Neatening sketched strokes using piecewise French Curves

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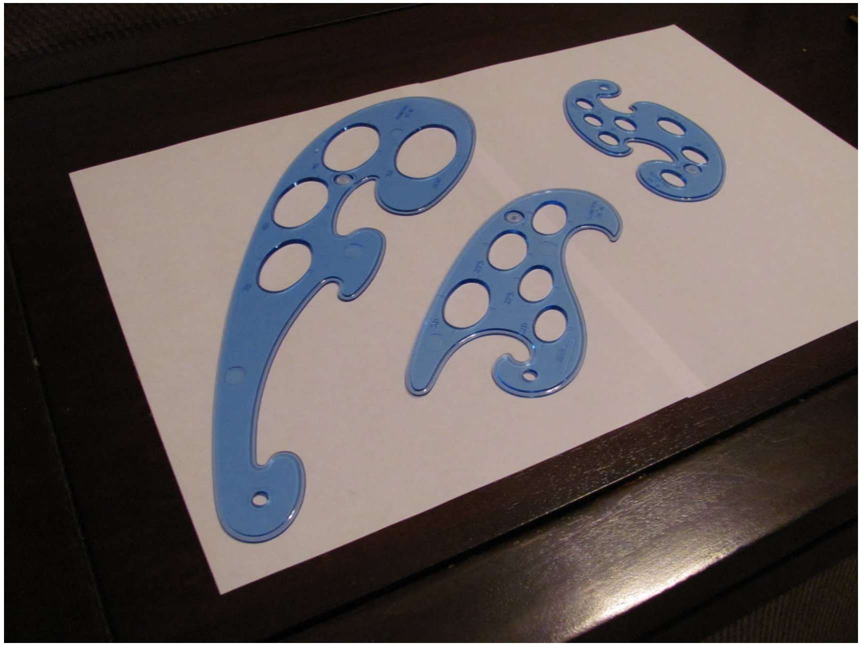
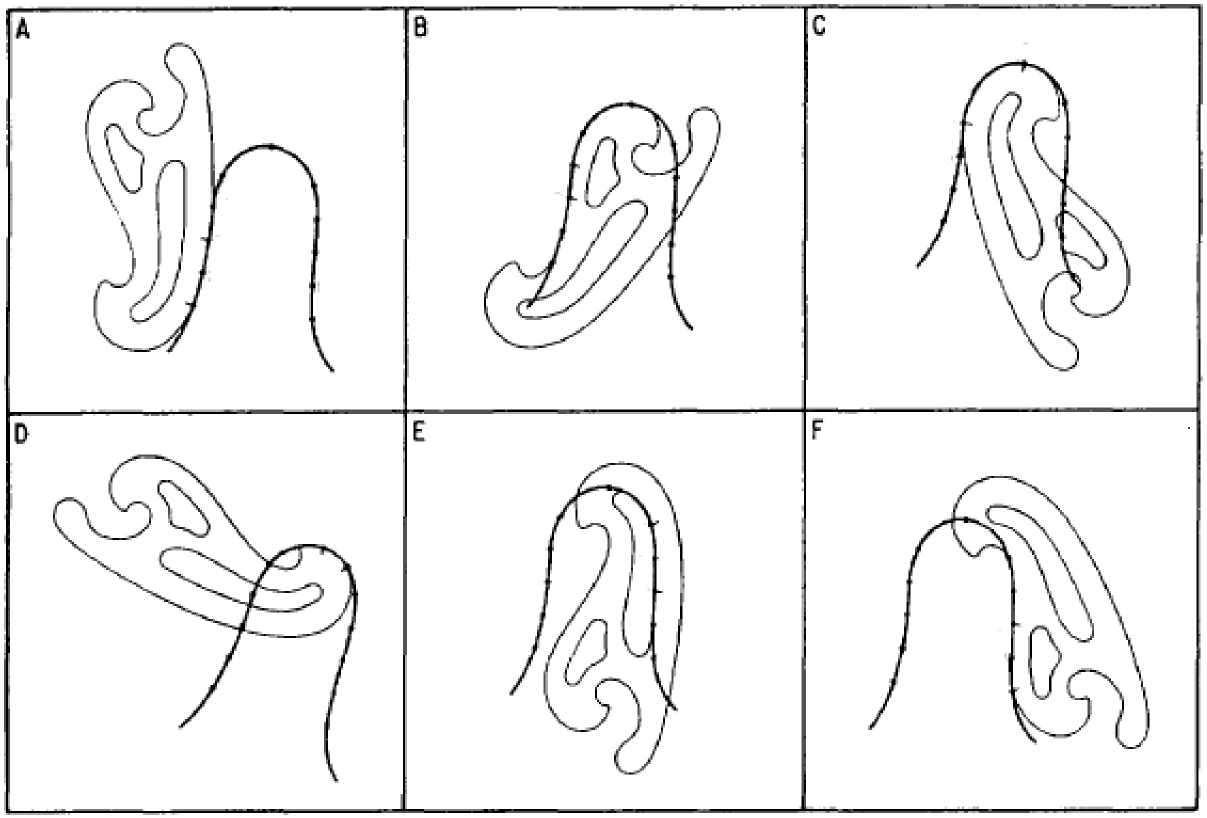
French Curves

