

subsurface scattering

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Pixar

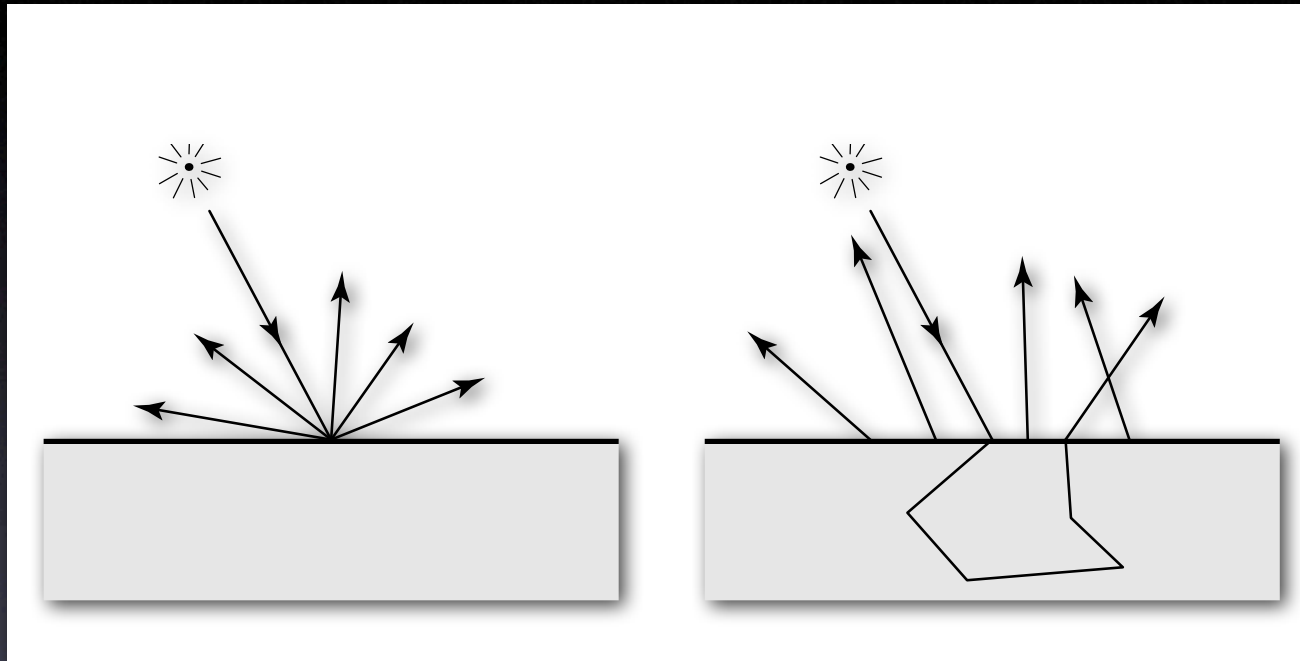


Dror Bar-Natan



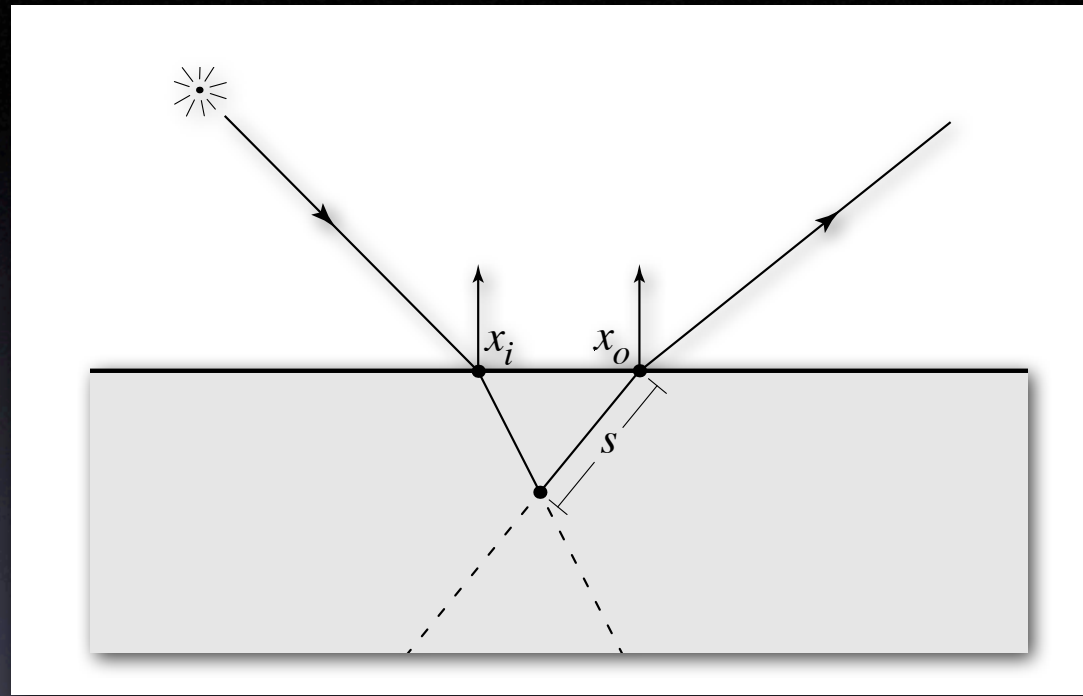
amazon.com

subsurface scattering



Jensen et al. 2001

BRDF vs. BSSRDF



Jensen et al. 2001

Fresnel effects

- absorption
- emission
- out scattering
- in scattering
- extinction

scattering processes

$$dL_o(\mathbf{p}, \mathbf{w}) = -\sigma_a(\mathbf{p}, \mathbf{w})L_i(\mathbf{p}, -\mathbf{w})dt$$

$$e \int_0^d \sigma_a(\mathbf{p} + t\mathbf{w}) \mathbf{w} dt$$

absorption

$$Q(p, w)$$

emission

$$dL_o(\mathbf{p}, \mathbf{w}) = -\sigma_s(\mathbf{p}, \mathbf{w})L_i(\mathbf{p}, -\mathbf{w})dt$$

out scattering

$$\sigma_t(\mathbf{w}, \mathbf{w}) = \sigma_a(\mathbf{w}, \mathbf{w}) + \sigma_s(\mathbf{w}, \mathbf{w})$$

extinction

$$tr(\mathbf{p} \rightarrow \mathbf{p}') = e^{-\int_0^d \sigma_t(\mathbf{p} + t\mathbf{w}) dt}$$

transmittance

$$\tau(p \rightarrow p') = \int_0^d \sigma_t(p + tw, w) dt$$

optical thickness

- distance between particles many times larger than radius
- describe with phase functions

in scattering

phase functions



$$(w \cdot \nabla)L(x, w) = -\sigma_t L(x, w) + \sigma_s \int_{4\pi} p(w, w')L(x, w')dw' + Q(x, w)$$

radiative transport

- bidirectional subsurface scattering reflectance distribution function
- bidirectional surface scattering distribution function
- bidirectional scattering surface reflectance distribution function

