Bayesian Learning for Computer graphics

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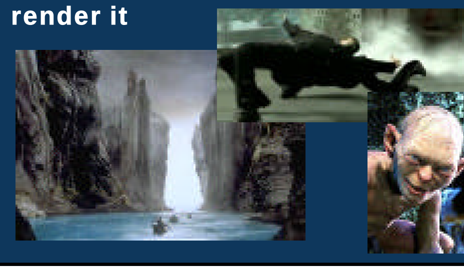
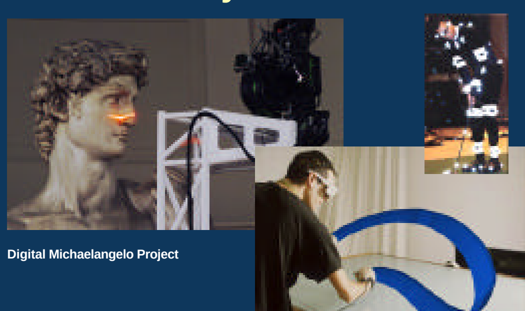
Computers are really fast

- If you can create it, you can render it



How do you create it?

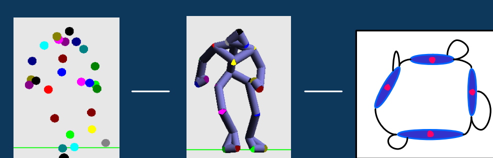


Digital Michaelangelo Project

Steven Schkolne

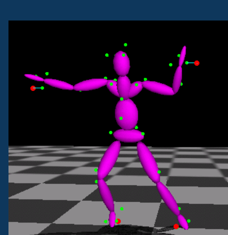
Two problems

- How do we get the data into the computer?
- How do we manipulate it?



Data fitting

$$\sum_{j,t} ||m_j(q_t) - s_{j,t}||^2 + w_1 \sum_{j,t} ||q_t - q_{t-1}||^2 + w_2 \sum_{j,t} ||\ldots||^2$$



Key questions

- How do you fit a model to data?
  - How do you choose weights and thresholds?
  - How do you incorporate prior knowledge?
  - How do you merge multiple sources of information?
  - How do you model uncertainty?

Bayesian reasoning provides a solution

Talk outline

- Bayesian reasoning
- Facial modeling example
- Non-rigid modeling from video

What is reasoning?

- How do people reason?
- How should computers do it?

Aristotelian Logic

- If A is true, then B is true
- A is true
- Therefore, B is true

A: My car was stolen
B: My car isn't where I left it

Real-world is uncertain

Problems with pure logic
- Don't have perfect information
- Don't really know the model
- Model is non-deterministic

Beliefs

- Let B(X) = "belief in X",
- B(¬X) = "belief in not X"

1. An ordering of beliefs exists
2. B(X) = f(B(¬X))
3. B(X) = g(B(X|Y),B(Y))

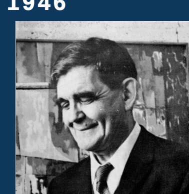
Cox axioms

R.T. Cox, "Probability, frequency, and reasonable expectation," American J. Physics, 14(1):1-13, 1946

$$p(\mathrm{TRUE}) = 1$$

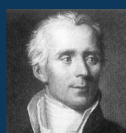
$$p(A) = \sum_{\{B=b_i\}} p(A, B)$$

$$p(A, B) = p(A|B)p(B)$$



“Probability theory is nothing more than common sense reduced to calculation.”

- Pierre-Simon Laplace, 1814



Bayesian vs. Frequentist

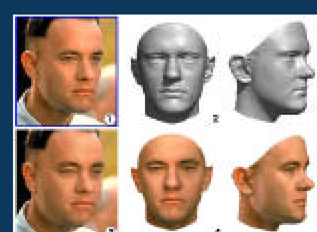
- Frequentist (“Orthodox”):
  - Probability = percentage of events in infinite trials
- Medicine, biology: Frequentist
- Astronomy, geology, EE, computer vision: largely Bayesian

Learning in a nutshell

- Create a mathematical model
- Get data
- Solve for unknowns

Face modeling

- Blanz, Vetter, “A Morphable Model for the Synthesis of 3D Faces,” SIGGRAPH 99



Generative model

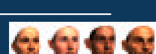
- Faces come from a Gaussian

$$p(S|\mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} e^{-(S-\mu)^T \Sigma^{-1} (S-\mu)/2}$$

$$p(\{S_i\}|\mu,\Sigma) = \prod_i p(S_i|\mu,\Sigma)$$

Learning

$$\arg\max_{\mu,\Sigma} p(\mu,\Sigma|\{S_i\})$$



S_i

Bayes Rule

$$P(A,B) = p(A|B)p(B)$$
$$= p(B|A)p(A)$$

$$\boxed{P(B|A) = p(A|B)p(B)/p(A)}$$

$$p(model|data) \propto p(data|model)p(model)$$

Often: $p(model|data) \propto p(data|model)$

Learning a Gaussian

$\arg\max_{\mu,\Sigma} p(\mu,\Sigma \mid \{S_i\})$

$= \arg\max_{\mu,\Sigma} p(\{S_i\} \mid \mu,\Sigma)$

$= \arg\max_{\mu,\Sigma} \prod_i p(S_i \mid \mu,\Sigma)$

$= \arg\max_{\mu,\Sigma} \prod_i \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} e^{-\frac{1}{2}(S_i-\mu)^T\Sigma^{-1}(S_i-\mu)/2}$

