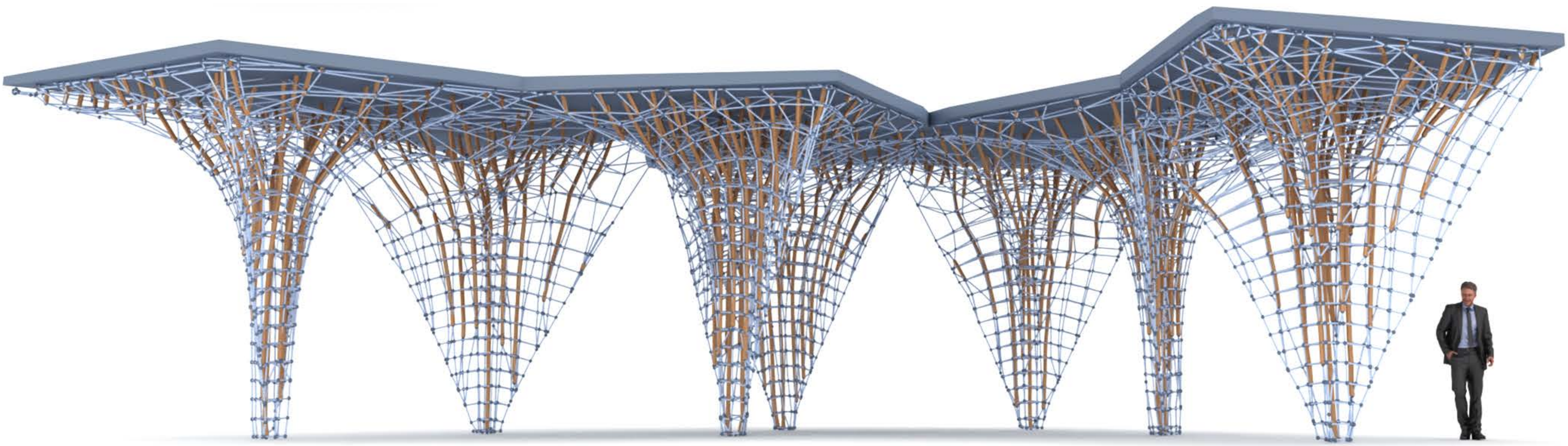
Results

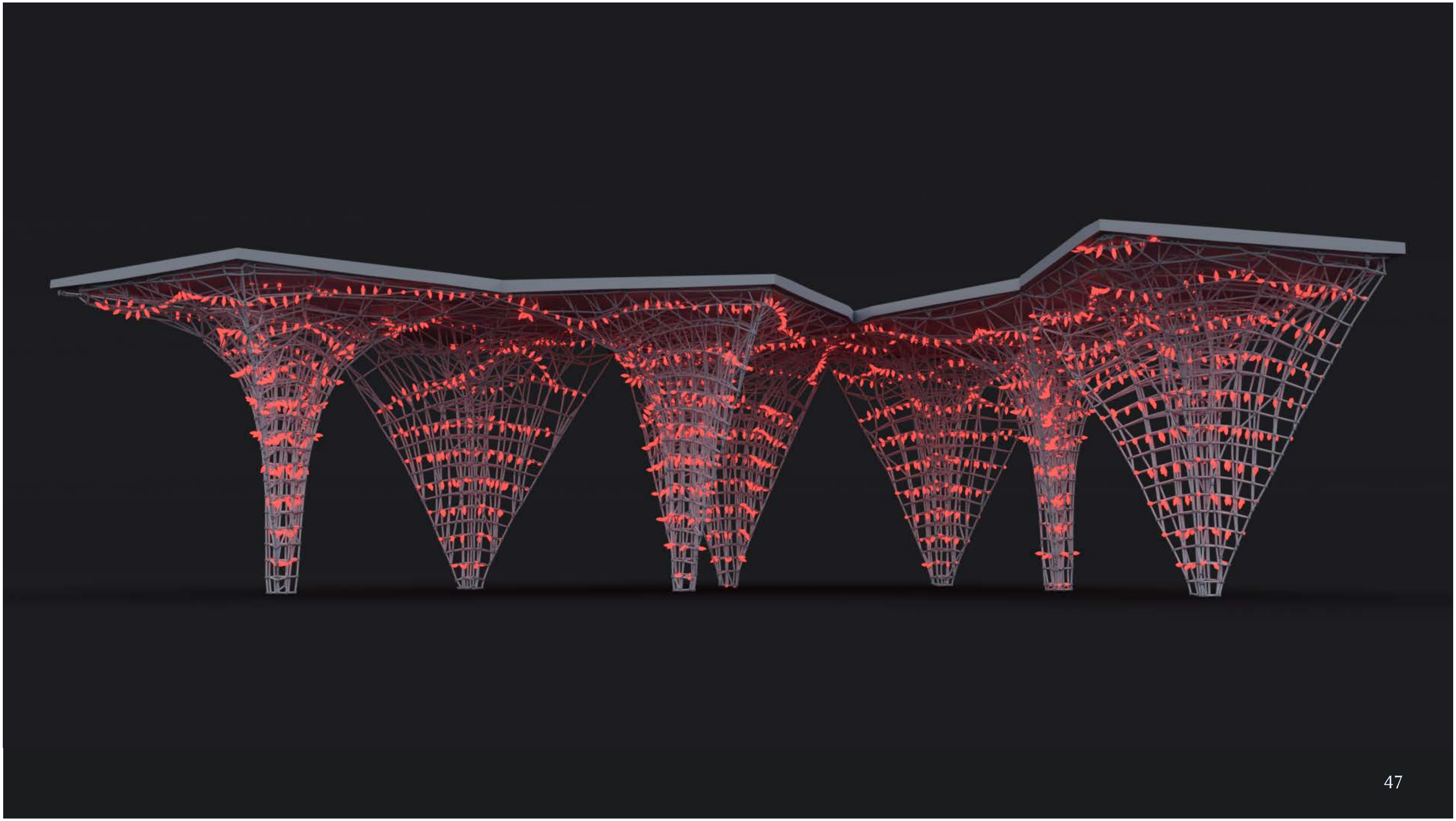
— d orthogonal families of smooth end-to-end curves