Physical tools, used to model curves

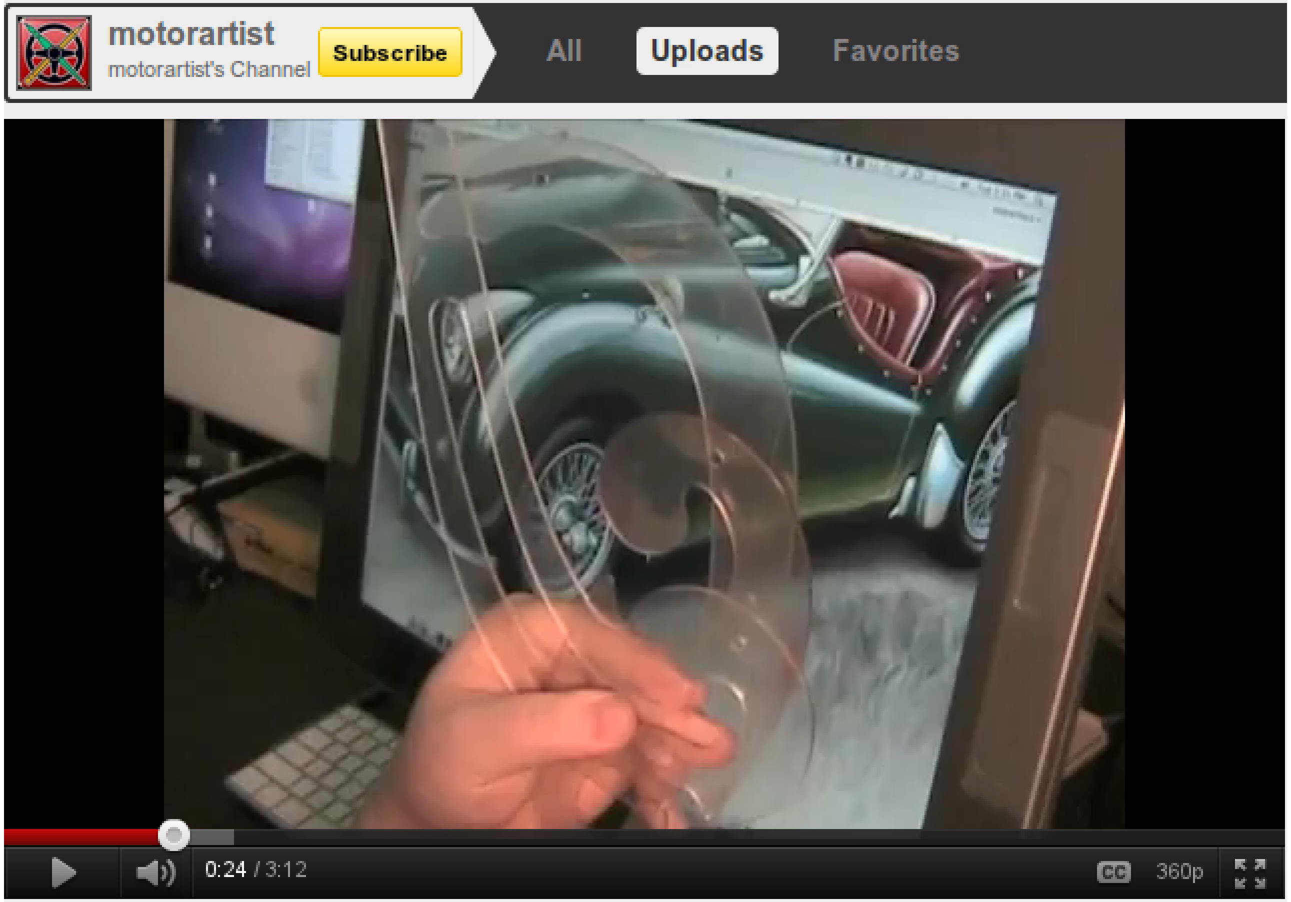


French Curves

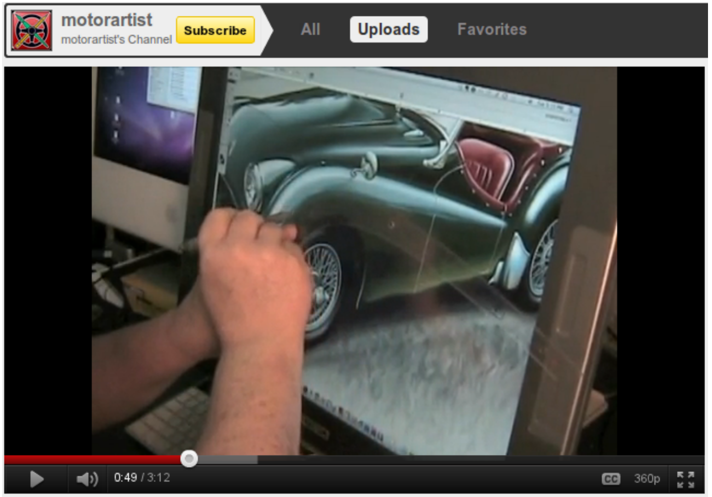
Smoothly connect pre-determined curve points

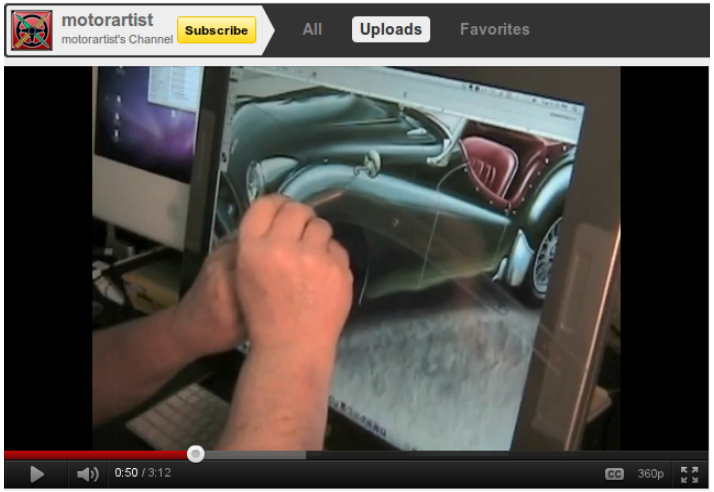


French Curves



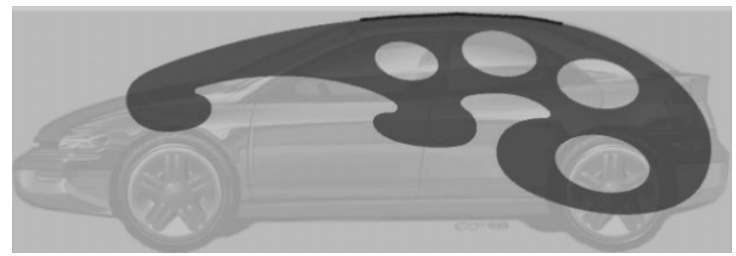
French Curves





Digital French Curves

Two-handed manipulation of digitized French curves (represented as cubic NURBS curves)



Karan Singh. 1999. *Interactive curve design using digital French curves*. Interactive 3D Graphics (I3D '99). ACM, New York, NY, USA, 23-30.

Motivation

The idea: French curves + sketch interface

Motivation

The idea: French curves + sketch interface

Why?

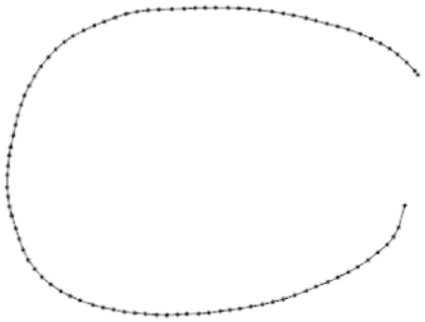
- Smooth, high quality
- Specific style/standard

- Fast to learn
- Easy curve modelling

Problem Statement

Specifically, given

input polyline



Problem Statement

Specifically, given

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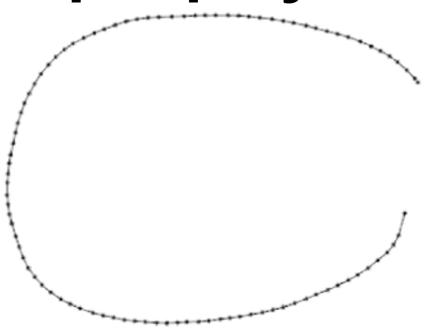
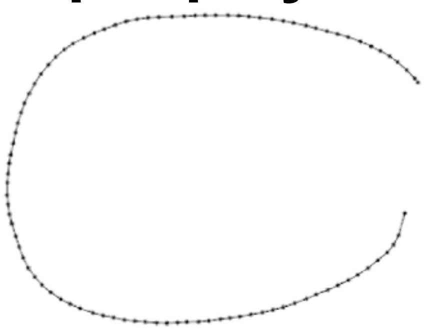