BSSRDF

$$L(x, w) = \int_{H^2} \int_S S(x', w'; x, w) L(x, w') \cos\theta' dw'$$

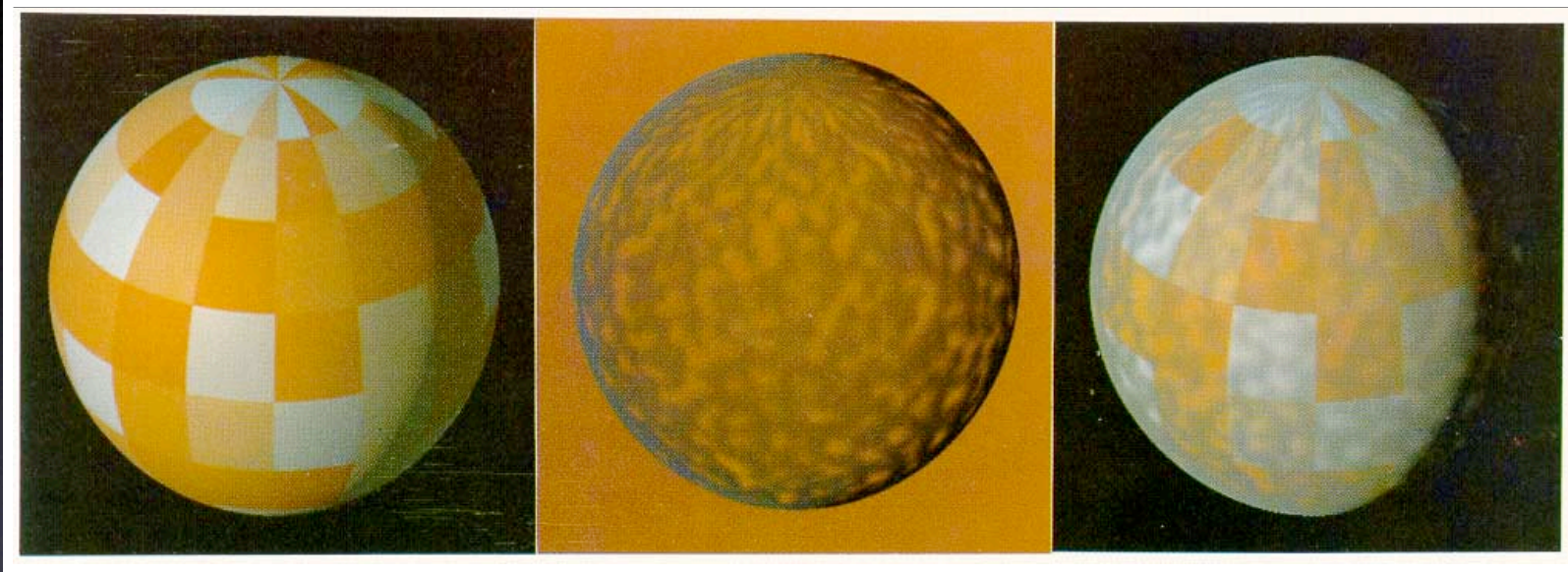
BSSRDF

- scattered light enters at one point
- based on linear transport theory
- infinite homogeneous plane
- constant illumination

analytic models

- HG phase function, Rayleigh scattering
- derived from a statistical model of scattering events
- layer of clouds or dust

