CSC 2521 Computer Graphics, Fall 2008: Assignment 2

Due October 21, in class

Written

Do any **five** of the following questions from the textbook: 9-8, 9-10, 9-13, 11-1, 11-4, 11-12, 11-17.

Notes: I recommend consulting the text and reading relevant sections in order to do these problems. For 9-10, see example 9.4, p. 343.

Programming

In this assignment, you will implement a rigid body simulator. Your simulator will include a single rigid body. You may use any formulation of rigid body mechanics you wish, but I recommend the following, from Baraff's SIGGRAPH 97 course notes, pages D4-D8. Represent the position and orientation of the object as a rigid transformation $\mathbf{R}(t)\mathbf{p} + \mathbf{x}(t)$, where $\mathbf{R}(t)$ is a time-varying 3×3 rotation matrix, and $\mathbf{x}$ is a time-varying translation vector. The object can be animated by applying this transformation to the object in its own (object) coordinates.

The equations of motion for a rigid object are:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} && (1) \\ \dot{\mathbf{v}} &= \frac{\mathbf{F}}{M} && (2) \\ \dot{\boldsymbol{\omega}} &= \mathbf{I}_{\text{world}}^{-1}\mathbf{N} && (3) \\ \dot{\mathbf{R}} &= \boldsymbol{\omega}^{\times}\mathbf{R} && (4) \end{aligned}$$

where $\mathbf{F}$ is the net external force, M is the total mass, ω is angular velocity, $\mathbf{I}$ is inertia tensor (written as a matrix), $\mathbf{N}$ is net torque, and

$$\begin{aligned} \boldsymbol{\omega}^{\times} &= \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} && (5) \\ \mathbf{I}_{\text{world}} &= \mathbf{R}\mathbf{I}_{\text{object}}\mathbf{R}^T && (6) \end{aligned}$$

You may use one of the integration methods from the first assignment, but be sure to choose a step-size so that you get reasonably accurate simulation. To avoid numerical drift, you may need to include a correction to make the rotation orthonormal. For the inertial tensor, you may use a diagonal matrix with three distinct inertias.

Generate animations to validate that your simulator works, including free flight with no external forces or torques, rotating free flight under gravity with no other forces, and free flight with occasional random forces/torque. Provide me with either animations or a live demo. Make sure your visualization and accompanying descriptions make it clear what's going on, e.g., render arrows to illustrate forces. If you're using MATLAB, make sure that the axes and visualization stay consistent.