French curve

Problem Statement

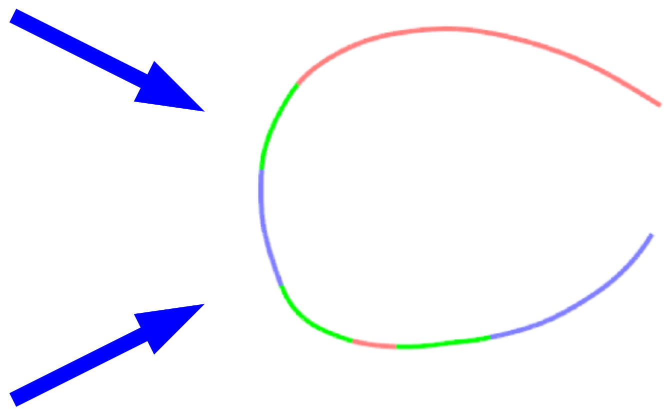
Specifically, given

input polyline



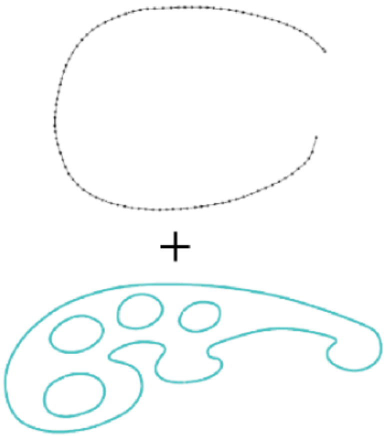
French curve

reconstruct the polyline

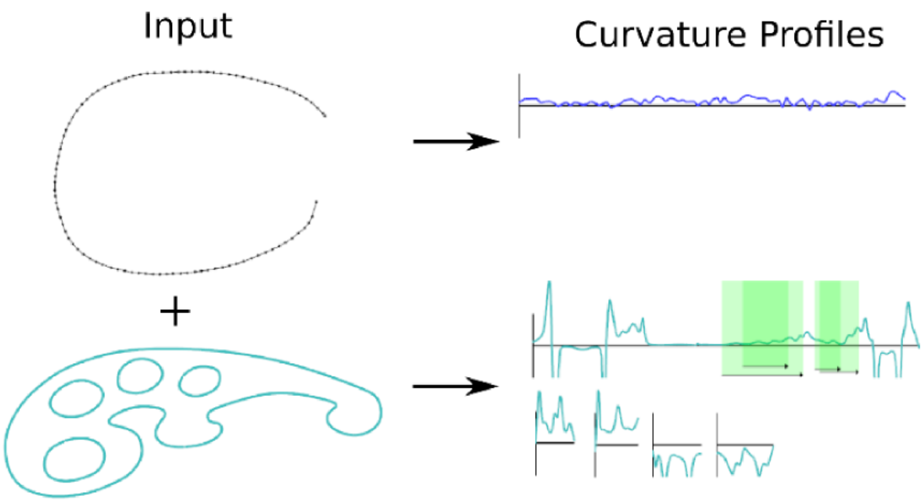


Approach

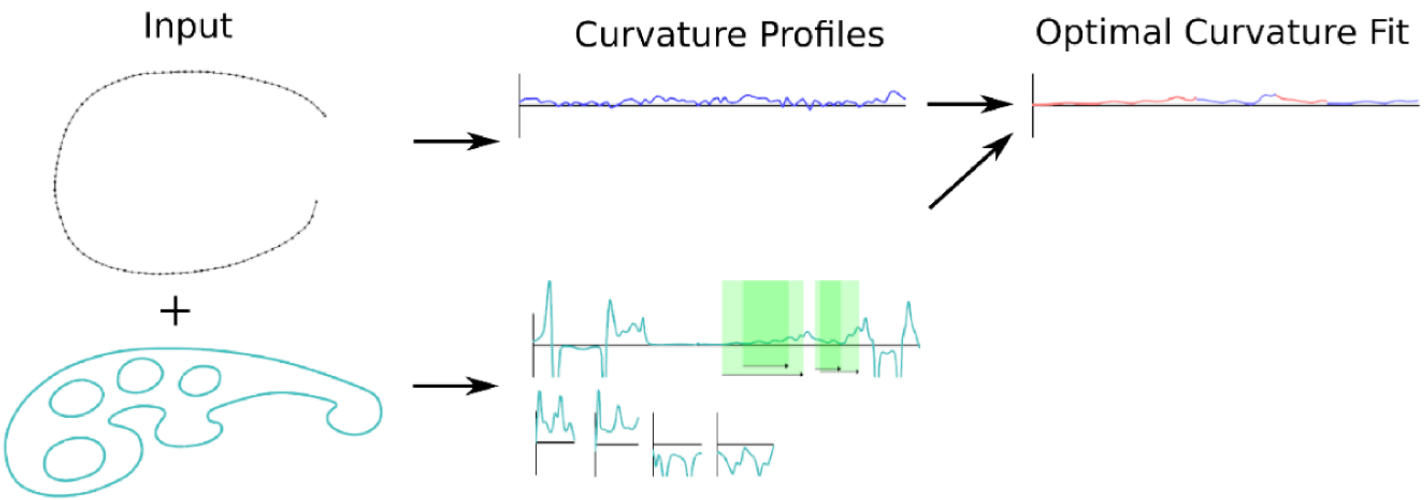
Input



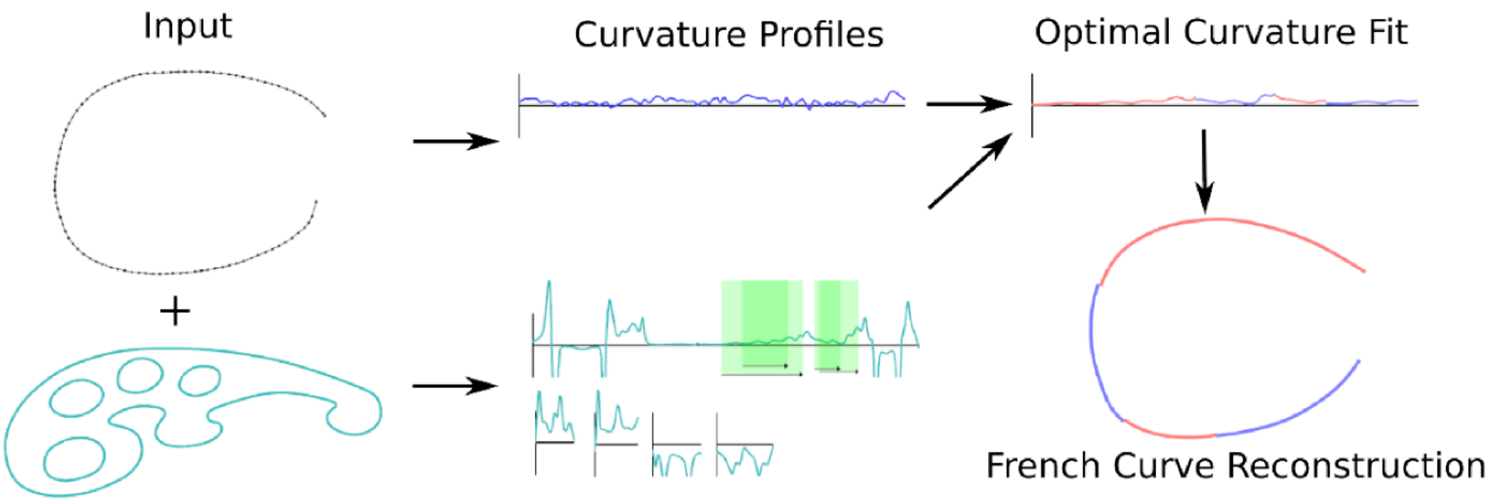
Approach



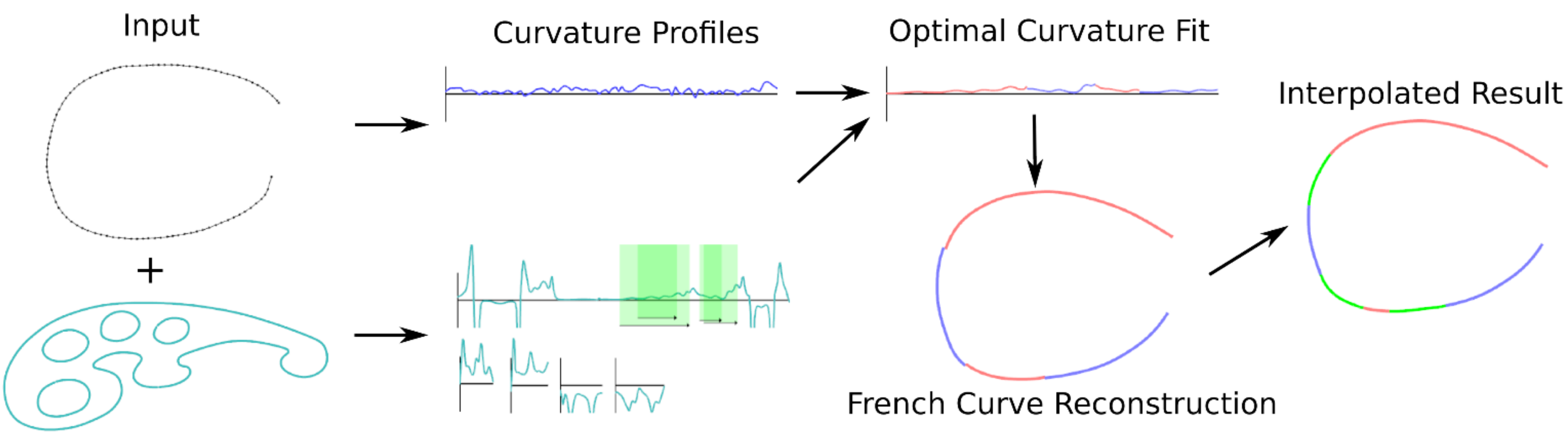
Approach



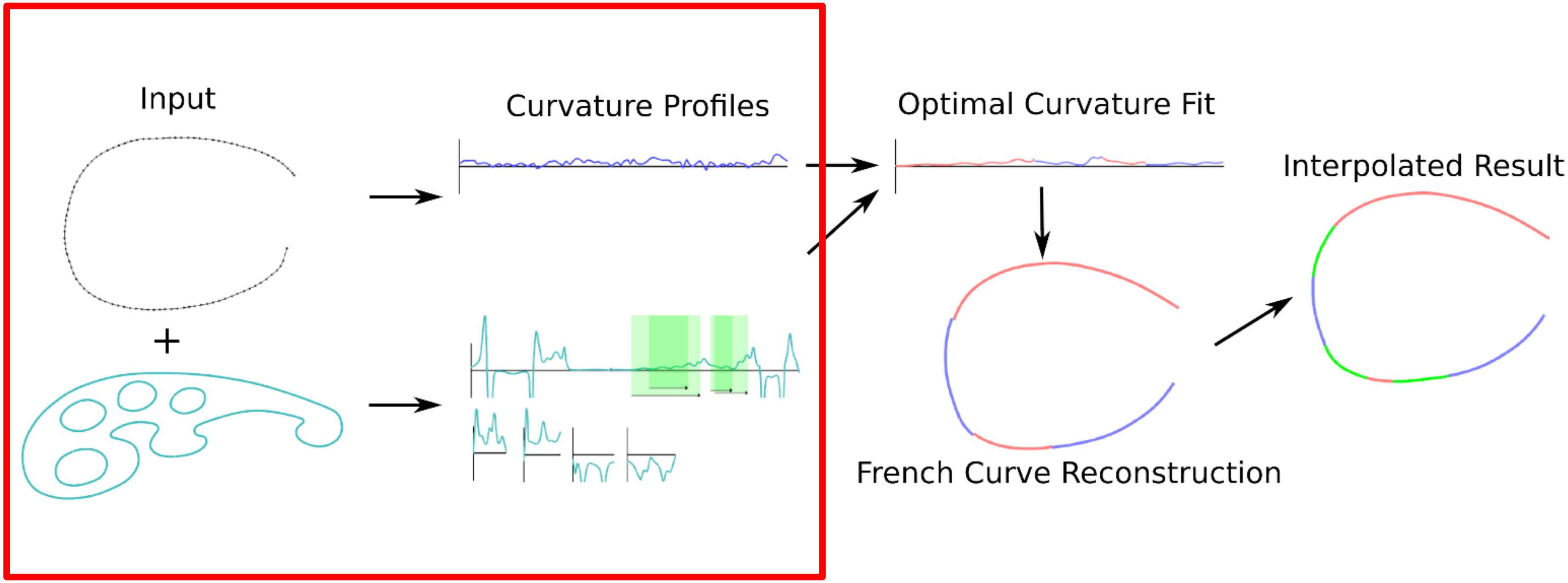
Approach


Input
Curvature Profiles
Optimal Curvature Fit
French Curve Reconstruction

Approach

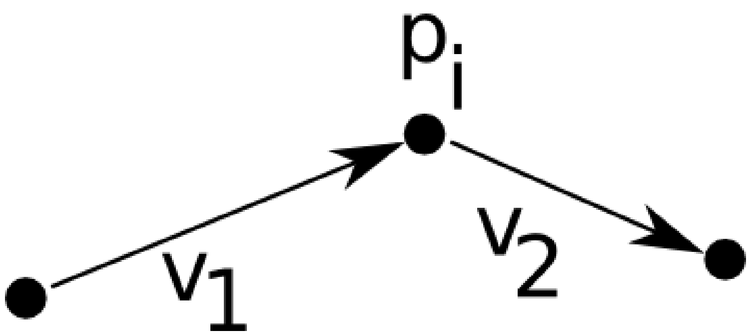

Input
+
Curvature Profiles
