Topic 6:

3D Transformations

- Homogeneous 3D transformations
- Scene Hierarchies
- Change of basis and rotations in 3D

Representing 2D transforms as a 3x3 matrix

Translate a point \([x y]\) by \([t_x t_y]\):

\[
\begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix}
\]

Rotate a point \([x y]\) by an angle \(t\):

\[
\begin{bmatrix}
\cos t & -\sin t & 0 \\
\sin t & \cos t & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Scale a point \([x y]\) by a factor \([s_x s_y]\):

\[
\begin{bmatrix}
s_x & 0 & 0 \\
0 & s_y & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Representing 3D transforms as a 4x4 matrix

Translate a point \([x y z]\) by \([t_x t_y t_z]\):

\[
\begin{bmatrix}
1 & 0 & 0 & t_x \\
0 & 1 & 0 & t_y \\
0 & 0 & 1 & t_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Rotate a point \([x y z]\) by an angle \(t\) around z axis:

\[
\begin{bmatrix}
\cos t & -\sin t & 0 & 0 \\
\sin t & \cos t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Scale a point \([x y z]\) by a factor \([s_x s_y s_z]\):

\[
\begin{bmatrix}
s_x & 0 & 0 & 0 \\
0 & s_y & 0 & 0 \\
0 & 0 & s_z & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Scene Hierarchies

Change of reference frame/basis matrix

\[p = ap' + bp' + cp' + o\]

\[p = \begin{bmatrix} a & b & c & o \\ 0 & 0 & 0 & 1 \end{bmatrix} p'\]

\[p' = \begin{bmatrix} a & b & c & o \\ 0 & 0 & 0 & 1 \end{bmatrix} p\]

Topic 7:

3D Viewing

- Camera Model
- Orthographic projection
- The world-to-camera transformation
- Perspective projection
- The transformation chain for 3D viewing
Camera model

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**Camera model: camera obscura**

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**Camera model**

Ideal pinhole camera

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Real pinhole camera

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**Camera model**

Real pinhole camera

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**Camera model**

Camera with a lens

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**Viewing Transform**

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Depth of Field
What is the difference between these images?
Orthographic projection

Orthographic projection

Perspective: Muller-Lyer Illusion

Orthographic projection

Orthographic projection

Orthographic projection

Orthographic projection

Cannonical view volume

Cannonical view volume

\[
p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} p
\]

\[
p = \begin{bmatrix} x & y & z & 1 \end{bmatrix}^T
\]

\[
p' = \begin{bmatrix} x & y & 1 \end{bmatrix}^T
\]

\[
p = \begin{bmatrix} x & y & z \end{bmatrix}^T
\]

\[
p' = \begin{bmatrix} x & y \end{bmatrix}^T
\]

\[
p = \begin{bmatrix} x \end{bmatrix}^T
\]

\[
m' = \frac{m + q}{2};
m = \frac{m' + q'}{2}
\]

\[
\text{Map 3D to a cube centered at the origin of side length 2!}
\]

\[
\text{Translate}(-\frac{l+r}{2}, -\frac{t+b}{2}, -\frac{n+f}{2})
\]

\[
\text{Scale}(\frac{2}{r-l}, \frac{2}{t-b}, \frac{2}{f-n})
\]
Camera model

Perspective Projection

Perspective projection

Simple Perspective

Simple Perspective

Simple Perspective

Simple Perspective
Simple Perspective

\[
\begin{pmatrix}
0 & 0 & 0 & a \\
0 & 0 & 0 & b \\
0 & 0 & 1/d & 1 \\
1 & 0 & 0 & 0
\end{pmatrix}
\]

Find \( a \) and \( b \) such that \( z' = -1 \) when \( z = d \) and \( z' = 1 \) when \( z = D \), where \( d \) and \( D \) are near and far clip planes.

Viewing volumes

Viewing Pipeline