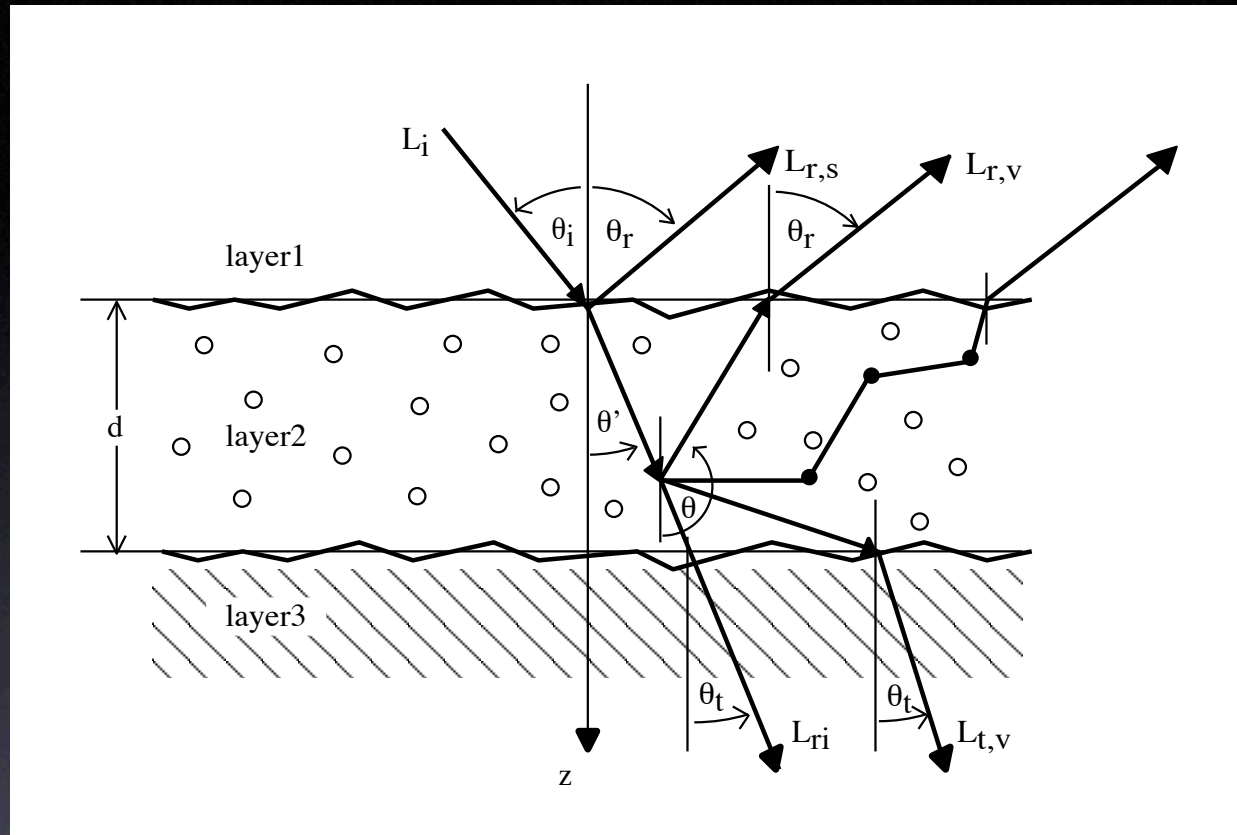
Blinn 1982



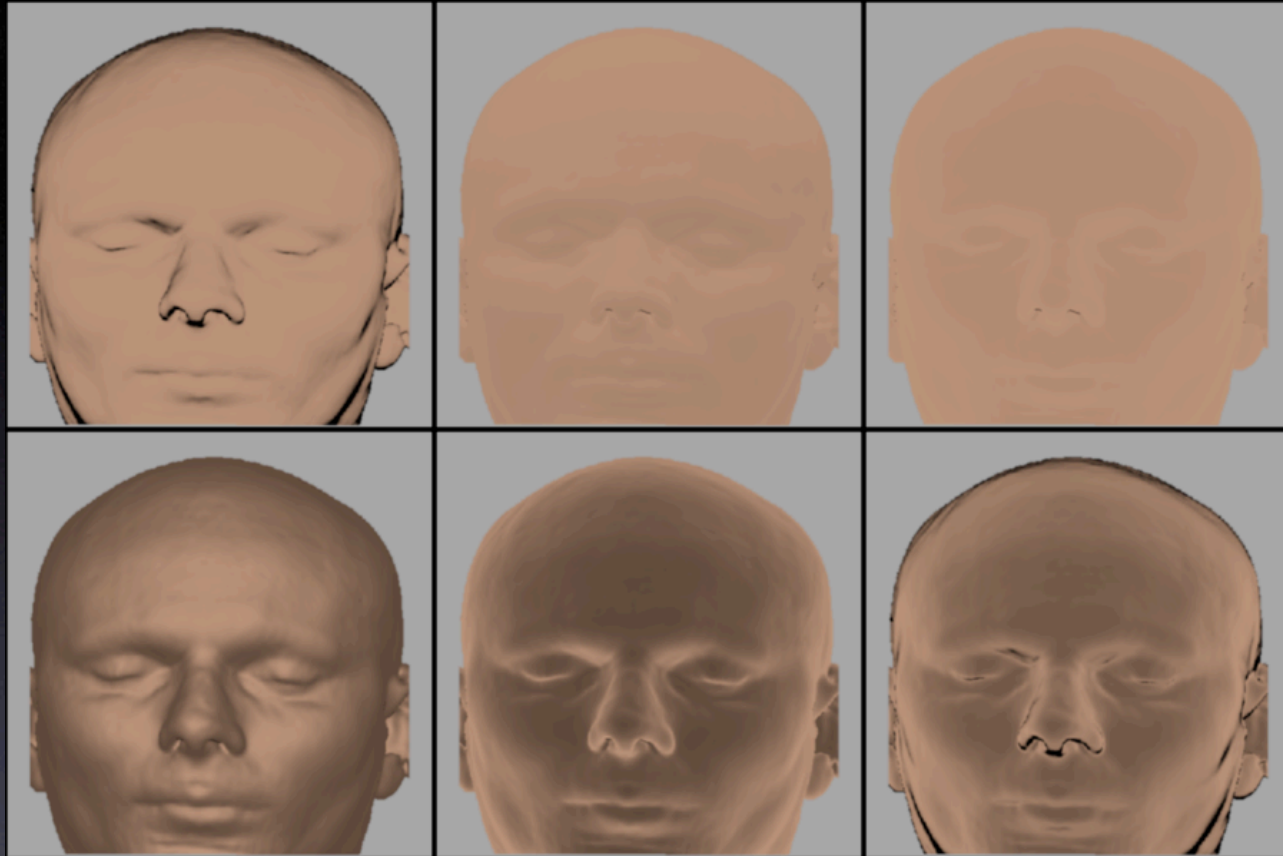
Blinn 1982

- derived from radiative transport theory
- multiple layers
- captures directional effects

