Volumetric Michell Trusses for Parametric Design & Fabrication

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with

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Structural Optimization

The design of *optimal* load-carrying structures

Fixed Boundary

Load

Structural optimization algorithm

Engineer image by GraphicMama-team from Pixabay.
Topology Optimization

Structural optimization methods that can introduce topological changes
A user-centric approach to structural optimization.

By generating a parametrized output, our method generates structures that can be easily controlled and edited a posteriori.
Background
[A.G.M. Michell. 1904. The limits of economy of material in frame-structures.]
Position material along the directions of **principal stresses**
Stress magnitude visualization
\[ \sigma \equiv d \times d \text{ symmetric matrix} \]

\[ d = 2 \text{ in 2D, } d = 3 \text{ in 3D} \]
Eigendecomposition of a stress matrix

\[ \sigma = Q \tilde{\Lambda} Q^T \]
\[ \sigma = Q \Lambda Q^T \]

Stress (Symmetric matrix)

Orientation (rotation matrix)

Scaling (diagonal matrix)

Eigendecomposition of a stress matrix

Principal stress directions

Computed eigenvectors

\( v_1 \)

\( v_2 \)
Computed eigenvectors

$\nu_1$ $\nu_2$
Cross-field symmetry

Computed eigenvectors

\( \mathbf{v}_1 \)

\( \mathbf{v}_2 \)
Cross-field symmetry

Computed eigenvectors
Cross-field symmetry

Computed eigenvectors
Isotropic stress tensor
(tensor field singularity)
Method
Algorithm: Overview

Step 0: Problem Specification

Step 1: Stress-field

Step 2: Stress-aligned frame-field

Step 3: Texture parametrization

Step 4: Truss layout

Finally: Editing & fabrication

Later...
1. Stress Field Computation

Problem description → Stress field
2. Stress-Aligned Frame-Field Generation

Stress field ➔ Smooth frame field
\[ R = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} \]

Frame (\(d \times d\) rotation matrix)

\(r_1, r_2, r_3\) are unit vectors
\( \sigma = Q \tilde{\Lambda} Q^T \)

\( \sigma_+ = Q \Lambda Q^T \)

- **Stress** (Symmetric matrix)
- **Orientation** (rotation matrix)
- **Scaling** (diagonal matrix)
- **Positive-definite matrix**
\[ R = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} \]

Frame (d×d rotation matrix)

\[ r_1, r_2, r_3 \text{ are unit vectors} \]

\[ E_{align}(R) = (r_1^T \sigma r_1)^{1/2} + (r_2^T \sigma r_2)^{1/2} + (r_3^T \sigma r_3)^{1/2} \]
\[ \lambda_1 \neq \lambda_2 \neq \lambda_3 \]
\[ \lambda_1 = \lambda_2 \neq \lambda_3 \]

or

\[ \lambda_1 \neq \lambda_2 = \lambda_3 \]
$\lambda_1 = \lambda_2 \neq \lambda_3$

or

$\lambda_1 \neq \lambda_2 = \lambda_3$
\[ \lambda_1 = \lambda_2 = \lambda_3 \]
\[ E = \sum_{i=1}^{\lvert T \rvert} E_{\text{align}}^i + \alpha E_{\text{smooth}} \]

Summed over all tets

Laplacian-based smoothness cost
3. Texture Parametrization

Frame field  →  Frame-aligned parametrization
\[ \phi : M \rightarrow \mathbb{R}^d \]

\[ \nabla \phi = R \]
4. Truss Layout Extraction

Global parametrization → Truss layout
Results
— $d$ orthogonal families of smooth end-to-end curves
— Curves in each family are identified with a pair of integers
— Each curve itself is parametrized
— $d$ orthogonal families of smooth end-to-end curves
— Curves in each family are identified with $d - 1$ integers
— Each curve itself is parametrized
Mars lander problem

Dense truss

Sparser variations
RodSteward: A Design-to-Assembly System for Fabrication using 3D-Printed Joints and Precision-Cut Rods

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Kindly presented by Rahul Arora
— $d$ orthogonal families of smooth end-to-end curves
— Curves in each family are identified with a pair of integers
— Each curve itself is parametrized
Entasis (Greek/Roman architecture)

(a) Selecting a curve family
(b) Exploring the entasis parameter
(c) Final structure with entasis

Structural Tests
Testing the cantilever beam

Ours

Regular grid (unoptimized)

GRAND3
[Zegard and Paulino 2015]
Human: 93 kg (205 lbs)

Bridge: 140 grams
Limitations and Future Work

— Manufacturing constraints not accounted for
  — Wire-bend each curve
  — Generate construction sequences for dowel assembly

— Sizing optimization
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Open-source! (MIT License)

https://github.com/rarora7777/VolumetricTruss
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Project page  https://www.dgp.toronto.edu/projects/michell
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