—Curves in each family are identified with a pair of integers

—Each curve itself is parametrized



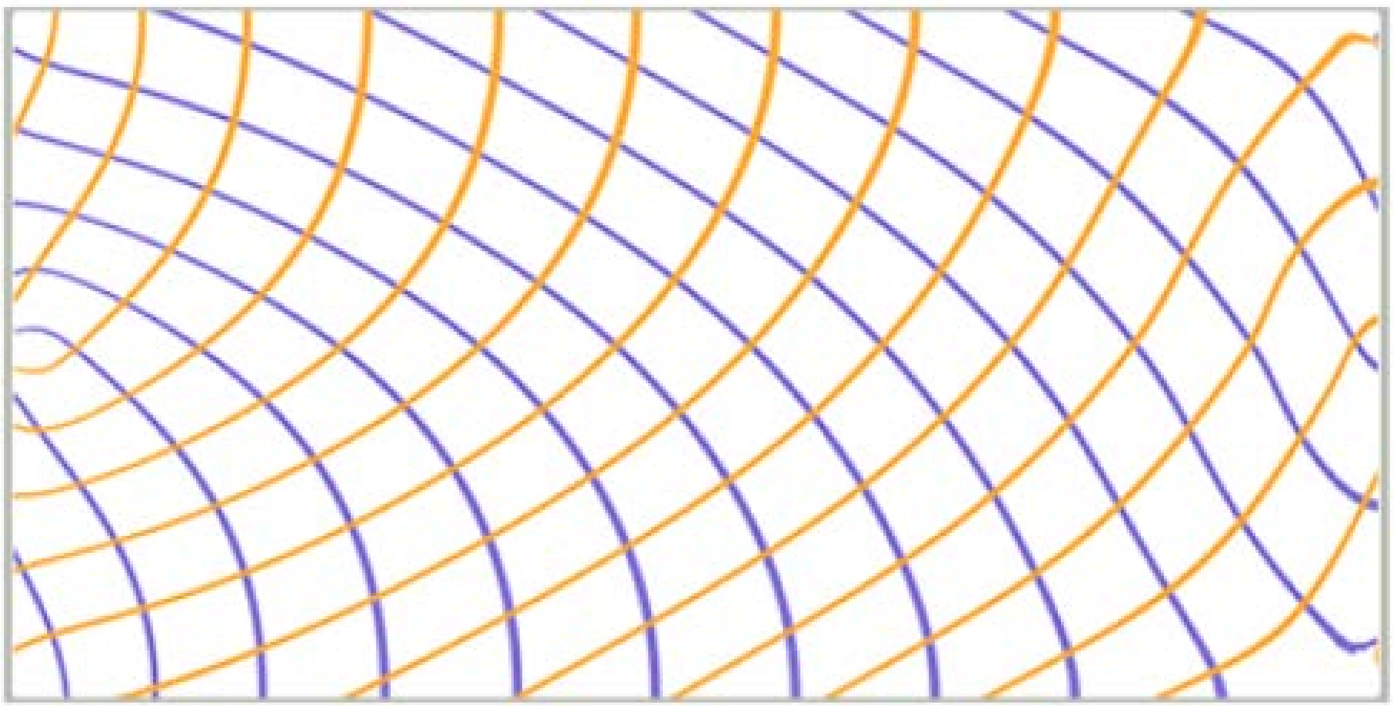


