Radiosity
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Greatly inspired by:  
- Radiosity & Global Illumination, F.X. Sillon & C. Puech
- SIGGRAPH’93 Education Slide Set
  - …
Lecture outline

Introduction
• Preliminaries
• Radiosity equation
• Solving the equation
• Optimization
• Extensions
• Conclusion
Introduction

- **Local illumination:**
  - source of illumination ≡ light

- **Global illumination:**
  - source of illumination ≡ any object
Lecture outline

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  - Solving the equation
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  - Extensions
  - Conclusion

What quantities are we dealing with?
Solid angle

What is the ‘size’ of an object from a point?

• Angle

• Solid angle

unit circle $\equiv 2\pi$ radians

unit sphere $\equiv 4\pi$ steradians (sr)
Solid angle

How is it related to other units?

- Solid angle in spherical coordinates
  \[ d\omega = \sin\theta d\varphi d\theta \]

- Solid angle subtended by area \( dy \)
  \[ d\omega = \frac{dy \cos \theta'}{r^2} \]
Radiance & Radiosity

What is the relevant quantity for light?

- Radiance \( L_R(x, \theta_R, \phi_R) \)

- Radiosity \( B(x) \)

- Ideal diffuse reflector case:
  \[
  L_R(x, \theta_R, \phi_R) \equiv L_R(x) = \frac{1}{\pi} B(x)
  \]

\[
B(x) = \int_{\Omega_R} L_R(x, \theta_R, \phi_R) \cos \theta_R d\omega_R
\]
How are described surfaces?

- **BRDF** (Bidirectional Reflectance Distribution Function)

\[
\rho_{bd}(x, \theta_R, \varphi_R, \theta_I, \varphi_I) = \frac{L_R(x, \theta_R, \varphi_R)}{L_I(x, \theta_I, \varphi_I) \cos \theta_I d\omega_I}
\]

- **DHRF** (Directional Hemispherical Reflectance Function)

\[
\rho_{dh}(x, \theta_R, \varphi_R) = \frac{\int_{\Omega_R} L_R(x, \theta_R, \varphi_R) \cos \theta_R d\omega_R}{L_I(x, \theta_I, \varphi_I) \cos \theta_I d\omega_I}
\]

- **Ideal diffuse reflector case:**

\[
\rho_{bd}^R(x, \theta_R, \varphi_R, \theta_I, \varphi_I) \equiv \rho_{bd}(x) = \frac{1}{\pi} \rho_{dh}(x)
\]

\[
\rho_{dh}^R(x) \in [0,1]
\]

\[
\rho_{dh}^G(x) \in [0,1]
\]

\[
\rho_{dh}^B(x) \in [0,1]
\]

\[\equiv 'color\ of\ surface'\]
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Global Illumination Equation

How can it lead to the Radiosity Equation?

\[ L_R(x, \theta_R, \phi_R) = L_E(x, \theta_R, \phi_R) + \int_{\Omega_I} \rho_{bd}(x, \theta_R, \phi_R, \theta_I, \phi_I)L_I(x, \theta_I, \phi_I) \cos \theta_I d\omega_I \]
Ideal diffuse reflectors

Equation

\[ L_R(x, \theta_R, \phi_R) = L_E(x, \theta_R, \phi_R) + \int_{\Omega_1} \rho_{bd}(x, \theta_R, \phi_R, \theta_1, \phi_1) L_I(x, \theta_1, \varphi_1) \cos \theta_1 d\omega_1 \]

\[ B(x) = E(x) + \rho_{dh}(x) \int_{\Omega_1} \frac{1}{\pi} B_I(x, \theta_1, \varphi_1) \cos \theta_1 d\omega_1 \]

Ideal diffuse:

\[ L_R(x, \theta_R, \phi_R) \equiv \frac{1}{\pi} B(x) \]

\[ \rho_{bd}(x, \theta_R, \phi_R, \theta_1, \phi_1) \equiv \frac{1}{\pi} \rho_{dh}(x) \]
Introducing Visibility

Can we replace the ‘incoming radiosity’ terms?

\[ B(x) = E(x) + \rho_{dh}(x) \int_{S} \frac{1}{\pi} B_I(x, \theta_1, \phi_1) \cos \theta_1 d\omega_I \]

\[ B(x) = E(x) + \rho_{dh}(x) \int_{S} \frac{1}{\pi} B(y)V(x, y) \cos \theta_x d\omega_y \]
Discrete Radiosity Equation

\[ B(x) = E(x) + \rho_{dh}(x) \int_{y \in S} \frac{1}{\pi} B(y)V(x, y) \cos \theta_x \, d\omega_y \]

\[ B(x) = E(x) + \rho_i \sum_{j=1}^{N} B_j \int_{y \in P_j} \frac{1}{\pi} V(x, y) \cos \theta_x \, d\omega_y \]

\[ B_i = \frac{1}{A_i} \int_{x \in P_i} B(x) \]

\[ E_i = \frac{1}{A_i} \int_{x \in P_i} E(x) \]
Radiosity Equation

- **Discrete formulation**

\[
B_i = E_i + \rho_i \sum_{j=1}^{N} B_j \frac{1}{A_i} \int_{x \in P_i} \int_{y \in P_j} V(x, y) \cos \theta_x d\omega_y \]

- **Solving radiosity:**
  - compute form factors
  - solve \(N\) equations
Form Factors

- Amount of energy

\[ F_{ij} \]