Hanrahan and Krueger
1990



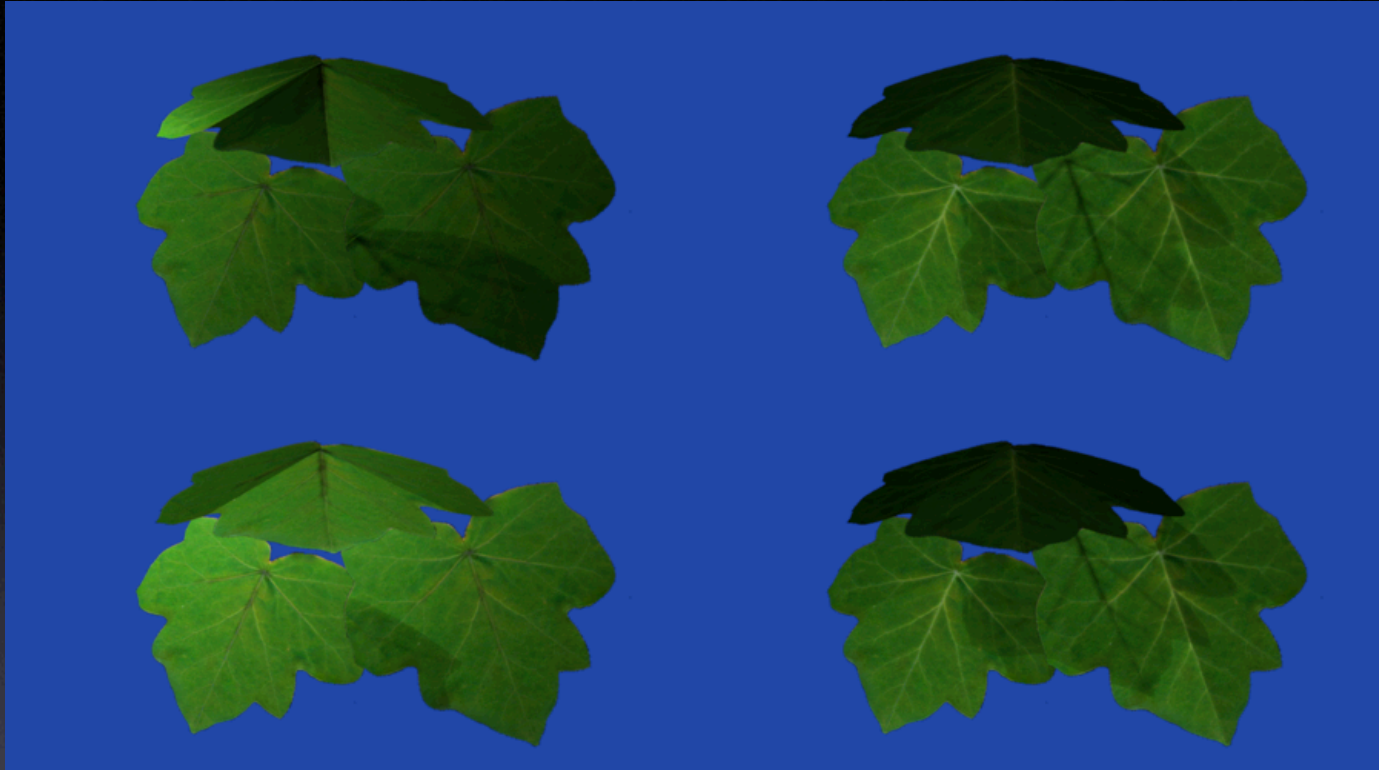
Hanrahan and Krueger 1990



Hanrahan and Krueger
1990



Hanrahan and Krueger
1990



Hanrahan and Krueger
1990

- convergence independent of dimension
- error decrease $O(\sqrt{N})$

Monte Carlo

- application: weathered stone layer
- cast photon map into layer
- sample along penetration depth

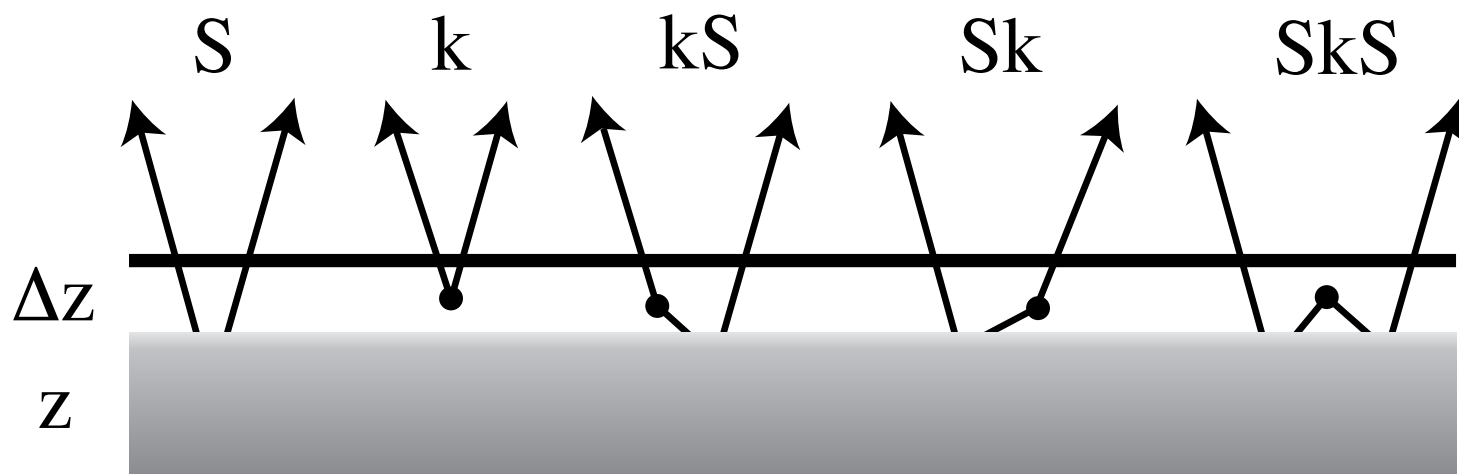
Dorsey 1999



Dorsey 1999

- abstraction of scattering functions
- layer-to-layer interaction described by a set of adding equations

Pharr 2000



Pharr 2000

$$\mathbf{S}(z) = \int_0^z e^{-\sigma_t(1/\mu_i+1/\mu_o)(z-z')} (\mathbf{k}(z') + \mathbf{k}(z')\mathbf{S}(z') + \mathbf{S}(z')\mathbf{k}(z') + \mathbf{S}(z')\mathbf{k}(z')\mathbf{S}(z')) dz'.$$

Pharr 2000

$$\begin{aligned}
S(z, r_i \rightarrow r_o) = & \int_0^z e^{-(\sigma_i(x_i)/\mu_i + \sigma_i(x_o)/\mu_o)(z-z')} \left(k(r_i(z') \rightarrow r_o(z')) + \frac{1}{4\pi} \int_{\mathcal{R}_{M^2(z')}^+} k(r_o \rightarrow -r') S(z', r_i \rightarrow r') \frac{dr'}{\mu_{r'}^2} + \right. \\
& \left. \frac{1}{4\pi} \int_{\mathcal{R}_{M^2(z')}^+} S(z', r' \rightarrow r_o) k(r_i \rightarrow r') \frac{dr'}{\mu_{r'}^2} + \frac{1}{16\pi^2} \int_{\mathcal{R}_{M^2(z')}^+} \int_{\mathcal{R}_{M^2(z')}^+} S(z', r'' \rightarrow r_o) k(-r' \rightarrow -r'') S(z', r_i \rightarrow r') \frac{dr'}{\mu_{r'}^2} \frac{dr''}{\mu_{r''}^2} \right) dz'
\end{aligned}$$