Optimal Curvature Fit
Interpolated Result
French Curve Reconstruction

Curvature Profiles



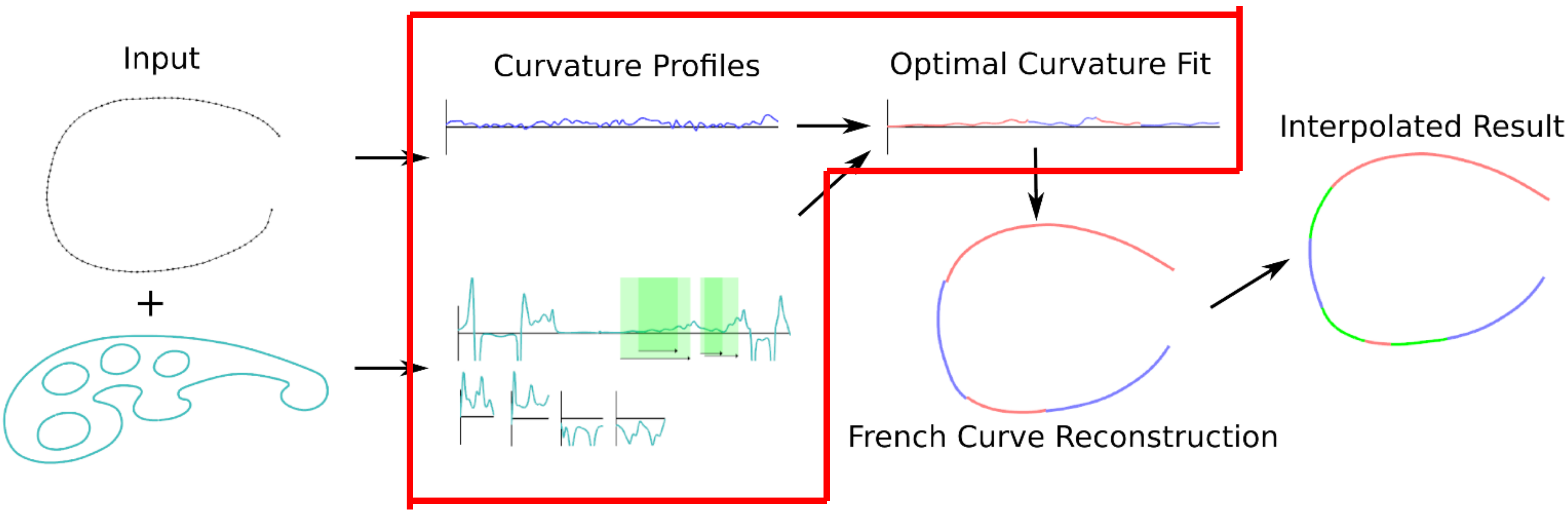
Curvature Profiles

Discrete curvature estimator:



$$\kappa = \frac{2 \cdot \vec{v_1} \times \vec{v_2}}{|| \vec{v}1 || + || \vec{v}2 || + || \vec{v}1 + \vec{v}2 ||}$$

Optimal Curvature Fit



Optimal Curvature Fit

Two parts:

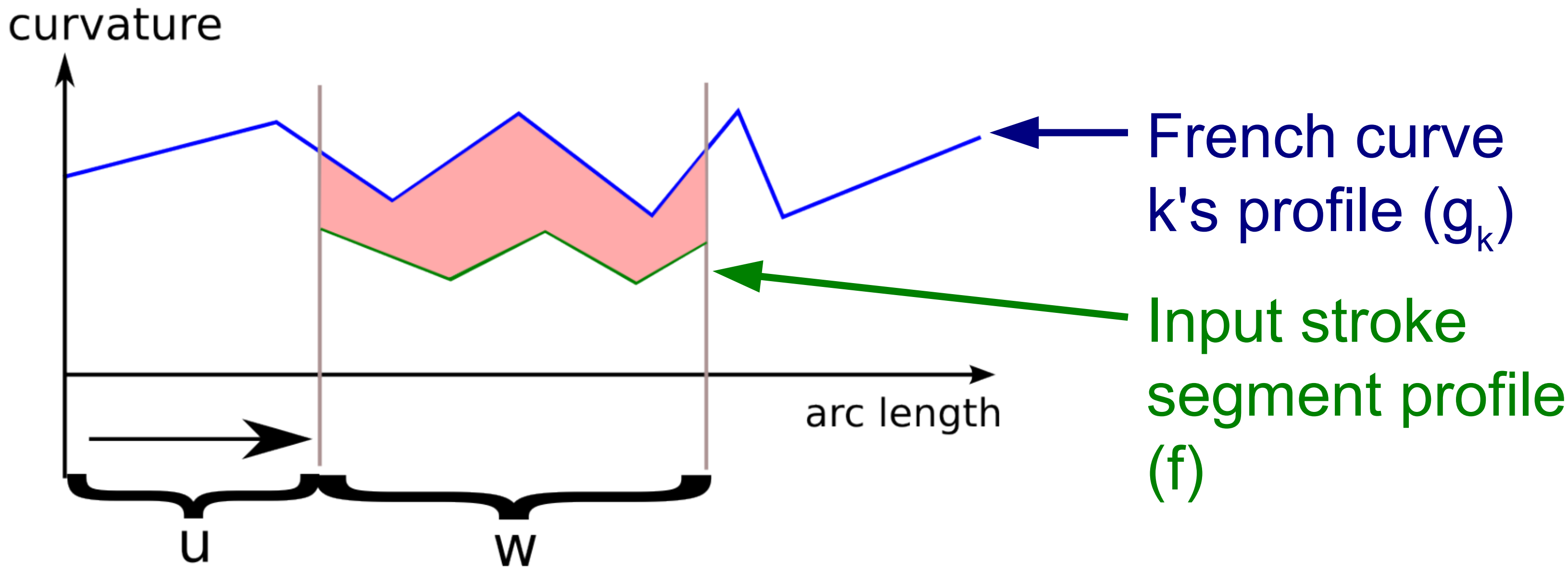
1. Optimal French curve piece for segment of input
2. Optimal segmentation of input curve profile

Optimal Curvature Fit

1. Optimal French curve piece for segment of input

Solution: **Iterate** over French curve profiles:

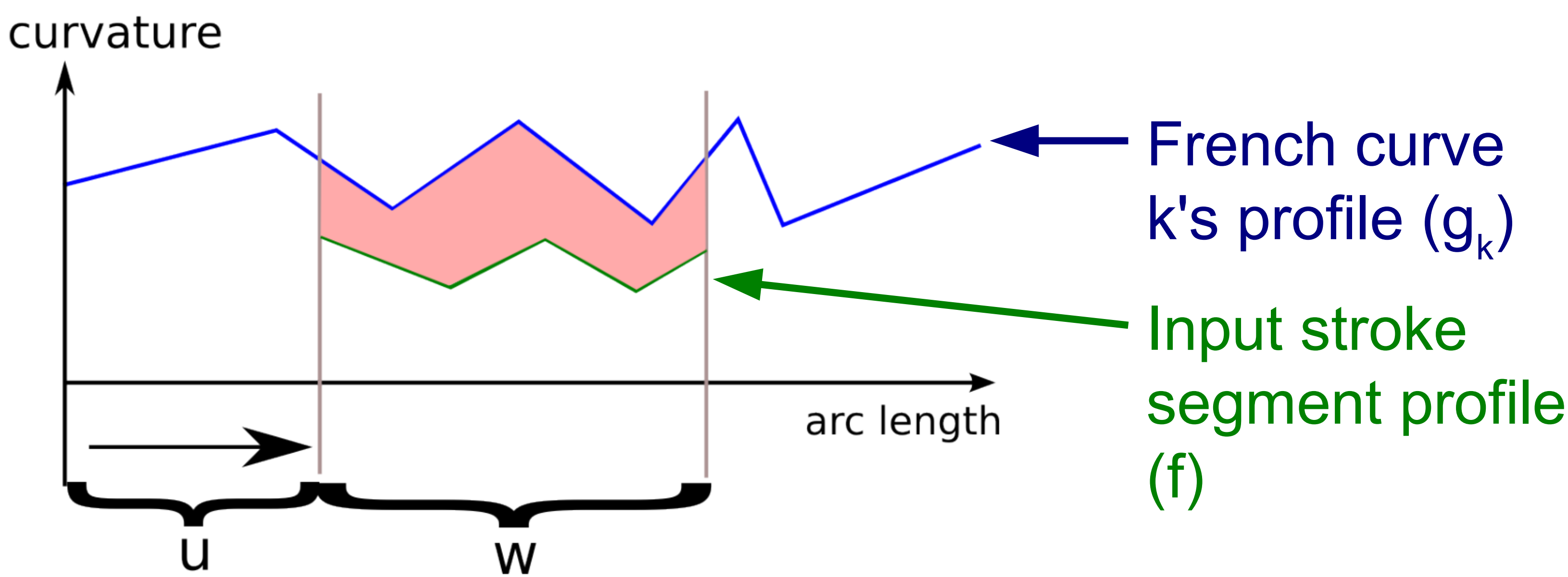
$$E_{fit}(i,j) = \min_u \int_0^w | f(s) - g_k(u+s) | \, ds$$



Optimal Curvature Fit

1. Optimal French curve piece for segment of input

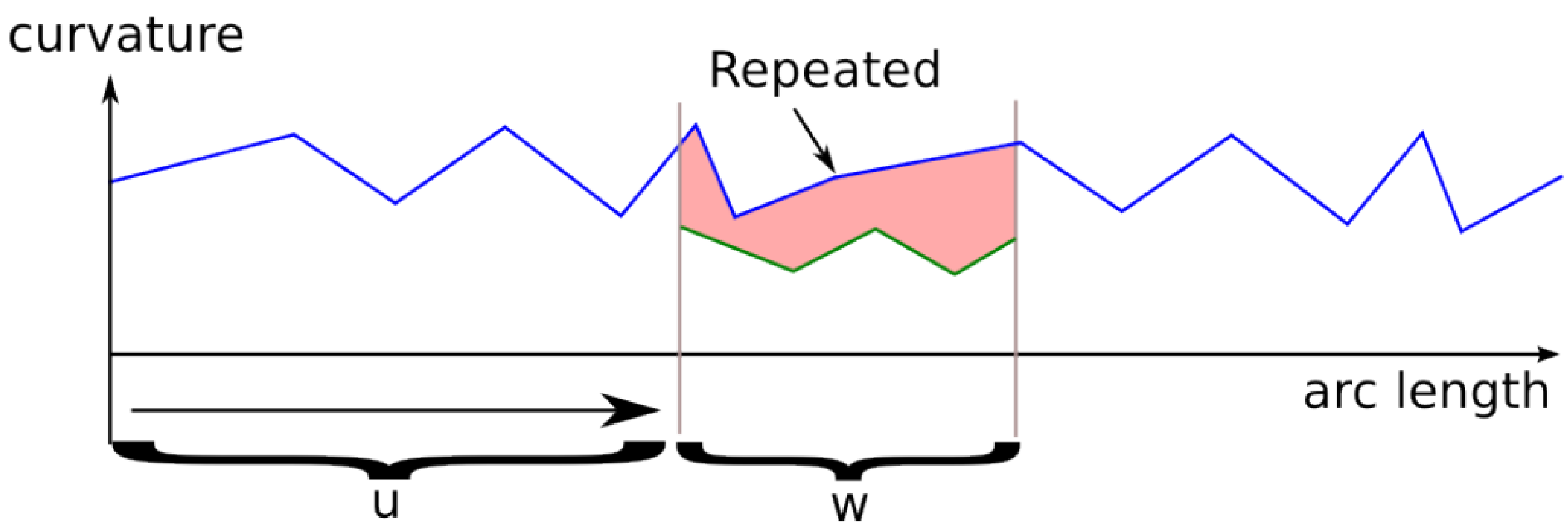
Q: What about **closed** curves (as all physical French curves would be)?



Optimal Curvature Fit

1. Optimal French curve piece for segment of input

A: Repeat French curve's profile



Optimal Curvature Fit

1. Optimal French curve piece for segment of input

Q: Physical French curves can be **flipped** upside down to produce other curves, address that?

Optimal Curvature Fit

1. Optimal French curve piece for segment of input

A: At each position, we perform a second evaluation of E_{fit}, negating curvature and reversing arc length direction:

$$E_{fit}(i,j) = \min_u \int_0^w |\, f(s) - g_k(u+s) \,|\, ds$$

$$\Big\downarrow \text{“flip” } g_k$$

$$E_{fit}(i,j) = \min_u \int_0^w |\, f(s) + g_k(u+w-s) \,|\, ds$$

Optimal Curvature Fit

Two parts:

1. ~~Optimal French curve piece for segment of input~~
2. Optimal segmentation of input curve profile

Optimal Curvature Fit

2. Optimal segmentation of input curve profile

Solution: Use **dynamic programming**:

$$\mathbf{M}(i,j) = \min\left\{E_{fit}(i,j) + E_{cost}, \min_{i<k<j}\{\mathbf{M}(i,k) + \mathbf{M}(k,j)\}\right\}$$

$E_{fit}(i,j)$: fit error of optimal French curve piece with points *i..j* of input curve

