Today’s Topics

11. Texture mapping
12. Introduction to ray tracing

Some slides and figures courtesy of Wolfgang Hürst, Patricio Simari
Some figures courtesy of Peter Shirley,
“Fundamentals of Computer Graphics”, 3rd Ed.

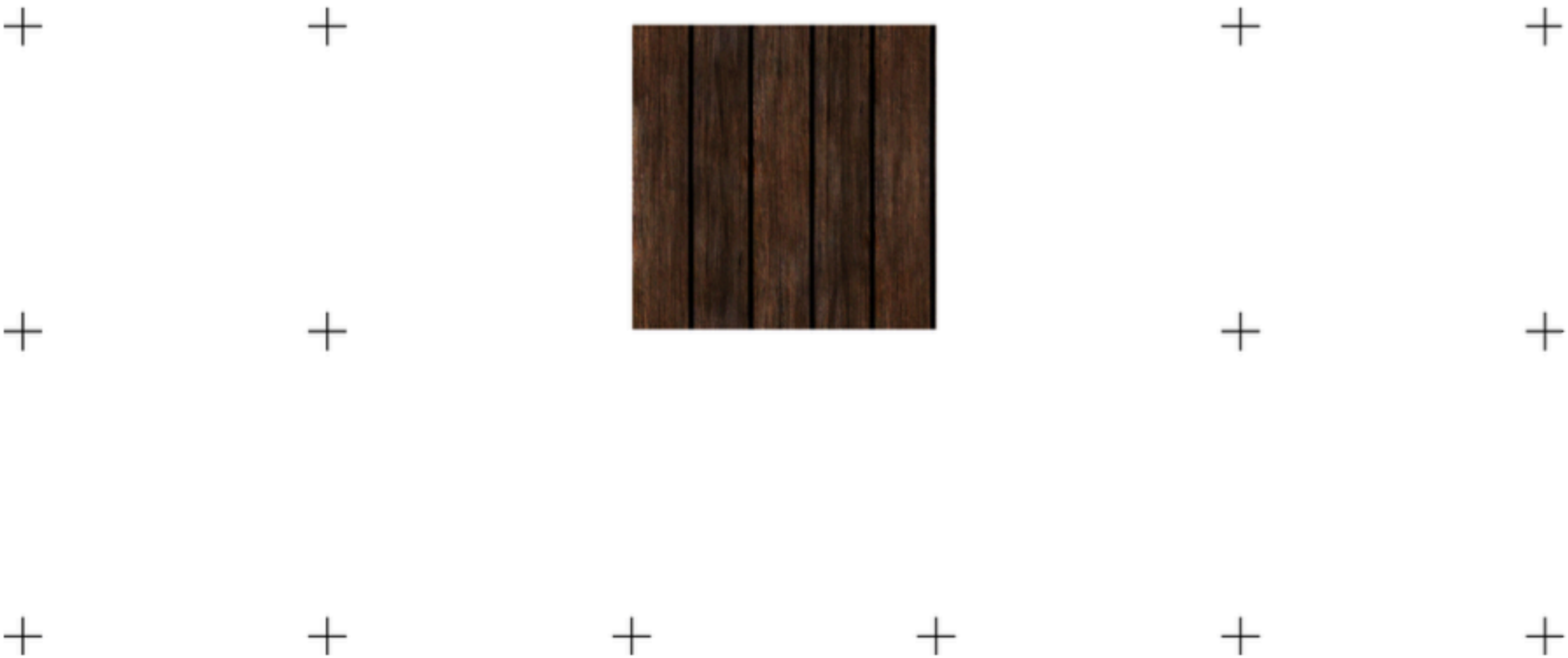
Topic 11:

Texture Mapping

- Motivation
- Sources of texture
- Texture coordinates
- {Bump, MIP, displacement, environmental} mapping

Motivation

- Adding lots of detail to our models to realistically depict skin, grass, bark, stone, etc., would increase rendering times dramatically, even for hardware-supported projective methods.