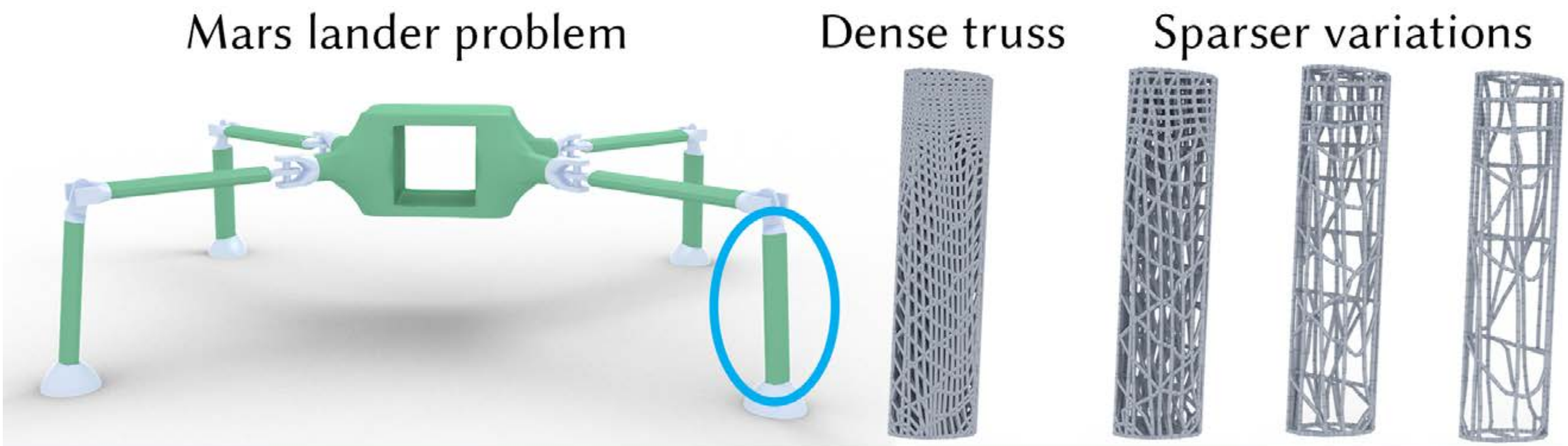


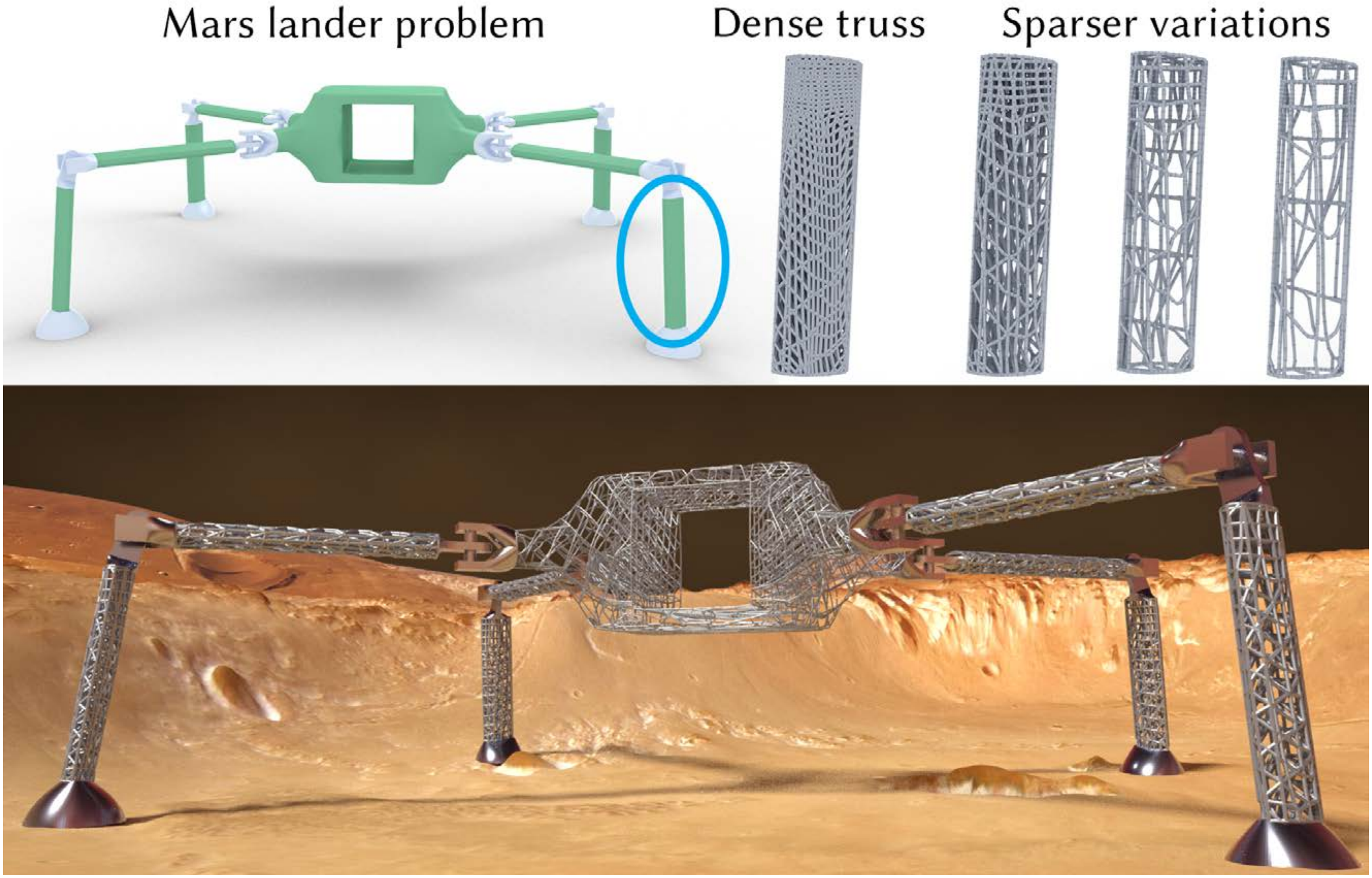
— d orthogonal families of smooth end-to-end curves