Pharr 2000



depth = 0.



depth = 0.2



depth = 0.8



depth = 2.0



Pharr 2000

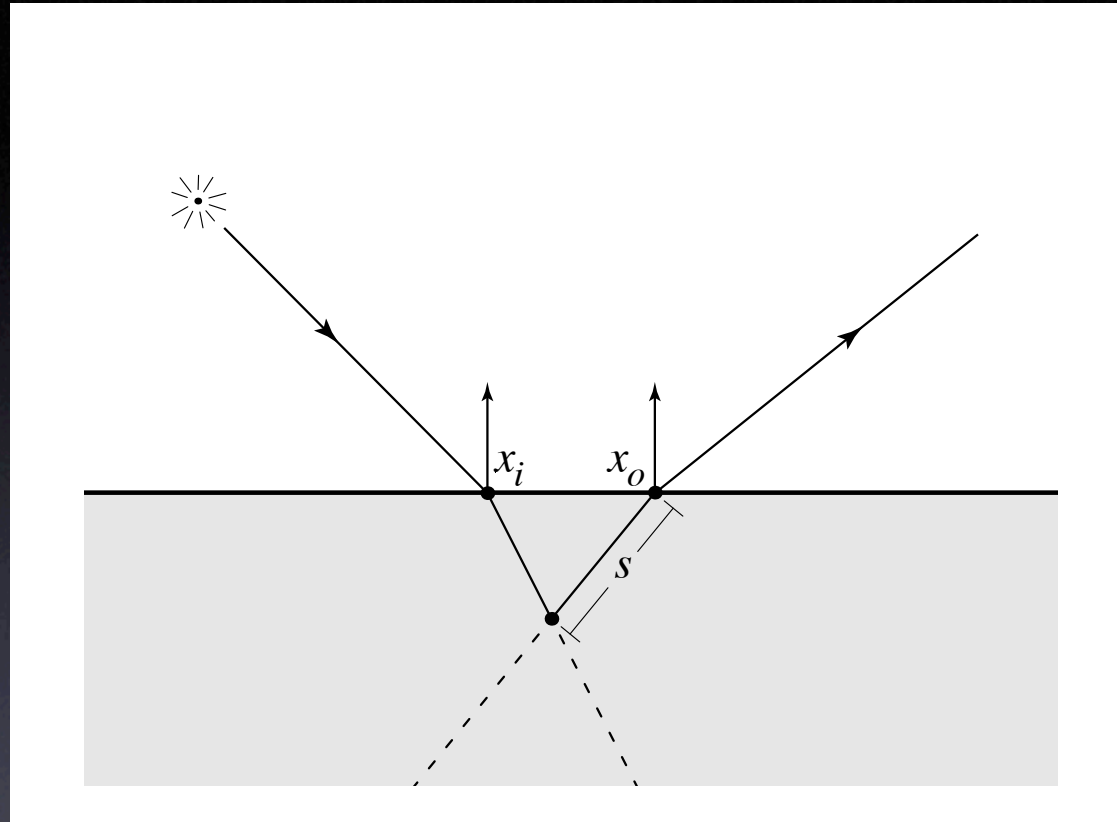
- assumes homogeneous media
- fast evaluation for multiple scattering with many events (milk, skin)
- diffusion for multiple scattering introduced by Stam in 95 for participating media

diffusion for multiple
scattering

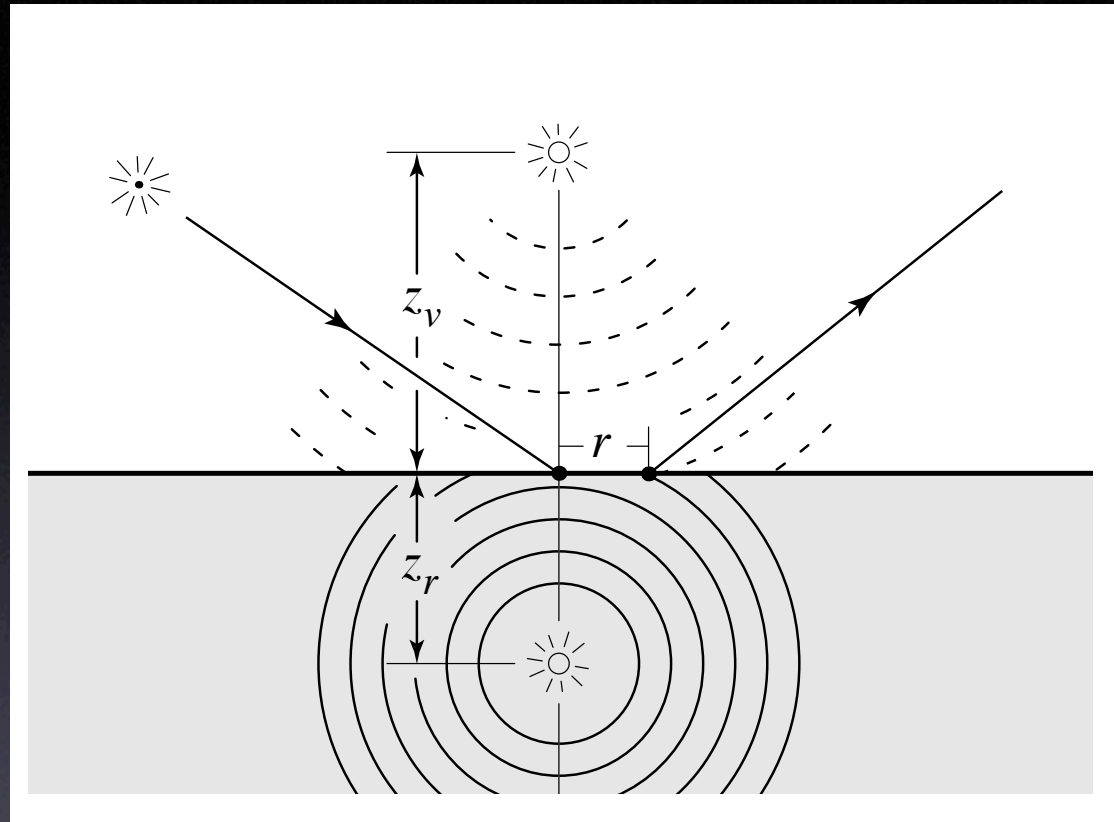
$$S(x_i, w_i; x_o, w_o) = S_d(x_i, w_i; x_o, w_o) + S^{(1)}(x_i, w_i; x_o, w_o)$$