Motivation

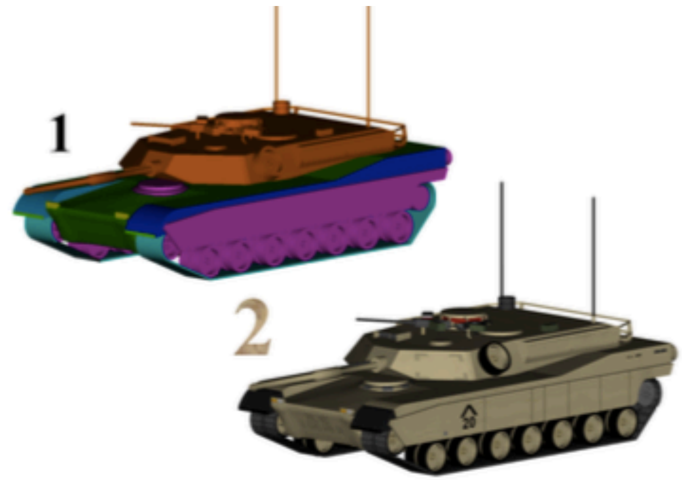
- Adding lots of detail to our models to realistically depict skin, grass, bark, stone, etc., would increase rendering times dramatically, even for hardware-supported projective methods.

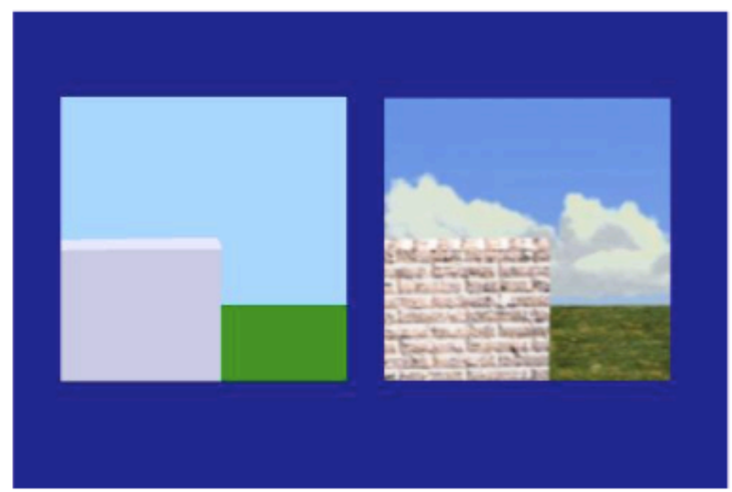
Motivation

Basic idea of texture mapping:

Instead of calculating color, shade, light, etc. for each pixel we just paste images to our objects in order to create the illusion of realism

Different approaches exist (e.g. tiling; cf. previous slide)





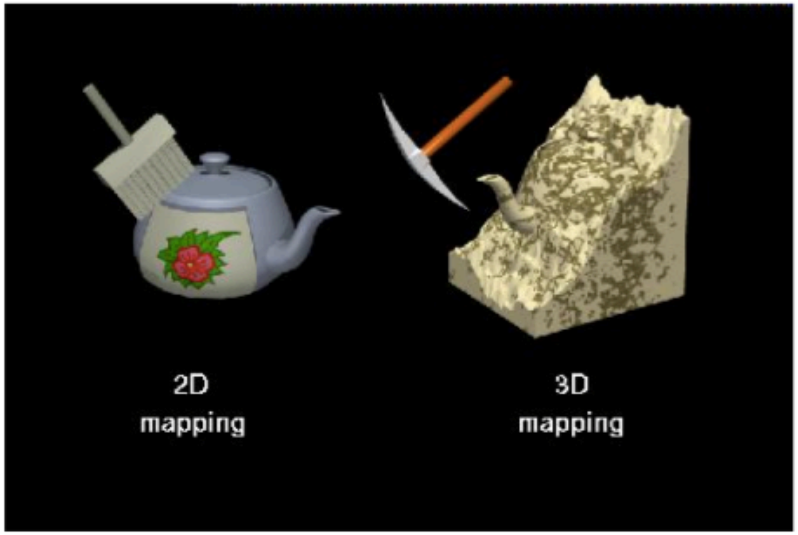
Motivation

In general, we distinguish between 2D and 3D texture mapping:

2D mapping (aka *image textures*): paste an image onto the object

3D mapping (aka *solid* or *volume textures*): create a 3D texture and "carve" the object