—Curves in each family are identified with $d - 1$ integers

—Each curve itself is parametrized




Mars lander problem
Dense truss
Sparser variations


Mars lander problem
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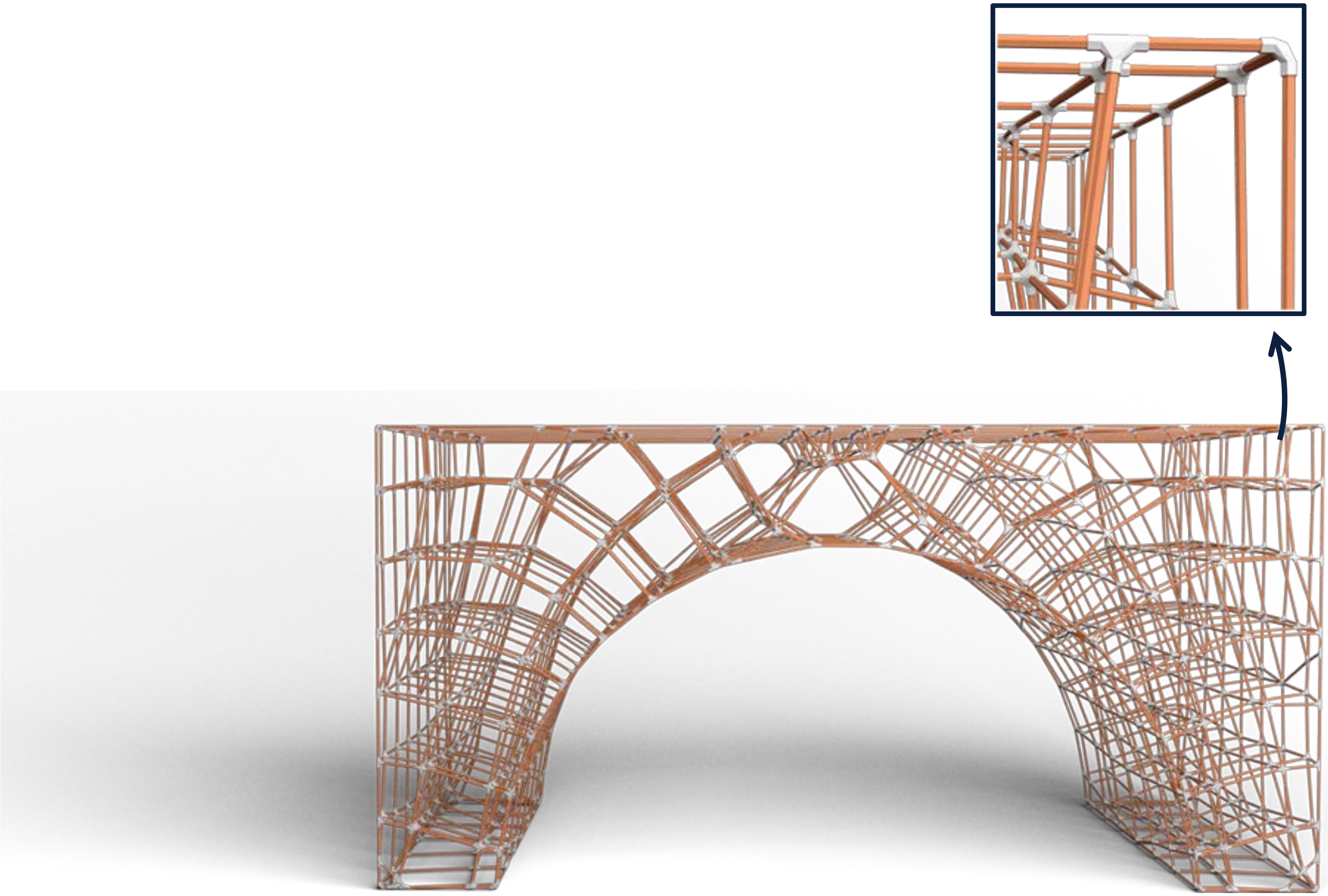
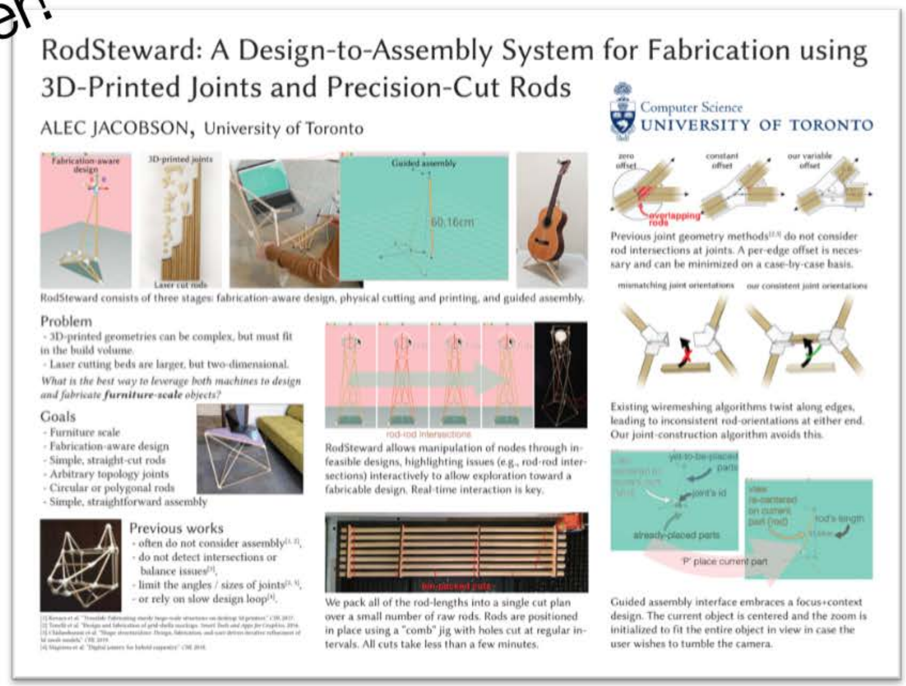