- Property

\[ A_i F_{ij} = A_j F_{ji} \]
Form Factors

- Nusselt’s analogy

\[
\frac{1}{A_i} \int_{x \in P_i} \int_{y \in P_j} \frac{V(x, y) \cos \theta_x d\omega_y}{\pi}
\]

\[
F_{ij} \approx \int_{y \in P_j} \frac{V(x, y) \cos \theta_x d\omega_y}{\pi}
\]

- Hemi-cube

\[
F_{ij} = \sum \Delta F
\]
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Radiosity Equation

\[ B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij} \]
Solving the equation

4 famous methods for solving the ‘classic’ radiosity

- Matrix inversion
- Jacobi relaxation
- Gauss-Seidel relaxation (gathering)
- Southwell relaxation (shooting)
Matrix inversion

**Radiosity Equation**

\[ B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij} \]

\[ B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i \quad i \in [1, N] \]

\[
\begin{pmatrix}
1 - \rho_{1} F_{11} & -\rho_{1} F_{12} & \cdots & -\rho_{1} F_{1N} \\
-\rho_{2} F_{21} & 1 - \rho_{2} F_{22} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_{N} F_{NN} & \cdots & \cdots & 1 - \rho_{N} F_{NN}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_N
\end{pmatrix}
= 
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_N
\end{pmatrix}
\]

\[ MB = E \]

Invert \( M \)

**Drawbacks:** all form factors required / no intermediate solution
Jacobi relaxation

Radiosity Equation

\[ B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij} \]

\[ B_i = \frac{E_i}{1 - \rho_i F_{ii}} + \rho_i \sum_{j=1}^{N} B_j \frac{F_{ij}}{1 - \rho_i F_{ii}} \]

vector \( B^t \)

vector \( B^{t+1} \)

\[
\begin{align*}
B_1 \\
B_2 \\
B_3 \\
\vdots \\
B_N
\end{align*}
\]

\[
\begin{align*}
B_1 \\
B_2 \\
B_3 \\
\vdots \\
B_N
\end{align*}
\]

compute \( N \) values

Drawbacks: all form factors required / two vectors \( B^t \) and \( B^{t+1} \) required
Gauss-Seidel relaxation

Radiosity Equation

\[ B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij} \]

\[ B_i = \frac{E_i}{1 - \rho_i F_{ii}} + \rho_i \sum_{j=1 \atop j \neq i}^{N} B_j \frac{F_{ij}}{1 - \rho_i F_{ii}} \]

Vector \( B^t = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_N \end{pmatrix} \)

update \( B_1 \)

update \( B_2 \)

update \( B_3 \)

update \( B_N \)

Drawbacks: all form factors are required for one complete iteration

Advantages: only one storage vector required / faster convergence than Jacobi
Gauss-Seidel relaxation

- Why is it called Gathering?

\[
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
\vdots \\
B_i \\
\vdots \\
B_N
\end{bmatrix} = \frac{E_i}{1 - \rho_i F_{ii}} + \rho_i \sum_{j=1}^{N} B_j \frac{F_{ij}}{1 - \rho_i F_{ii}}
\]
Southwell relaxation

• Idea: distribute the residual energy of the patch of most residual energy, $P_i$

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

Energy Equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{N} A_j B_j F_{ji}$$

Residuals energy

$$r_i = \varepsilon_i - \beta_i + \rho_i \sum_{j=1}^{N} \beta_j F_{ji}$$

1. $k = \arg \max_k (r_k)$

2. $r_{i\neq k} = \frac{\rho_i F_{ik}}{1 - \rho_k F_{kk}} r_k$, $r_k = 0$

3. $\beta_k = \frac{\varepsilon_k}{1 - \rho_k F_{kk}} + \rho_i \sum_{j=1}^{N} \frac{\beta_j F_{jk}}{1 - \rho_k F_{kk}}$, $B_i = \frac{\beta_i + r_i}{A_i}$
Southwell relaxation

- Why is it called **Shooting**?

\[
    r_{i \neq k} + = \frac{\rho_i F_{ik}}{1 - \rho_k F_{kk}} r_k
\]

\[
    r_k = 0
\]

Ambient correction
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Optimizing

- **Sampling tradeoff**
  - too coarse: ugly solution
  - too fine: time consuming

- The necessity to emit energy at different levels:
Hierarchical Radiosity

- Allow exchanges at different levels
- Links between two hierarchical patches:
  \[ \Rightarrow \text{threshold + splitting criterion} \]

Gathering radiosity for a patch \( P_i \):
1. Compute gathered radiosities \( B_G \) for patch \( P_i \) and its subpatches
2. Update the radiosities \( B \) of the tree (bidirectional sweep)
Decreasing tolerance
(Blocks are breaking)
Importance

- View dependent refinement

Radiosity solution  Importance solution  Superposition, in yellow

Importance Equation

\[ I_i = R_i + \sum_{j=1}^{N} \rho_j F_{ji} I_j \]
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Basic algorithms fail at transitions

$$\text{Diffuse} \rightarrow \text{Specular} \ | \ \text{Specular} \rightarrow \text{Diffuse}$$

Basic Raytracing
$$LS^*E$$

Basic Radiosity
$$LD^*E$$

Global Illumination
$$L(D|S)^*E$$
Directional radiosity

- Global Illumination:

- Radiosity:
  radiance is constant

→ Directional radiosity:
  radiance is piecewise constant
Two-pass approach

- Two-pass approach
  1. Radiosity: LD*E
  2. Raytracing LDS*E

- Limitation: misses LS*DE
Complete* two-pass approach
(* Ideal diffuse & ideal specular surfaces)

- Two-pass approach with extended form factors
  Idea: Include $D \rightarrow S^* \rightarrow D$ in the first pass
For the eyes
Cornell University Program of Computer Graphics
END