3D Object



2D texture $\longleftrightarrow$ 3D texture

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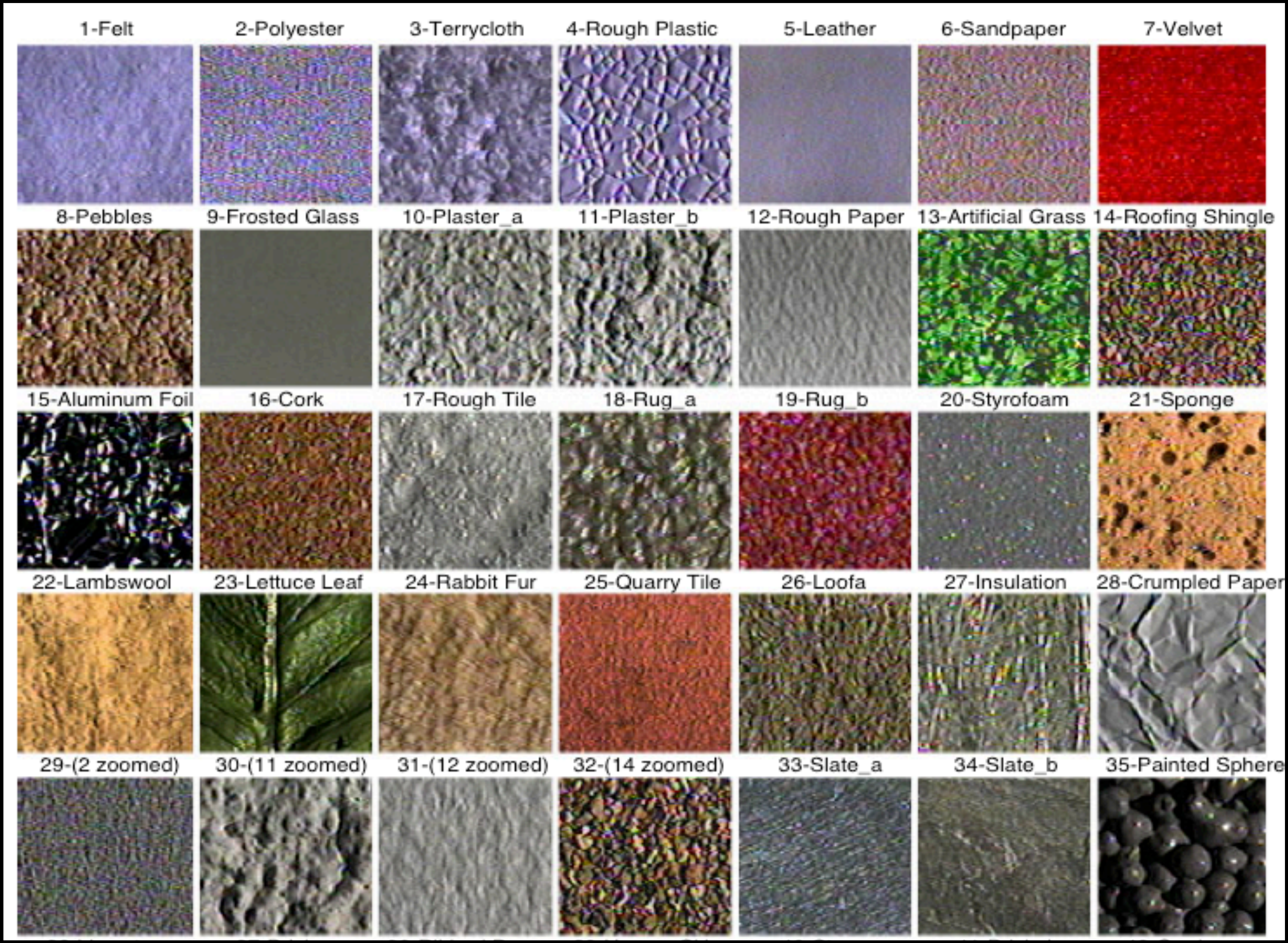
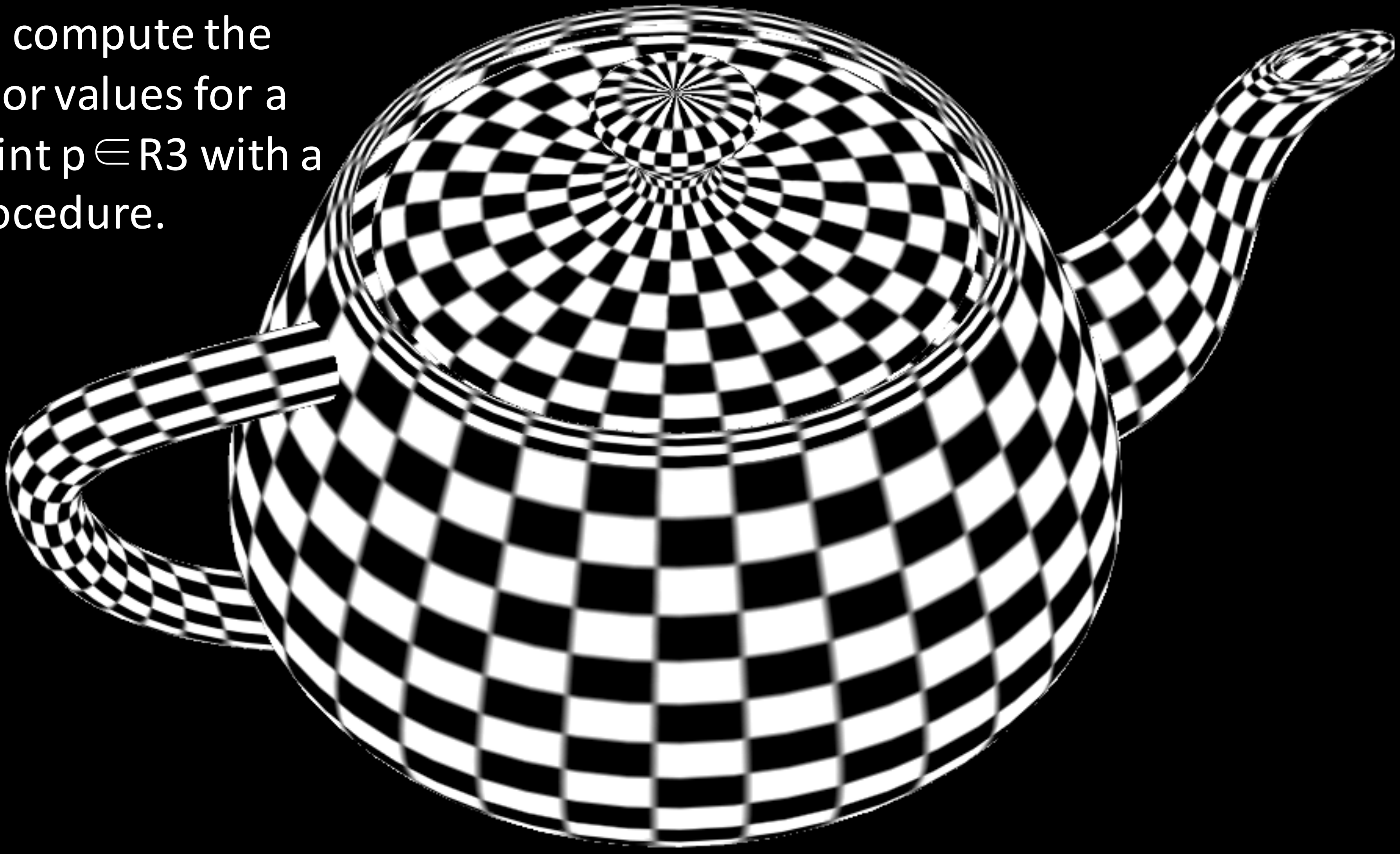
Photos of real materials