RodSteward:
A Design-to-Assembly System for Fabrication using 3D-Printed Joints and Precision-Cut Rods

Come see the poster!



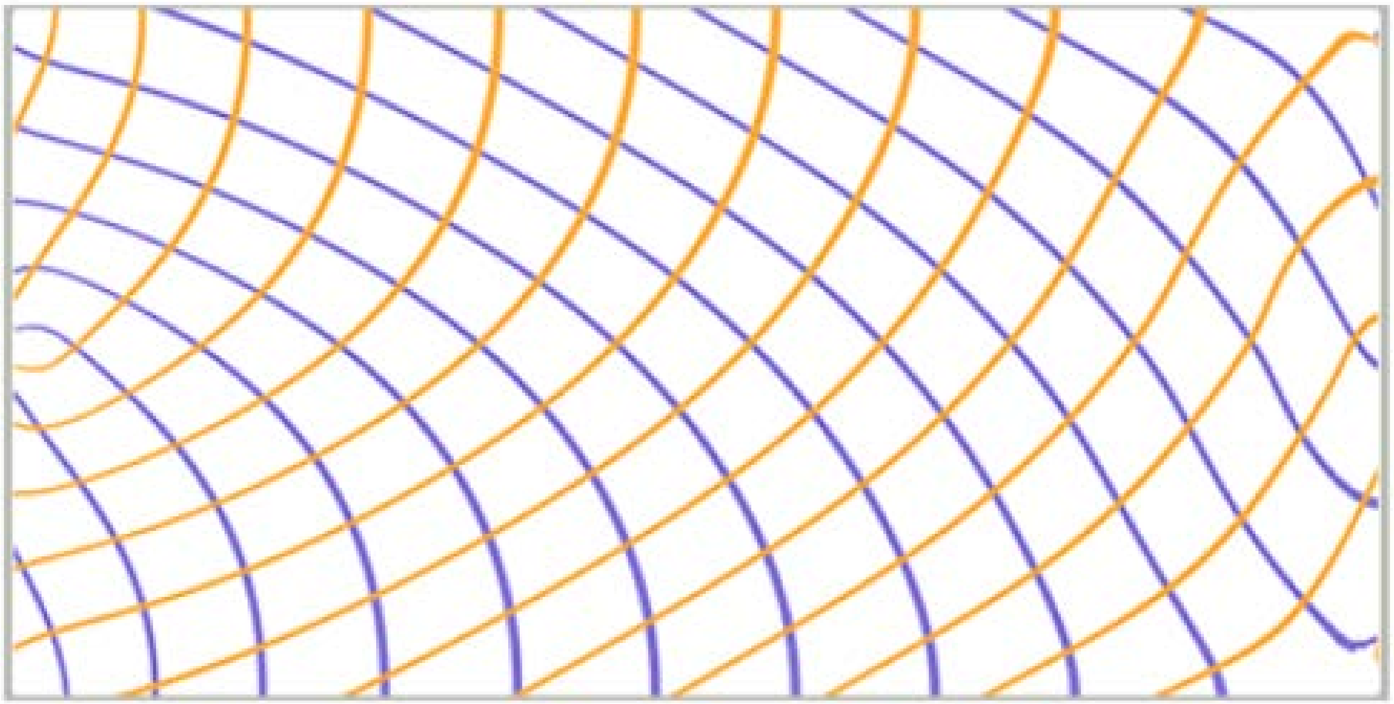
Alec Jacobson
jacobson@cs.toronto.edu
University of Toronto