- exact single scattering
- dipole approximation of diffusion model for multiple scattering

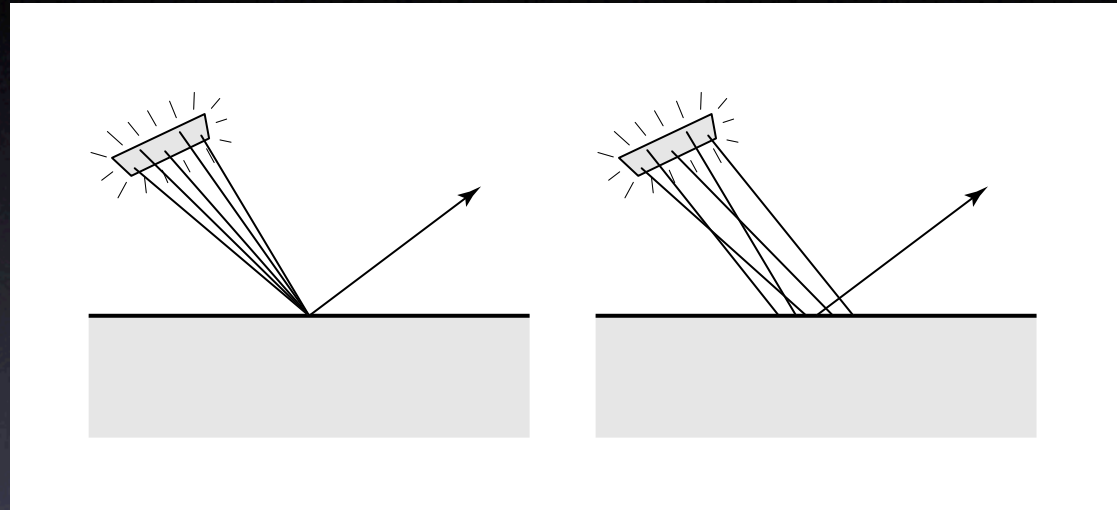
Jensen 2001



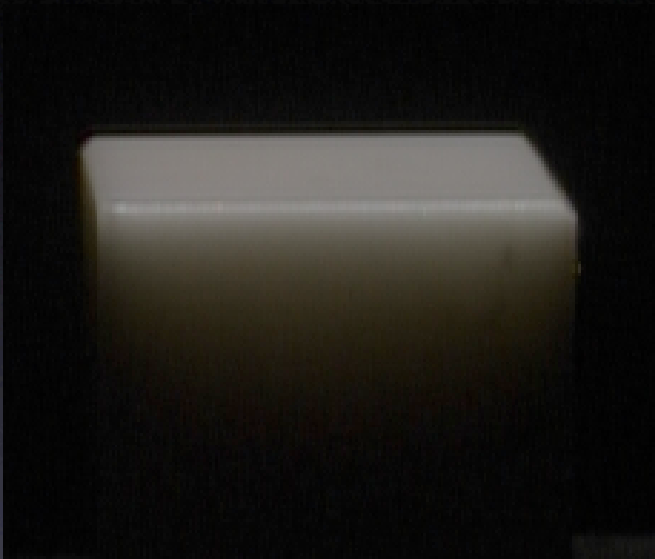
single scattering



multiple scattering



MC evaluation



Jensen 01



Jensen 01



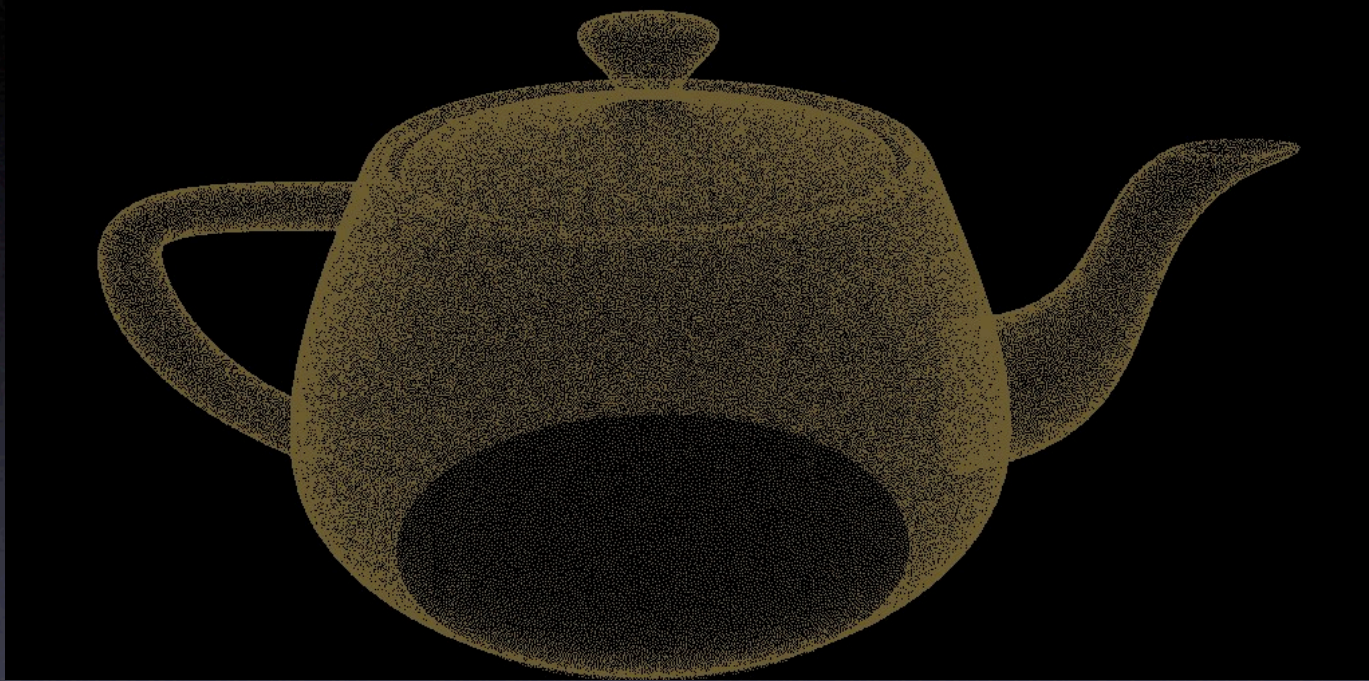
Jensen 01



Jensen 01

- decouple sample irradiance computation from diffuse approximation computation
- hierarchical evaluation of multiple scattering term

Jensen 02



Jensen 02

- build octree model of (irradiance, centroid, total area)
- depth criteria: solid angle for evaluation point to total area $< \epsilon$.