E_{cost}: penalty for using additional French curve piece

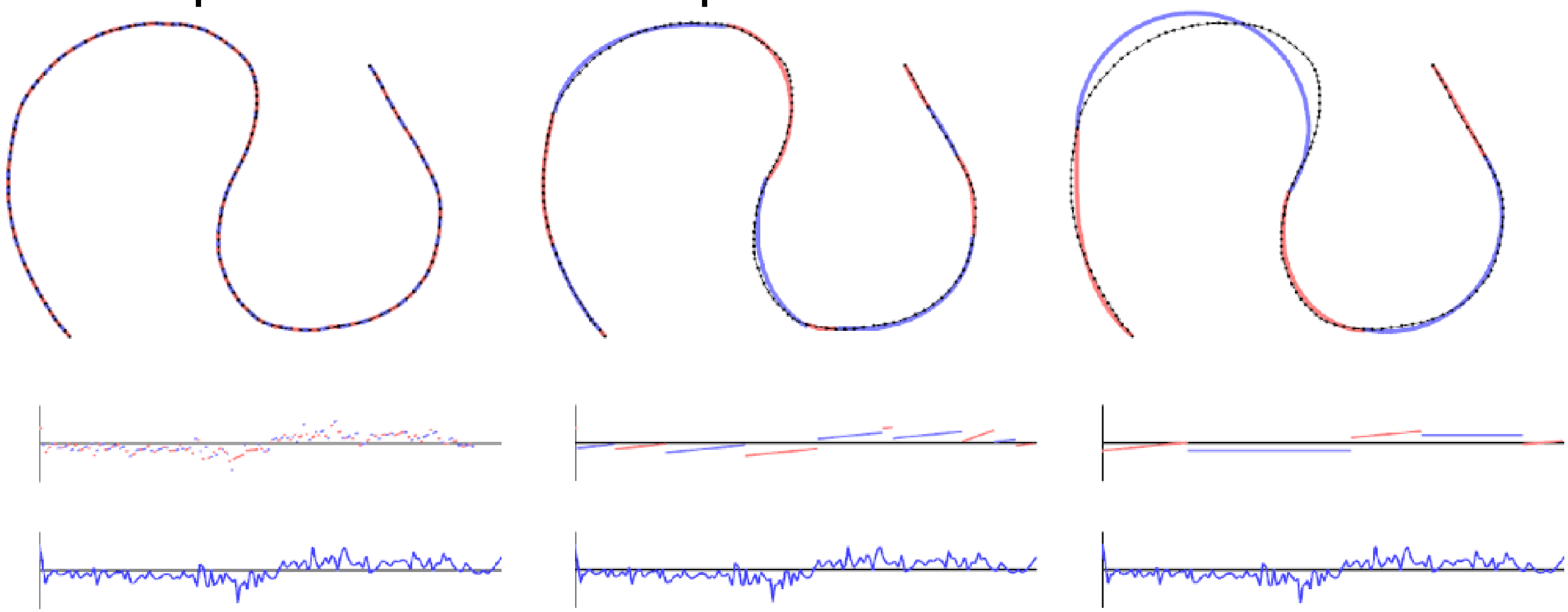
Optimal Curvature Fit

2. Optimal segmentation of input curve profile

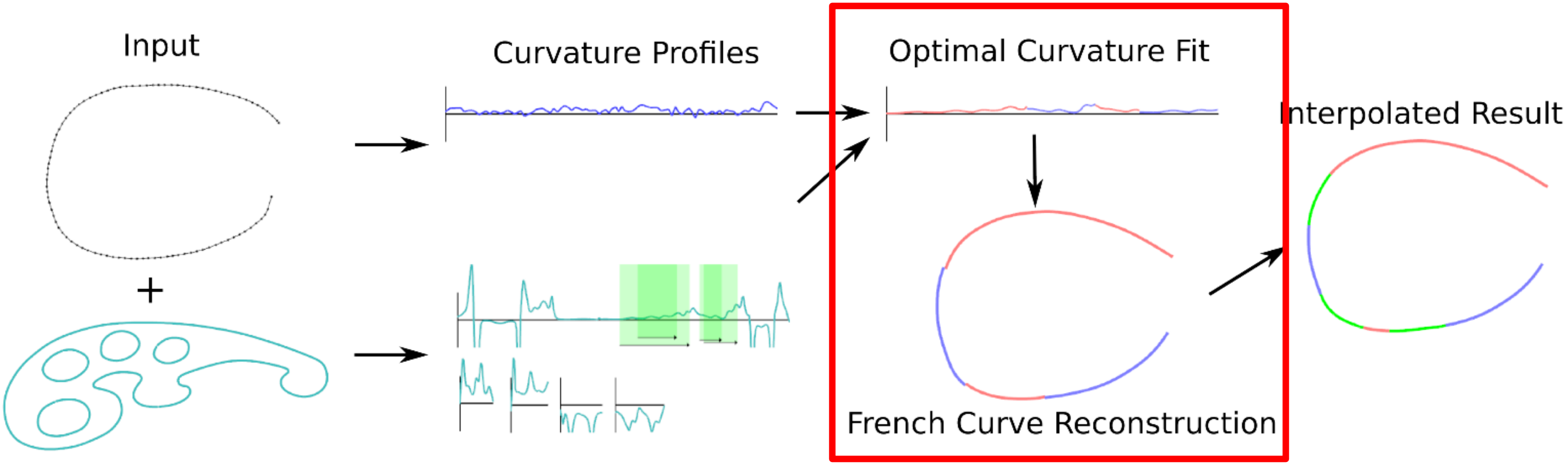
$E_{cost} = 0.0$
50+ pieces

$E_{cost} = 0.2$
10 pieces

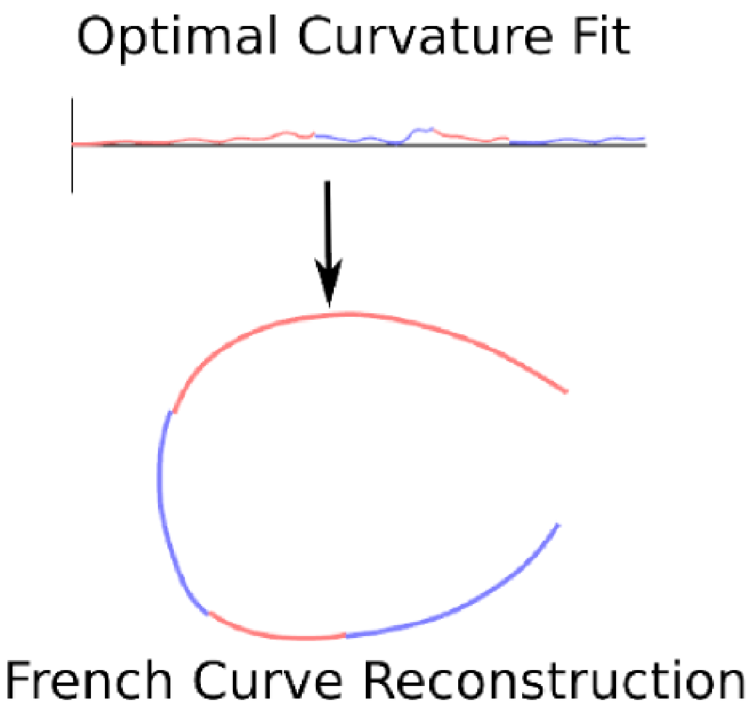
$E_{cost} = 0.4$
5 pieces



French Curve Reconstruction



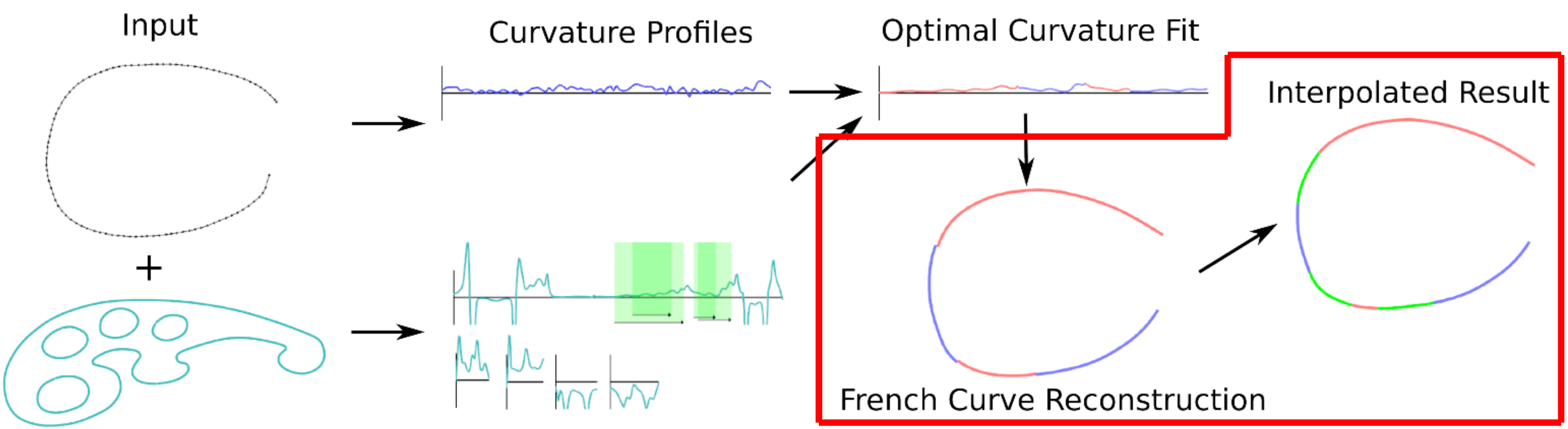
French Curve Reconstruction



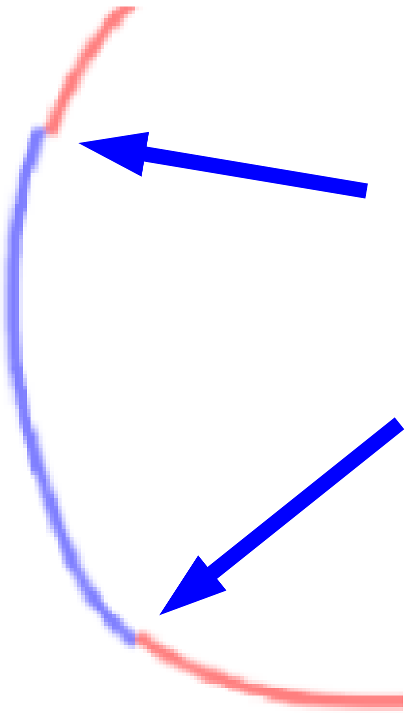
- Rotate/translate optimal pieces to input segment endpoints
- French curve pieces are piecewise clothoid*, each G^2 continuous

*Refer to: James McCrae, Karan Singh. *Sketching piecewise clothoid curves*, SBIM 2008.