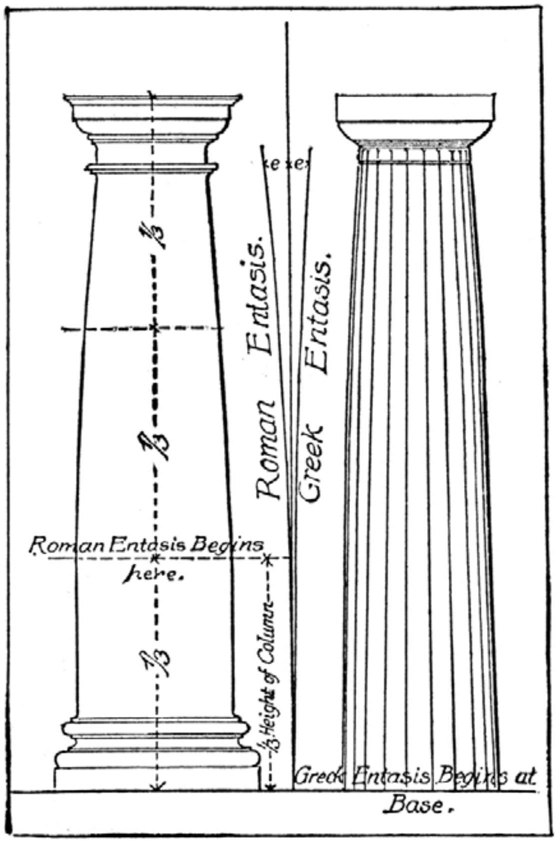
Kindly presented by Rahul Arora

— d orthogonal families of smooth end-to-end curves

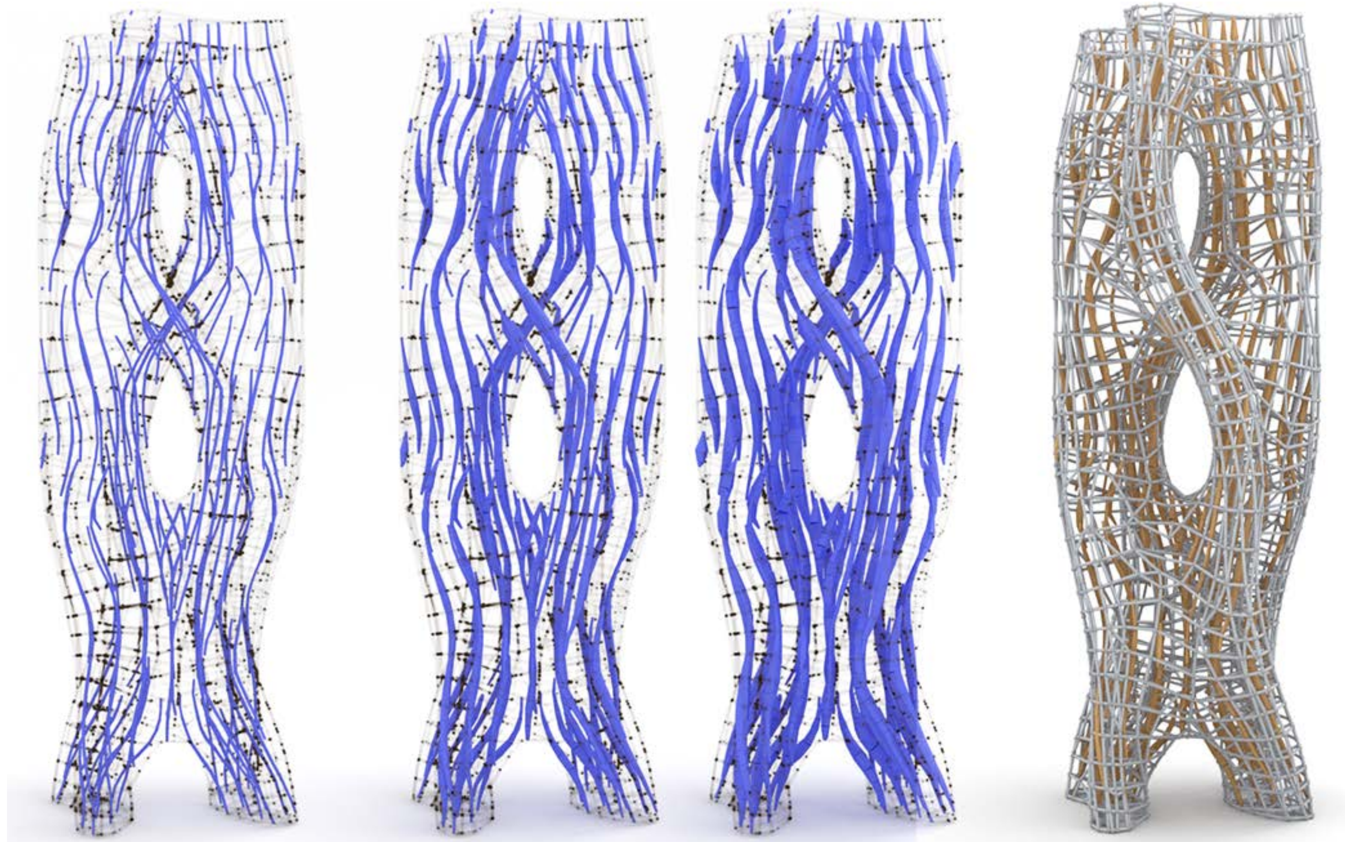
—Curves in each family are identified with a pair of integers