Jensen 02



Jensen 02



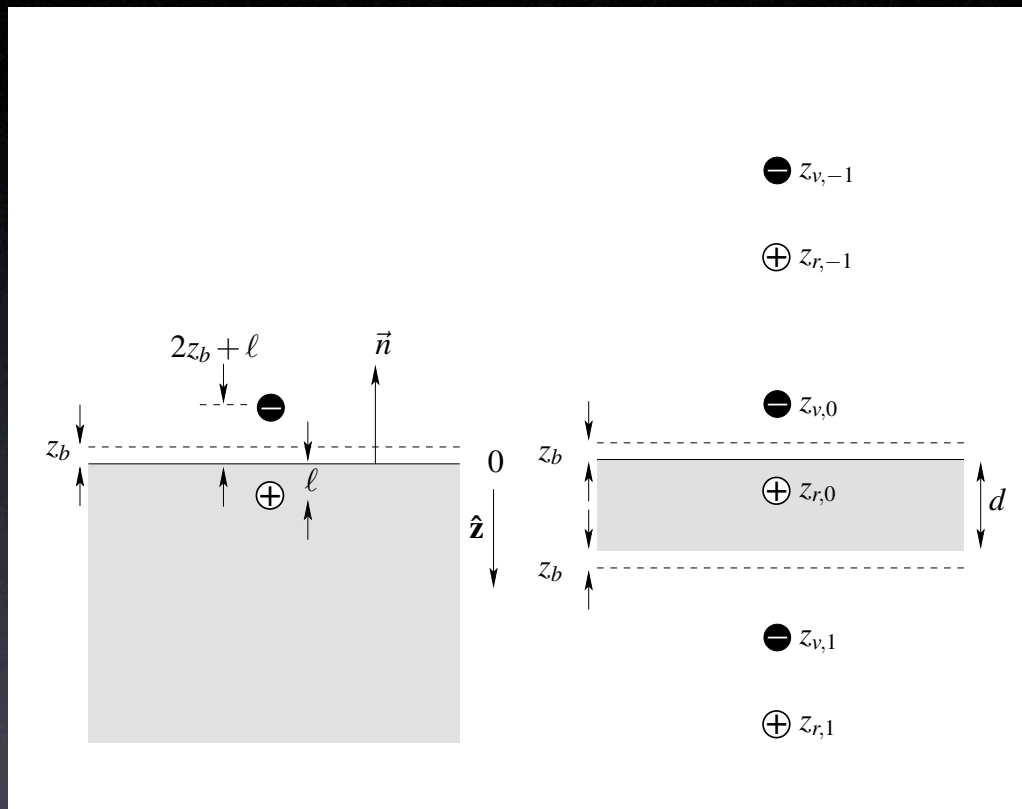
Jensen 02

- dipole diffusion approximation breaks down
- assumption of infinite depth: no transmittance though to opposite side

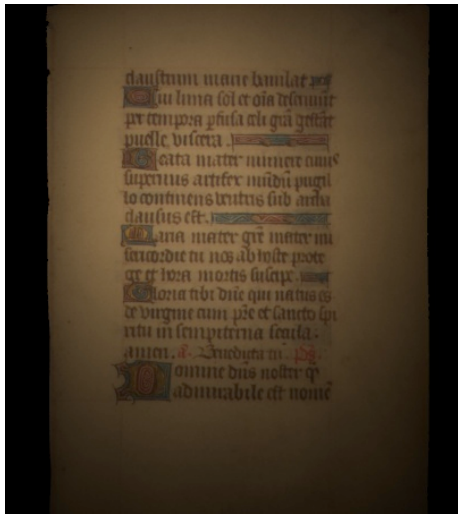
thin objects

- multipole approximation
- frequency space Kubelka Munk model for interaction among multiple layers
- rough surfaces: replace fresnel attenuation with a BRDF