$\mu = \sum_i S_i/N$

$\Sigma = \sum_i (S_i-\mu)(S_i-\mu)^T/N$

Maximization trick

- Maximize $p(x)$
  <-> minimize $-\ln p(x)$

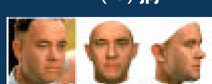
Fitting a face to an image

Generative model

$p(S \mid \mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} e^{-(S-\mu)^T\Sigma^{-1}(S-\mu)/2}$

$I = \mathrm{Render}(S,\theta) + n$

$\Rightarrow p(I \mid S,\theta,\sigma^2) = \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} e^{-\|I-\mathrm{Render}(S,\theta)\|^2/2\sigma^2}$



Fitting a face to an image

Maximize

$p(S,\theta \mid I,\mu,\Sigma,\sigma^2)$

minimize

$-\ln p(S,\theta \mid I,\mu,\Sigma,\sigma^2) =$

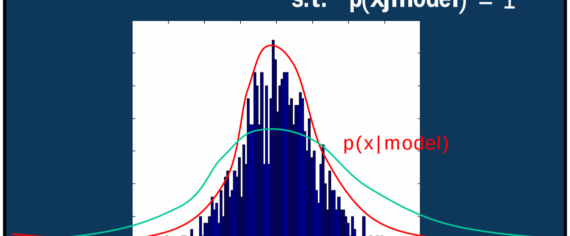
$\|I-\mathrm{Render}(S,\theta)\|^2/2\sigma^2 + (S-\mu)^T\Sigma^{-1}(S-\mu)/2$

$+ \frac{N}{2}\ln 2\pi\sigma^2 + \frac{1}{2}\ln(2\pi)^d|\Sigma|$

Why does it work?

$p(\{x_i\} \mid \mathrm{model}) = \prod p(x_i \mid \mathrm{model})$

$\text{s.t.} \int p(x \mid \mathrm{model}) = 1$



General features

- Models uncertainty
- Applies to any generative model
- Merge multiple sources of information
- Learn *all* the parameters

Caveats

- Still need to understand the model
- Not necessarily tractable
- Potentially more involved than *ad hoc* methods

Applications in graphics

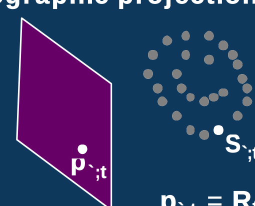
- Shape and motion capture
- Learning styles and generating new data

Learning Non-Rigid 3D Shape from 2D Motion

Joint work with Lorenzo Torresani and Chris Bregler (Stanford, NYU)

Camera geometry

Orthographic projection



$$p_{\ell,t} = R_t S_{\ell,t} + T_t$$

Non-rigid reconstruction

Input: 2D point tracks

Output: 3D nonrigid motion

$$p_t = R_t S_t + T_t$$

Totally ambiguous!

Shape reconstruction

Least-squares version

$$p_t = R_t S_t + T_t$$

minimize

$$\sum_t ||p_t - R_t S_t - T_t||^2 + w \sum_t ||S_t - \bar{S}||^2$$

Bayesian formulation

$$p(S_t|\bar{S};\Phi) = \frac{1}{\sqrt{(2\pi)^d|\Phi|}} e^{-(S_t-\bar{S})^T\Phi^{-1}(S_t-\bar{S})/2}$$

$$p_{\ell,t} = R_t S_{\ell,t} + T_t + n \qquad n \sim N(0;\sigma^2)$$

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maximize $p(R;S;T;\bar{S};\Phi|P)$

$\propto p(P|R;S;T)p(S|\bar{S};\Phi)$

How does it work?

Maximize $p(R;S;T;\bar{S};\Phi|P)$

$\propto p(P|R;S;T)p(S|\bar{S};\Phi)$

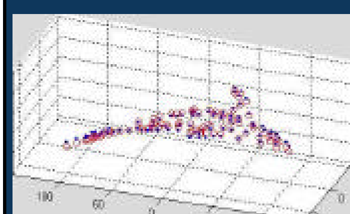
Minimize

$$-\sum_t \ln p(P_t|R_t;S_t;T_t) - \sum_t \ln p(S_t|\bar{S};\Phi)$$

$$= \sum_t ||p_t - R_t S_t - T_t||^2/2\sigma^2 + \frac{N}{2}\ln\sigma^2 + \sum_t \frac{1}{2}(S_t-\bar{S})^T\Phi^{-1}(S_t-\bar{S}) + \frac{T}{2}\ln|\Phi|$$

(actual system is slightly more sophisticated)

Results



Input

Least -squares, no reg. View 2

Least -squares, reg.

Gaussian

LDS

Conclusions

- Bayesian methods provide unified framework
- Build a model, and reason about it
- The future of data-driven graphics