

CSC418/2504F: Midterm Test: SAMPLE  
Wednesday,

**Family Name:** \_\_\_\_\_

**First Name:** \_\_\_\_\_

**Student ID:** \_\_\_\_\_

**Instructions:**

Attempt **all** questions.  
There are five questions.  
The total mark is 24.

You have 60 minutes to complete the test.

Aids allowed: Calculators  
Textbooks and notes are NOT allowed.

1: /5

2: /8

3: /2

4: /12

\_\_\_\_\_

Total: /27

## 1. Clipping and scan conversion [5 marks]:

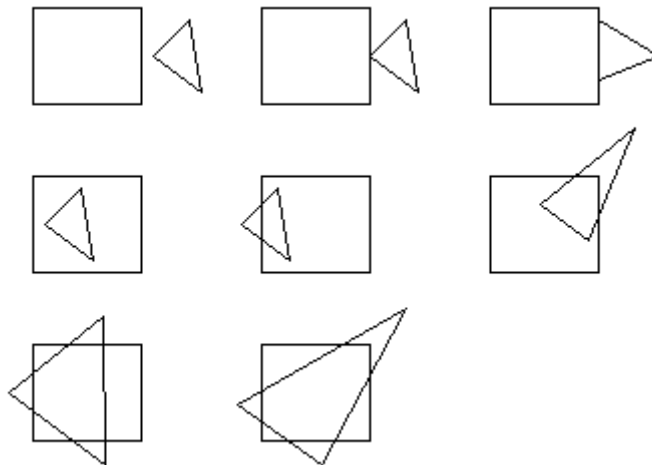
- a. [2 marks] List the pixels produced by scan-converting the line segment from (0,4) to (6,0).

(0,4)(1,3)(2,3)(3,2)(4,1)(5,1)(6,0)

- b. [3 marks] Suppose we clip a polygon with  $n$  sides against a rectangular window. The clipped polygon will have  $m$  sides, and often  $n \neq m$ .

If  $n = 3$ , what values can  $m$  have? Give a sketch for each value of  $m$ .

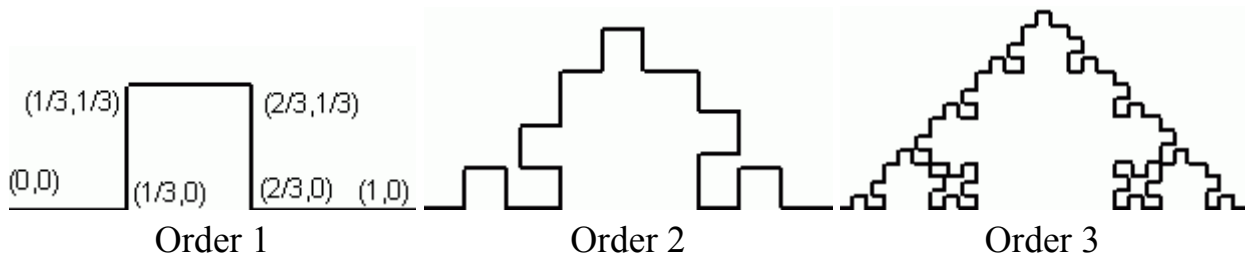
- $m$  can range from 0 to 7. Here is a sketch for each case:



2. Modeling: [8 marks] For this question, use the following OpenGL-like function calls:

```
trans(tx,ty)  scale(sx,sy)  rotz(ang)
glBegin()      push()           pop()
```

**In this question, you will write code to draw a fractal “meander”.**



- a. [3 marks] Write code to draw an order-1 meander.

```
order1 () {
    glBegin(GL_LINE_STRIP);
    vertex(0,0);
    vertex(0.3333,0);
    vertex(0.3333,0.3333);
    vertex(0.6666,0.3333);
    vertex(0.6666,0);
    vertex(1,0);
    glEnd();
}
```

- c. [5 marks] As you can see, an order-1 meander consists of 5 **segments**, each length  $1/3$ . If we **replace each segment** with a whole **meander scaled down by  $1/3$** , we obtain a higher-order meander. Write a function **meander(int n)** which draws an order-**n** meander. It is easiest to define the function recursively.

```
meander (int n) {
    if (n == 1) {
        order1();
    }
    else {
        push();
        scale(0.3333,1);
        meander(n-1);
        pop();
    }
}
```

```

    trans(0.3333,0);
    rotz(90);

    push();
    scale(0.3333,1);
    meander(n-1);
    pop();
    trans(0.3333,0);
    rotz(-90);

    push();
    scale(0.3333,1);
    meander(n-1);
    pop();
    trans(0.3333,0);
    rotz(-90);

    push();
    scale(0.3333,1);
    meander(n-1);
    pop();
    trans(0.3333,0);
    rotz(90);

    push();
    scale(0.3333,1);
    meander(n-1);
    pop();
}
}

```

3. True or False with a reason: [2 marks]

- a. [1 mark] Real cameras use a lens to compensate for the fact that apertures larger than a pin-hole make the image blurry.