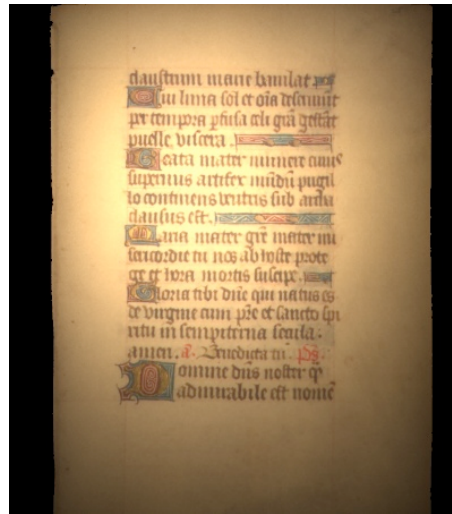
Donner 05



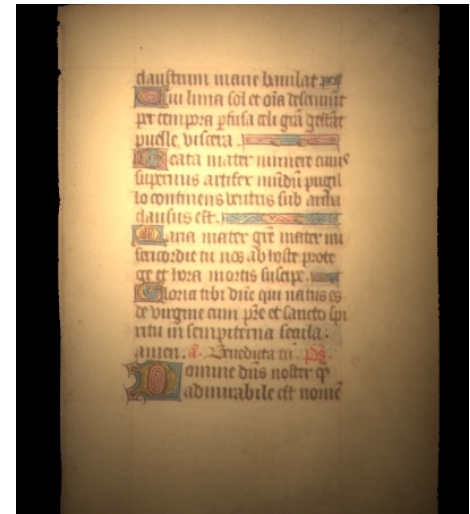
Donner 05



Dipole model

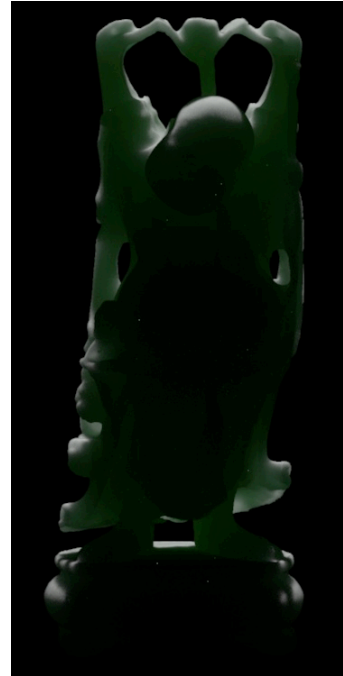
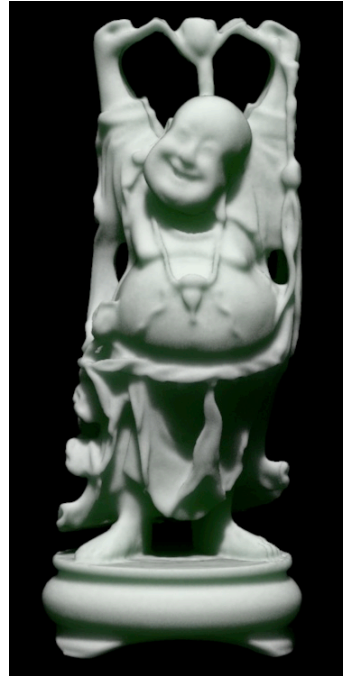


Multipole model



Monte Carlo reference

Donner 05



Jade

Jade + paint

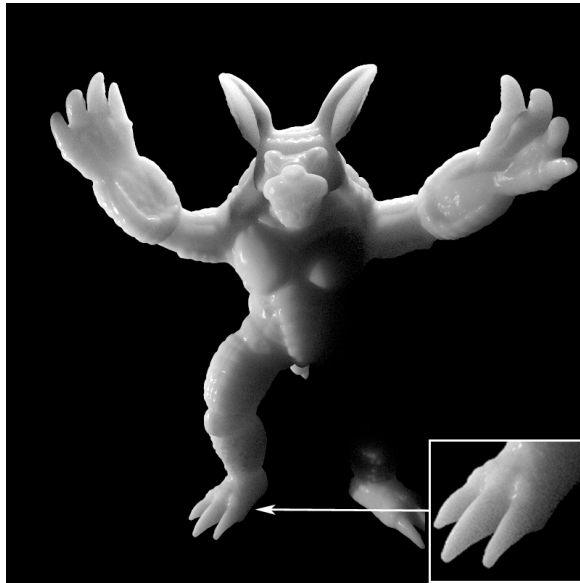
Donner 05



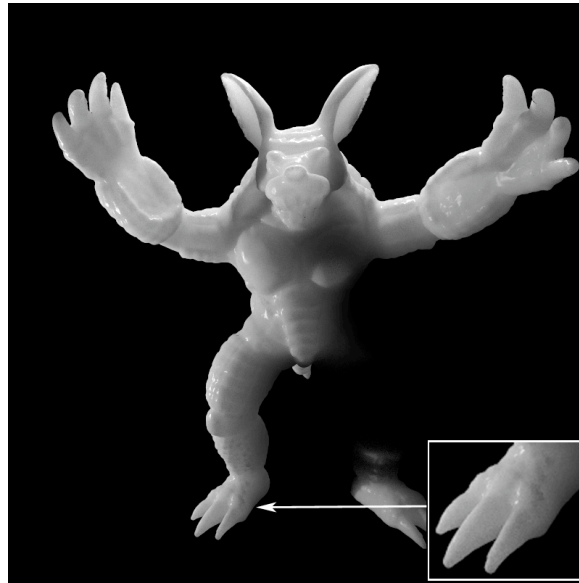
Donner 05

- MC evaluation until ray passes a depth threshold
- diffusion approximation in core

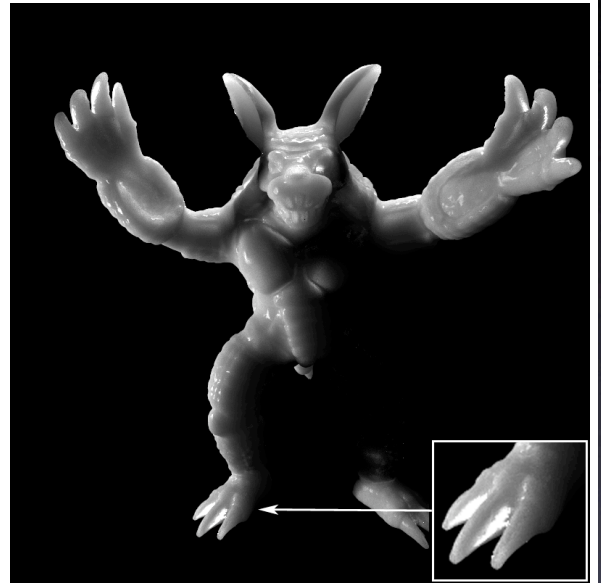
Li 05



a. Monte Carlo (246 min)



b. Hybrid method (33 min)



c. Jensen et al. (10 min)

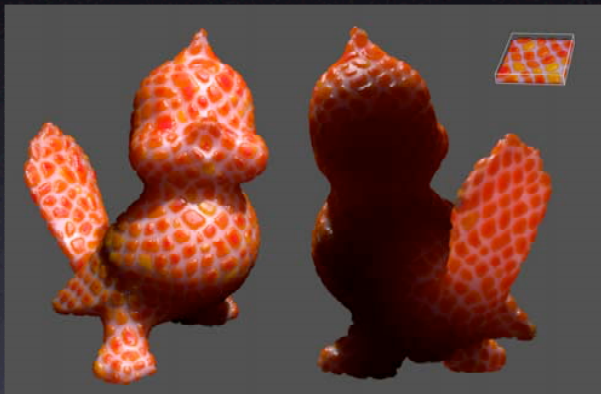
Li 05

- diffusion approximation for multiple scattering blurs out local variation

nonhomogeneities

- model mesostructure and volumetric nonhomogeneity in region near surface
- fit “shell texture function” to high quality data in volume
- evaluate core using diffusion approximation
- evaluate shell with STF

Chen 04



Chen 04

- two scale model: fine nonhomogeneities, course diffusion approximation
- local scattering effects: local reflectance, mesostructure entrance function, mesostructure exit function
- texture synthesis of local model.

Tong 05



Tong 05



Tong 05