—Each curve itself is parametrized





Entasis
(Greek/Roman architecture)



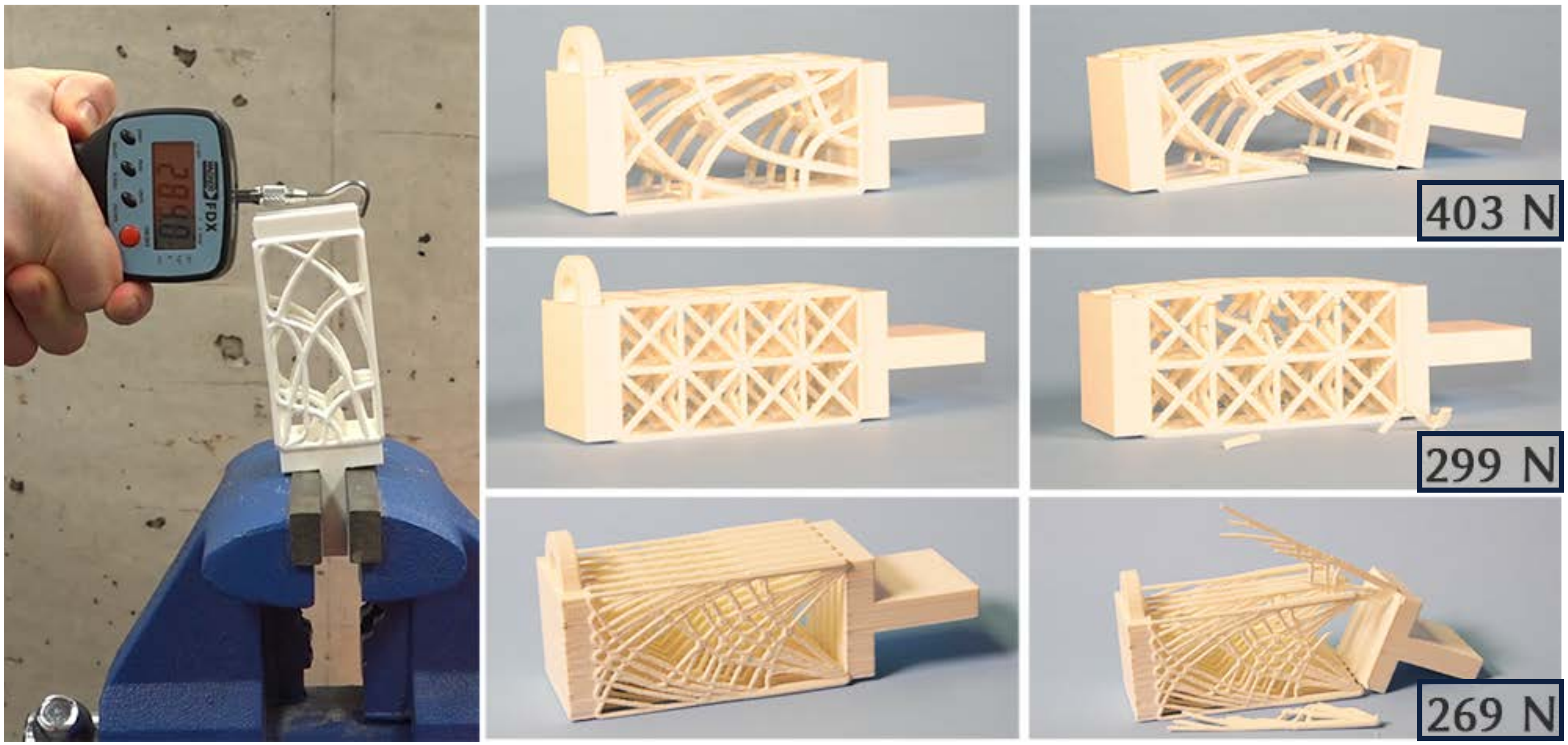
(a) Selecting a curve family

(b) Exploring the *entasis* parameter

(c) Final structure with *entasis*

Image Source: William Dwight Whitney *The Century Dictionary: An Encyclopedic Lexicon of the English Language* (New York, NY: The Century Co., 1911)

