Neatening sketched strokes using piecewise French Curves

James McCrae, Karan Singh
French Curves

Physical tools, used to model curves
French Curves

Smoothly connect pre-determined curve points
French Curves
French Curves
Digital French Curves

Two-handed manipulation of digitized French curves (represented as cubic NURBS curves)

Motivation

The idea: French curves + sketch interface
Motivation

The idea: French curves + sketch interface

Why?

• Smooth, high quality
• Specific style/standard

• Fast to learn
• Easy curve modelling
Problem Statement

Specifically, given

input polyline
Specifically, given

input polyline

French curve
Problem Statement

Specifically, given

input polyline

French curve

reconstruct the polyline
Approach
Approach
Approach
Approach
Approach
Curvature Profiles

Input → Curvature Profiles → Optimal Curvature Fit → Interpolated Result

French Curve Reconstruction
Curvature Profiles

Discrete curvature estimator:

\[ \kappa = \frac{2 \cdot \vec{v}_1 \times \vec{v}_2}{\| \vec{v}_1 \| + \| \vec{v}_2 \| + \| \vec{v}_1 + \vec{v}_2 \|} \]
Optimal Curvature Fit

Curvature Profiles

Optimal Curvature Fit

Input

Interpolated Result

French Curve Reconstruction
Optimal Curvature Fit

Two parts:

1. Optimal French curve piece for segment of input
2. Optimal segmentation of input curve profile
1. Optimal French curve piece for segment of input

Solution: **Iterate** over French curve profiles:

\[
E_{\text{fit}}(i, j) = \min_u \int_0^W | f(s) - g_k(u + s) | \, ds
\]
1. Optimal French curve piece for segment of input

Q: What about **closed** curves (as all physical French curves would be)?

![Diagram](image)

- French curve $k$'s profile ($g_k$)
- Input stroke segment profile ($f$)
Optimal Curvature Fit

1. Optimal French curve piece for segment of input

A: Repeat French curve's profile
Optimal Curvature Fit

1. Optimal French curve piece for segment of input

Q: Physical French curves can be flipped upside down to produce other curves, address that?
1. Optimal French curve piece for segment of input

**A:** At each position, we perform a second evaluation of $E_{\text{fit}}$, negating curvature and reversing arc length direction:

$$E_{\text{fit}}(i, j) = \min_u \int_0^w | f(s) - g_k(u + s) | \, ds$$

“flip” $g_k$

$$E_{\text{fit}}(i, j) = \min_u \int_0^w | f(s) + g_k(u + w - s) | \, ds$$
Optimal Curvature Fit

Two parts:

1. Optimal French curve piece for segment of input
2. Optimal segmentation of input curve profile
2. Optimal segmentation of input curve profile

Solution: Use dynamic programming:

\[ M(i, j) = \min \left\{ E_{\text{fit}}(i, j) + E_{\text{cost}}, \min_{i < k < j} \left\{ M(i, k) + M(k, j) \right\} \right\} \]

\( E_{\text{fit}}(i, j) \): fit error of optimal French curve piece with points \( i..j \) of input curve

\( E_{\text{cost}} \): penalty for using additional French curve piece
2. Optimal segmentation of input curve profile

\[ E_{\text{cost}} = 0.0 \]
50+ pieces

\[ E_{\text{cost}} = 0.2 \]
10 pieces

\[ E_{\text{cost}} = 0.4 \]
5 pieces
French Curve Reconstruction
French Curve Reconstruction

- Rotate/translate optimal pieces to input segment endpoints
- French curve pieces are piecewise clothoid*, each $G^2$ continuous

Interpolating Reconstruction

Input → Curvature Profiles → Optimal Curvature Fit → Interpolated Result

French Curve Reconstruction
Interpolating Reconstruction

- Adjacent pieces may not have perfect alignment
Interpolating Reconstruction

Blending function:

\[ f(s) = s^3(6s^2 - 15s + 10) \]

Produces \( G^2 \) continuity between French curve pieces
Interpolating Reconstruction

Interpolation used for “nearly closed” input
Results
Results
Results
Results
Results
Results
We present an algorithm to use French curves with a sketch interface

Our approach:

- Creates a globally optimal input segmentation
- Selects curvature-optimal French curve pieces
- Balances number of French curve pieces and global curvature error
- Produces $G^2$ continuous curves
- Runs interactively (for reasonable lengths)
Thanks

We will be releasing source code and a demo application online soon!

http://www.dgp.toronto.edu/~mccrae/

Thank you!