CSC418/2504F: Midterm Test: SAMPLE
Wednesday,

Family Name: ____________________________
First Name: ____________________________
Student ID: ____________________________

Instructions:
Attempt all questions.
There are five questions.
The total mark is 24.

You have 60 minutes to complete the test.

Aids allowed: Calculators
Textbooks and notes are NOT allowed.

1: /5
2: /8
3: /2
4: /12

_________
Total: /27
1. Clipping and scan conversion [5 marks]:
   a. [2 marks] List the pixels produced by scan-converting the line segment from (0,4) to (6,0).

   (0,4)(1,3)(2,3)(3,2)(4,1)(5,1)(6,0)

   b. [3 marks] Suppose we clip a polygon with \( n \) sides against a rectangular window. The clipped polygon will have \( m \) sides, and often \( n \neq m \).

   If \( n = 3 \), what values can \( m \) have? Give a sketch for each value of \( m \).

   - \( m \) can range from 0 to 7. Here is a sketch for each case:
2. Modeling: [8 marks] For this question, use the following openGL-like function calls:

\[
\text{trans(t}_x, \text{t}_y) \quad \text{scale(s}_x, \text{s}_y) \quad \text{rot}_z(\text{ang}) \\
\text{glBegin()} \quad \text{push()} \quad \text{pop()}
\]

In this question, you will write code to draw a fractal “meander”.

![Diagram of meander](image)

a. [3 marks] Write code to draw an order-1 meander.

```cpp
order1 () {
    glBegin(GL_LINE_STRIP);
    vertex(0,0);
    vertex(0.3333,0);
    vertex(0.3333,0.3333);
    vertex(0.6666,0.3333);
    vertex(0.6666,0);
    vertex(1,0);
    glEnd();
}
```

c. [5 marks] As you can see, an order-1 meander consists of 5 segments, each length 1/3. If we replace each segment with a whole meander scaled down by 1/3, we obtain a higher-order meander. Write a function `meander(int n)` which draws an order-n meander. It is easiest to define the function recursively.

```cpp
meander (int n) {
    if (n == 1) {
        order1();
    }
    else {
        push();
        scale(0.3333,1);
        meander(n-1);
        pop();
    }
}
```
3. True or False with a reason: [2 marks]
   a. [1 mark] Real cameras use a lens to compensate for the fact that apertures larger than a pin-hole make the image blurry.

   True. Lenses cause rays through the aperture to focus towards a smaller region on the image plane making the image sharper.

   b. [1 mark] The result of multiple rotations to a point is order dependent but combinations of scaling and translations can be applied in any order with the same result.

   False. Take a point at the origin and scale it 2x and translate it to (5,5).

   If translated and then scaled the resulting point is (10,10).
4. Projection, drawing primitives: [12 marks] An ellipse defined by semi-major and semi-minor axes vectors $\mathbf{A}, \mathbf{B}$ is to be drawn using orthographic projection with view direction $\mathbf{V}$:

![Diagram of orthographic projection]

a. [4 marks] Explain intuitively why the projected curve must also be an ellipse or a straight line.

Possible answers (you will get partial credit for a reasonable attempt at an answer):

The view space transform is a change of basis rotate and translate transform that leaves an equation in $x,y,z$ to have the same or lesser degree. Orthographic projection sets $z=0$. A quadratic thus projects to a quadratic or linear equation in $x,y$. If quadratic the ellipse being closed projects to a closed quadratic that is an ellipse. (2 marks).

When $(\mathbf{A} \times \mathbf{B}).\mathbf{V}=0$ or the plane of the ellipse is perpendicular to the view plane the curve projects to a line. (1 mark).

Ellipse eq. $x=a\cos t$, $y=b\sin t$, $z=0$. Any view space rotation can be seen as a sequence of 3 rotation around $x,y,z$ axes. Ignore rotation of ellipse around $z$ axes which is the projection plane (ellipse rotated in 2D is an ellipse). After rotation around $y$ by angle $m$, eq. is $x'=x$, $y'=ycosm$, $z'=xsinm$. After rotation around $y$ by angle $n$ eq. is $x''=xcosn- xsinm*ssin$, $y''=ycosm$, $z''=xsinn+xsinn*cosn$. Throw away $z''$ in the equation. We are left with the projected curve $x''= a(cosn-sinm*ssin)cost$, $y''= b(cosm)sint$ which is still the parametric eq. of an ellipse. (2 marks).
b. [4 marks] Under what conditions can the projected curve be a circle and what is the radius of that circle as a function of $A$, $B$, $V$.

$A' = \sqrt{||A||^2 - (A.V)^2}$. $B' = \sqrt{||B||^2 - (B.V)^2}$. The curve is a circle when $A' = B' = \text{radius of the circle}$. 

(1 mark for getting the $A'$, $B'$ values, 1.5 marks for saying that when the new semi-major, semi-minor axes are equal we get a circle, 0.5 marks for pointing out that the radius is $A'$ or $B'$.

c. [4 marks] Write pseudo code for a loop that plots the ellipse defined as $(x/a)^2 + (y/b)^2 = 1$ as a sequence of $n$ line segments.

ang = $2\pi(n-1)/n$; px = a*cos(ang); py = b*sin(ang);

For (i=0;i<n;i++) {ang = 2*pi*i/n; x = a*cos(ang); y = b*sin(ang); line(x,y,px,py); x = px; y = py;}

(1 mark for using the parametric eq. of ellipse, 1 mark for loop str., 1 mark for getting the angle incr. right). Same marking scheme if you use the explicit equation for 4 quadrants.

d. EXTRA CREDIT [2 marks] What property of the curve should determine where to sample points on the ellipse so that the $n$ line segments that connect them best approximate the ellipse. What part of your pseudo-code in part c needs to change to reflect this.

The arc length of the ellipse between sample points should be roughly equal for better approximation. Curvature of the ellipse can be used as a measure of this. (1.5 marks). The angle increment is now not fixed but depends on the arc length between samples (0.5 marks).