Structural Tests

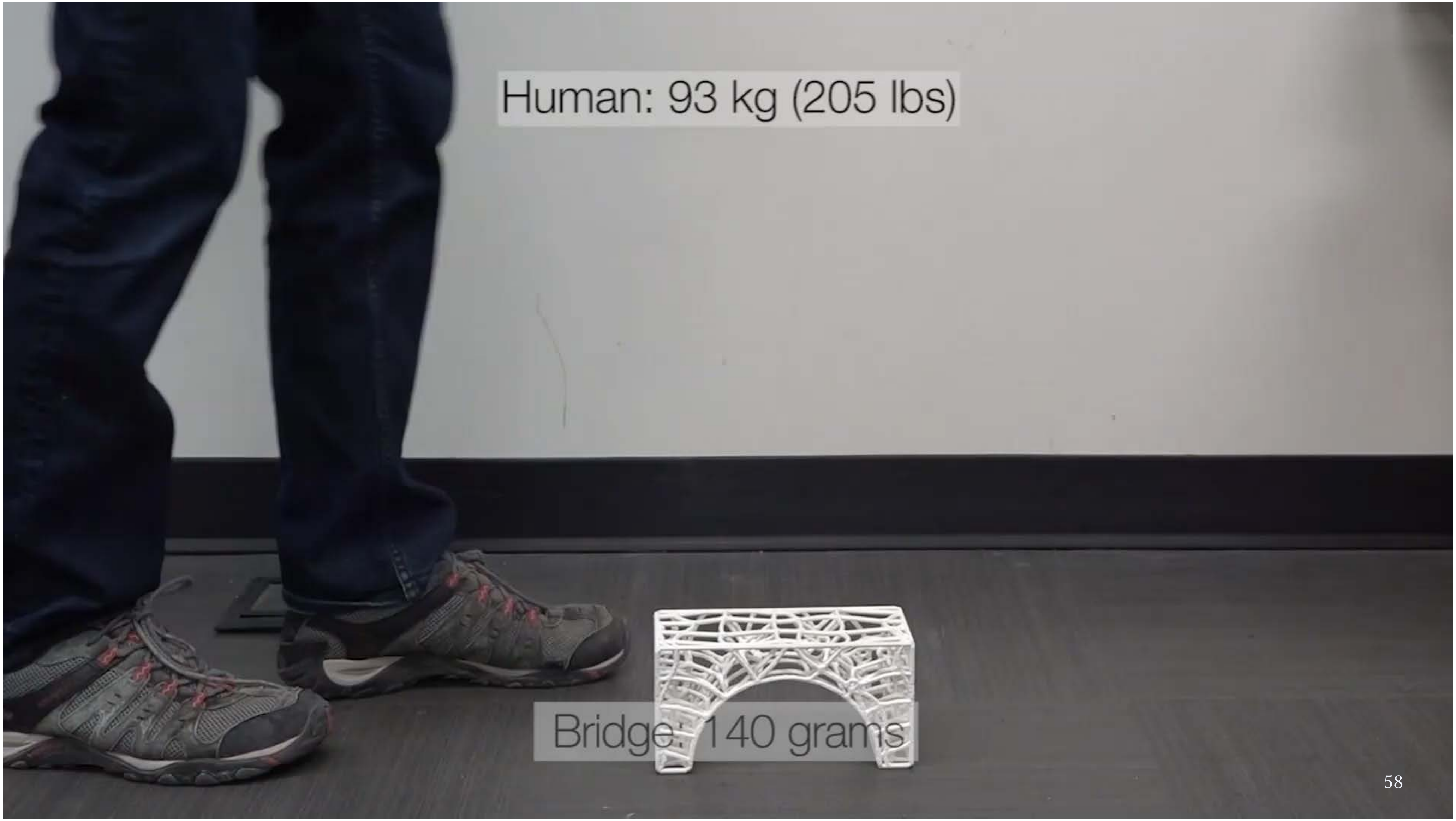


Ours

Regular grid (unoptimized)

GRAND3 [Zegard and Paulino 2015]

Testing the cantilever beam


Human: 93 kg (205 lbs)
Bridge: 140 grams

Limitations and Future Work

- Manufacturing constraints not accounted for
  - Wire-bend each curve
  - Generate construction sequences for dowel assembly

- Sizing optimization

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https://github.com/rarora7777/VolumetricTruss

Volumetric Michell Trusses for Parametric Design & Fabrication

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Project page	https://www.dgp.toronto.edu/projects/michell
Code	https://github.com/rarora7777/VolumetricTruss
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