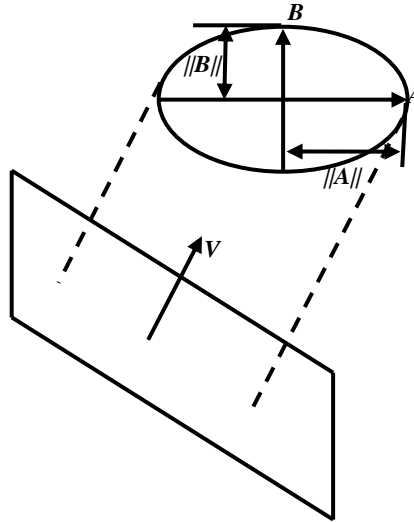
True. Lenses cause rays through the aperture to focus towards a smaller region on the image plane making the image sharper.

- b. [1 mark] The result of multiple rotations to a point is order dependent but combinations of scaling and translations can be applied in any order with the same result.

False. Take a point at the origin and scale it 2x and translate it to (5,5).

If translated and then scaled the resulting point is (10,10).

4. Projection, drawing primitives: [12 marks] An ellipse defined by semi-major and semi-minor axes vectors  $A$ ,  $B$  is to be drawn using orthographic projection with view direction  $V$  :



- a. [4 marks] Explain intuitively why the projected curve must also be an ellipse or a straight line.

Possible answers (you will get partial credit for a reasonable attempt at an answer):

The view space transform is a change of basis rotate and translate transform that leaves an equation in  $x,y,z$  to have the same or lesser degree. Orthographic projection sets  $z=0$ . A quadratic thus projects to a quadratic or linear equation in  $x,y$ . If quadratic the ellipse being closed projects to a closed quadratic that is an ellipse. (2 marks).

When  $(A \times B) \cdot V = 0$  or the plane of the ellipse is perpendicular to the view plane the curve projects to a line. (1 mark),

Ellipse eq.  $x=a \cdot \cos t, y=b \cdot \sin t, z=0$ . Any view space rotation can be seen as a sequence of 3 rotation around  $x,y,z$  axes. Ignore rotation of ellipse around  $z$  axes which is the projection plane (ellipse rotated in 2D is an ellipse). After rotation around  $y$  by angle  $m$ , eq. is  $x'=x, y'=y \cos m, z'=x \sin m$ . After rotation around  $x$  by angle  $n$  eq. is  $x''=x \cos n - x' \sin n \cdot \sin m, y''=y \cos m, z''=x \sin n + x' \sin m \cdot \cos n$ . Throw away  $z''$  in the equation. We are left with the projected curve  $x''=a(\cos n - \sin n \cdot \sin m) \cos t, y''=b(\cos m) \sin t$  which is still the parametric eq. of an ellipse. (2 marks).

- b. [4 marks] Under what conditions can the projected curve be a circle and what is the radius of that circle as a function of  $A$ ,  $B$ ,  $V$ .

$A' = \sqrt{A^2 - (A \cdot V)^2}$ ,  $B' = \sqrt{B^2 - (B \cdot V)^2}$ . The curve is a circle when  $A' = B' = \text{radius of the circle}$ .

(1 mark for getting the  $A'$ ,  $B'$  values, 1.5 marks for saying that when the new semi-maj, semi-minor axes are equal we get a circle, 0.5 marks for pointing out that the radius is  $A'$  or  $B'$ ).

- c. [4 marks] Write pseudo code for a loop that plots the ellipse defined as  $(x/a)^2 + (y/b)^2 = 1$  as a sequence of  $n$  line segments.

$\text{ang} = 2\pi(n-1)/n$ ;  $\text{px} = a \cdot \cos(\text{ang})$ ;  $\text{py} = b \cdot \sin(\text{ang})$ ;

For ( $i=0$ ;  $i < n$ ;  $i++$ ) {  $\text{ang} = 2\pi \cdot i/n$ ;  $x = a \cdot \cos(\text{ang})$ ;  $y = b \cdot \sin(\text{ang})$ ;  $\text{line}(x, y, \text{px}, \text{py})$ ;  $x = \text{px}$ ;  $y = \text{py}$ ; }

(1 mark for using the parametric eq. of ellipse, 1 mark for loop str., 1 mark for getting the angle incr. right). Same marking scheme if you use the explicit equation for 4 quadrants.

- d. EXTRA CREDIT [2 marks] What property of the curve should determine where to sample points on the ellipse so that the  $n$  line segments that connect them best approximate the ellipse. What part of your pseudo-code in *part c* needs to change to reflect this.

The arc length of the ellipse between sample points should be roughly equal for better approximation. Curvature of the ellipse can be used as a measure of this. (1.5 marks). The angle increment is now not fixed but depends on the arc length between samples (0.5 marks).