Interpolating Reconstruction

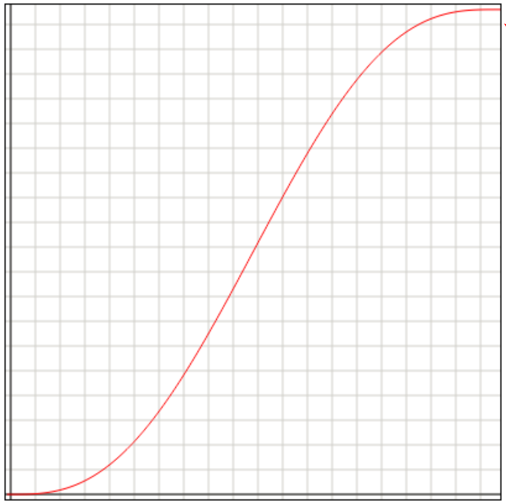


Interpolating Reconstruction



• Adjacent pieces may not have perfect alignment

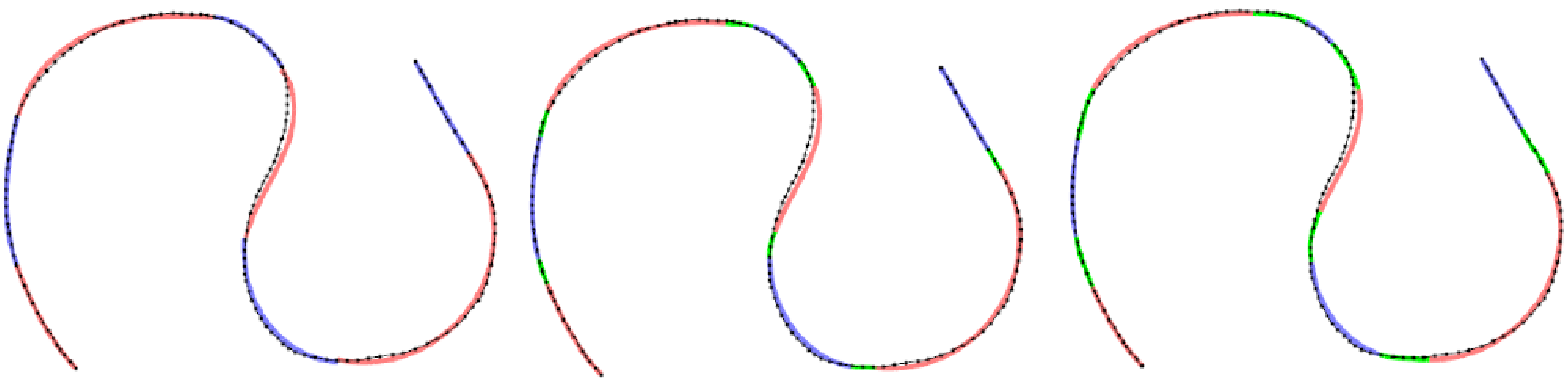
Interpolating Reconstruction



Blending function:

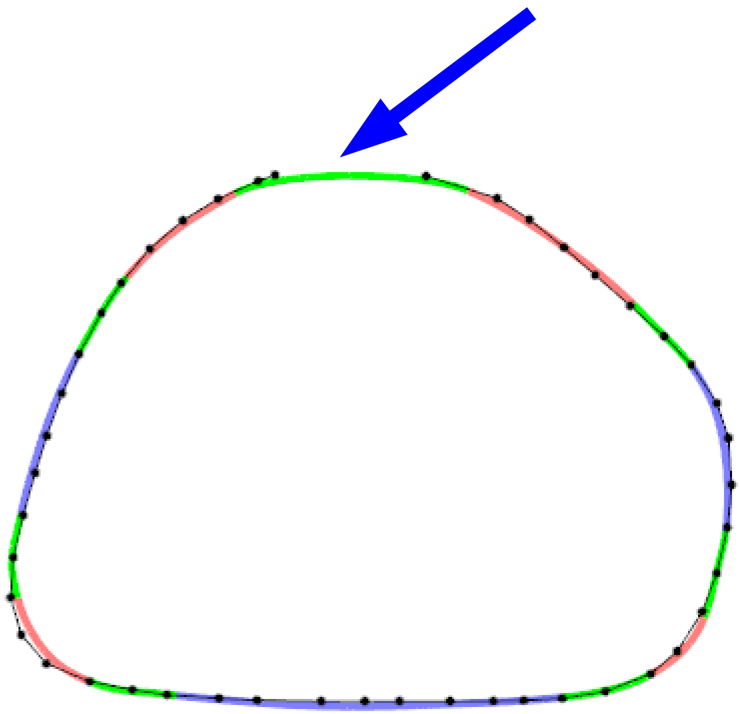
$$f(s) = s^3(6s^2 - 15s + 10)$$

Produces G^2 continuity between French curve pieces

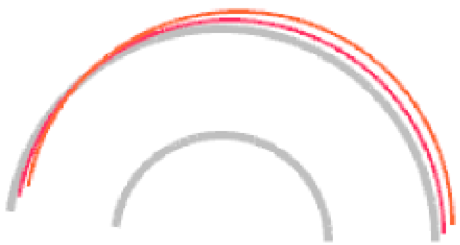


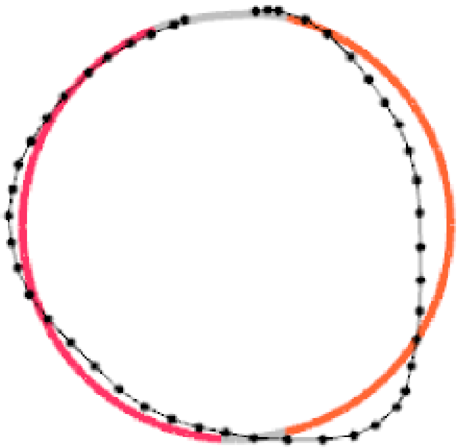
Interpolating Reconstruction

Interpolation used for “nearly closed” input

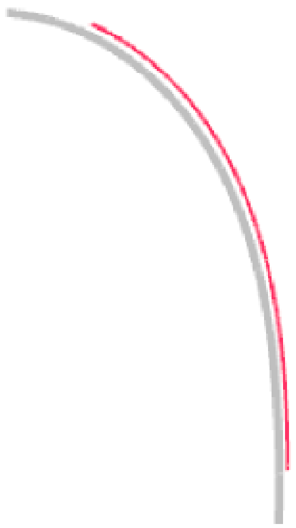


Results

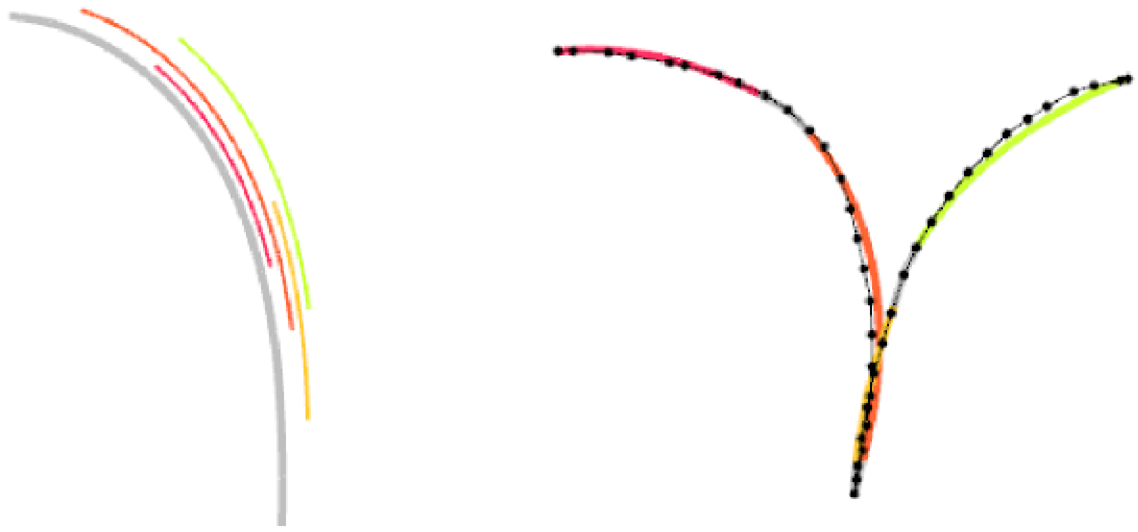




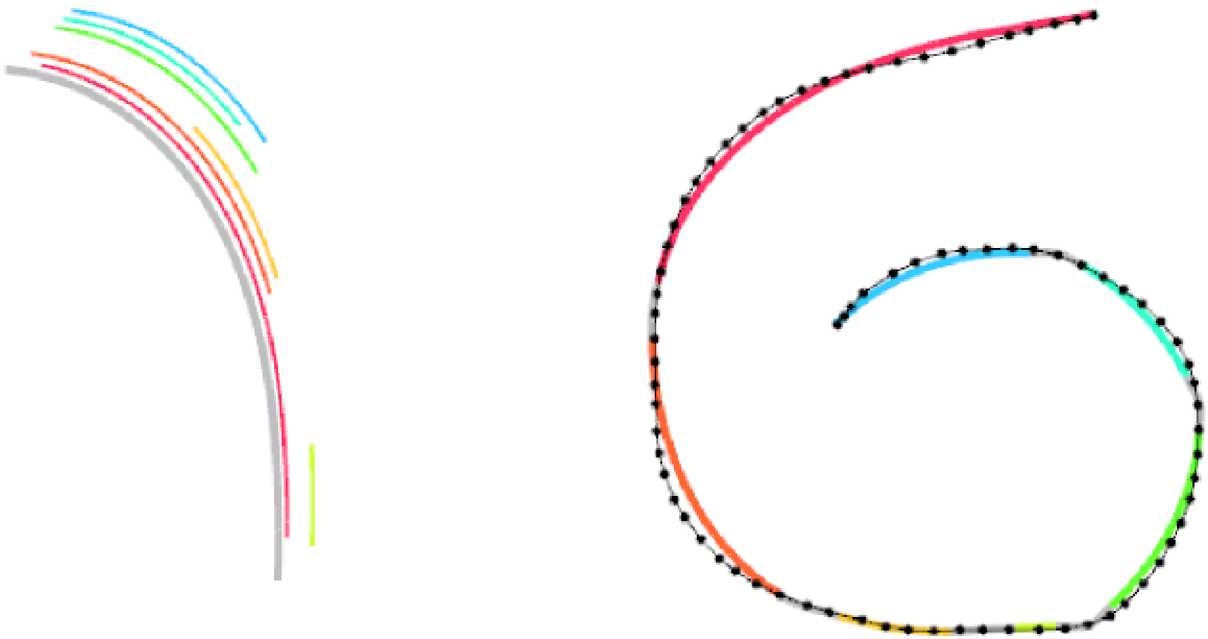
Results



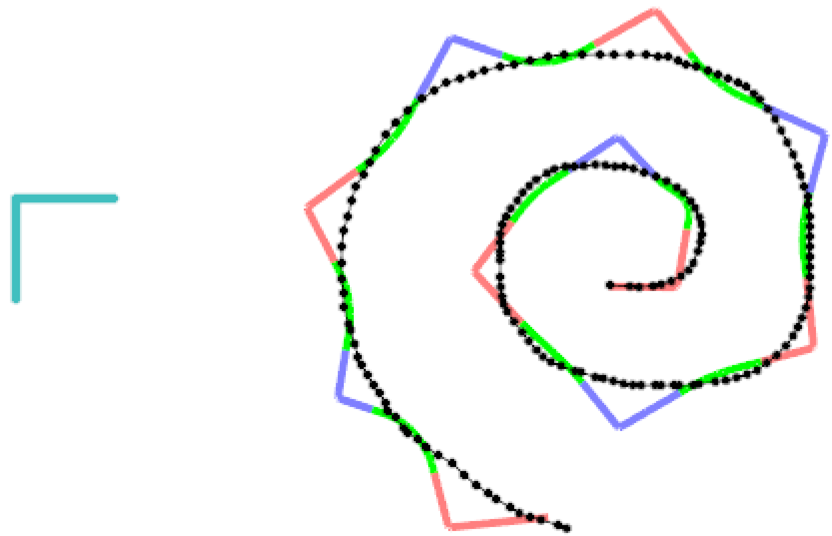
Results



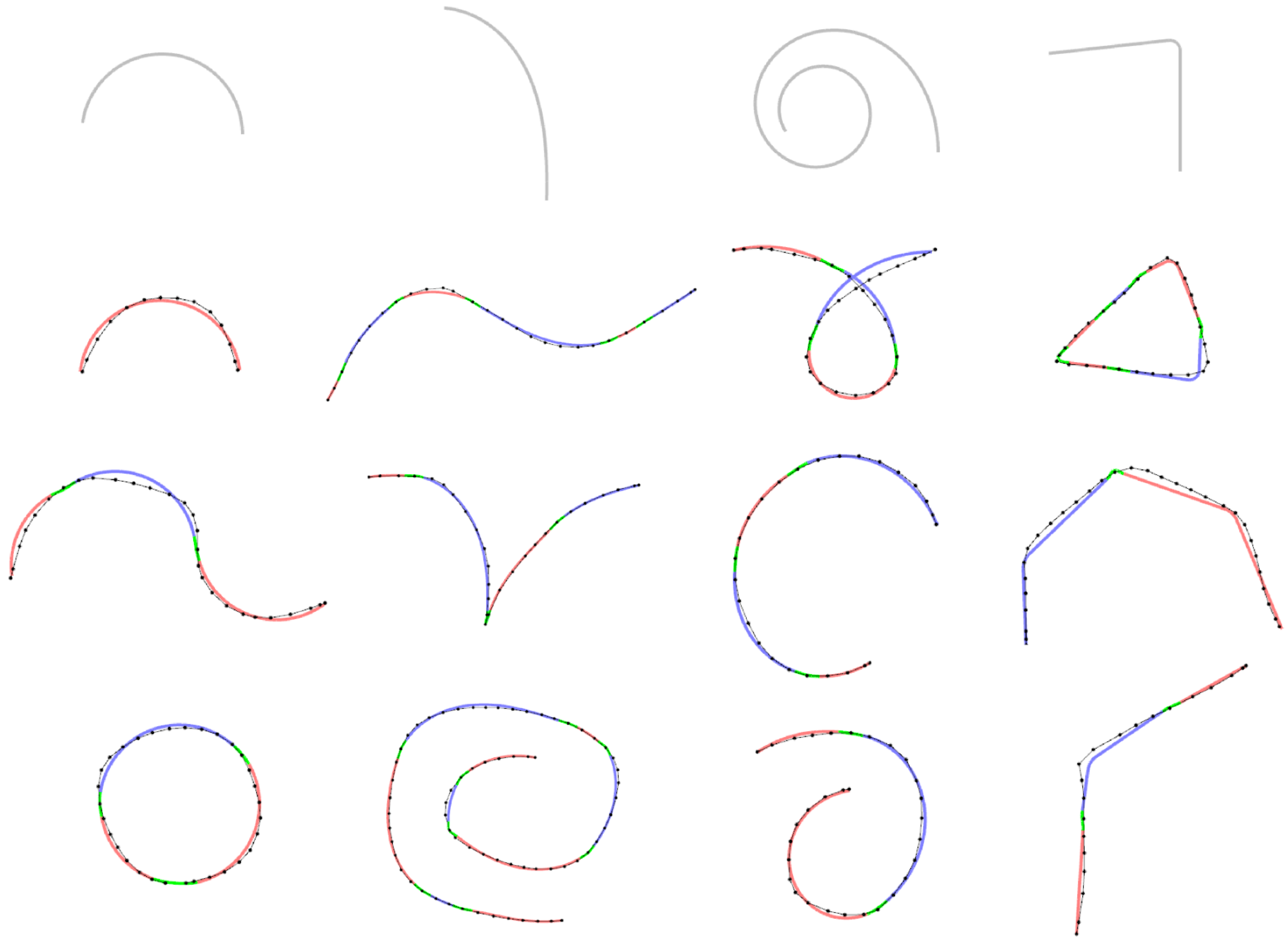
Results



Results



Results



Summary

We present an algorithm to use French curves with a sketch interface

Our approach:

- Creates a globally optimal input segmentation
- Selects curvature-optimal French curve pieces
- Balances number of French curve pieces and global curvature error
- Produces G^2 continuous curves
- Runs interactively (for reasonable lengths)

Thanks

We will be releasing source code and a demo application online soon!

http://www.dgp.toronto.edu/~mccrae/

Thank you!