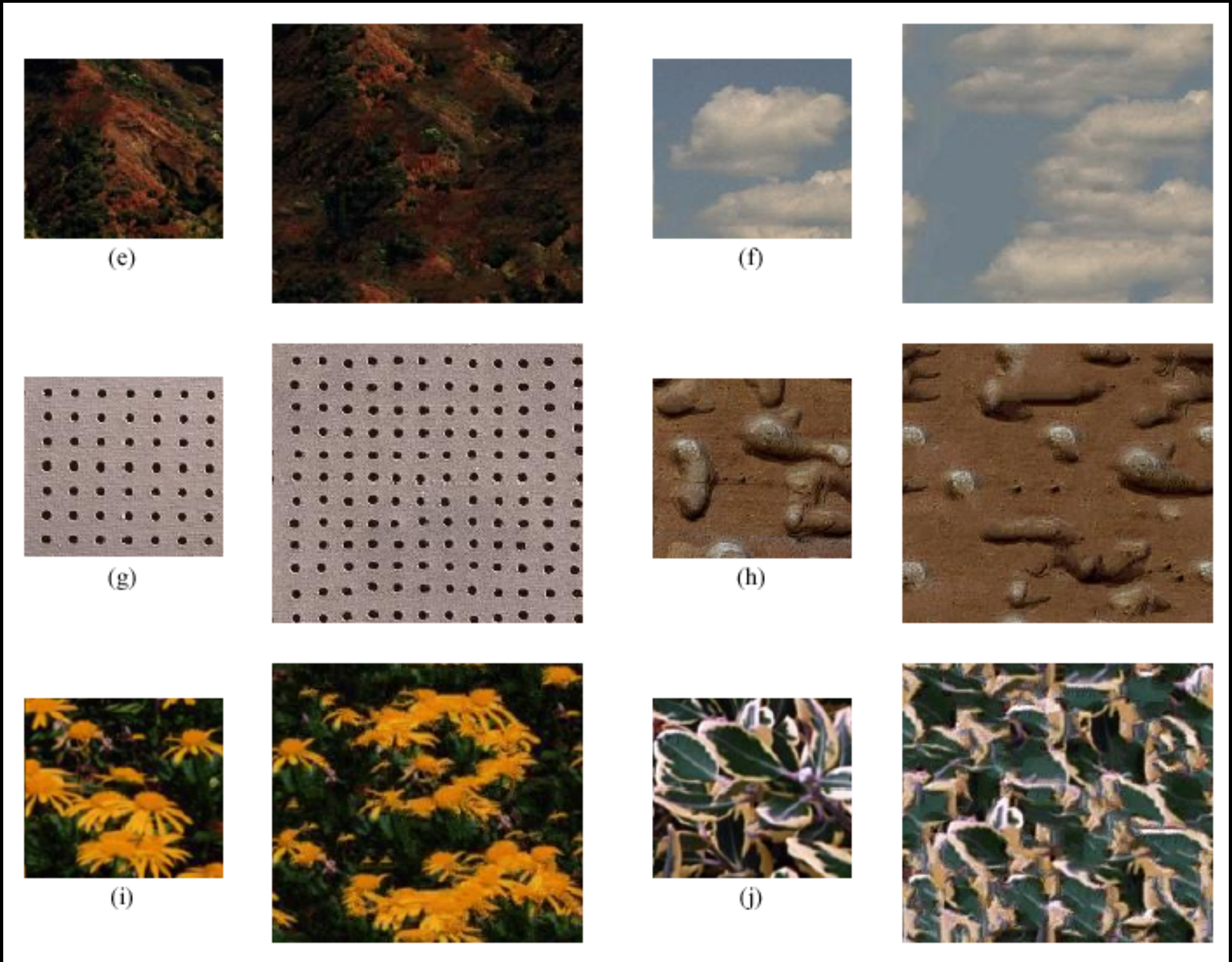
Dana et al, TOG '99

Procedurally-defined textures

It is called procedural because we compute the color values for a point p ∈ R3 with a procedure.



Texture Synthesis



Kwatra et al, SIGGRAPH '05

Texture Synthesis

Original

Synthesized

Original

Synthesized

Kwatra et al, SIGGRAPH '05

Texture Synthesis

Original Synthesized

Kwatra et al, SIGGRAPH '05

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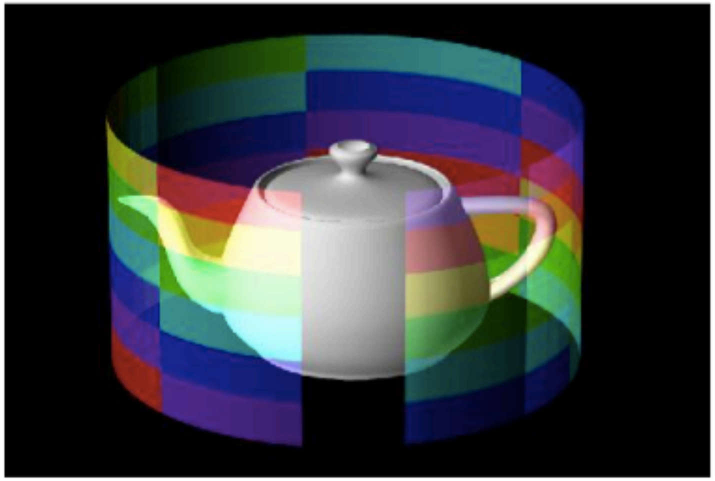
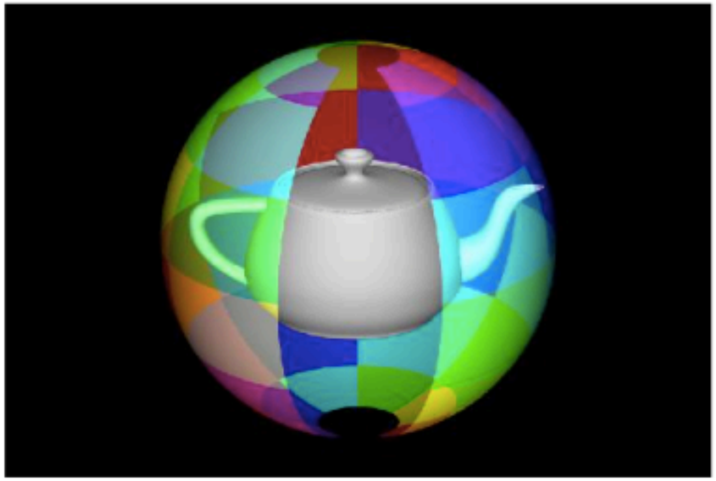
Texture coordinates

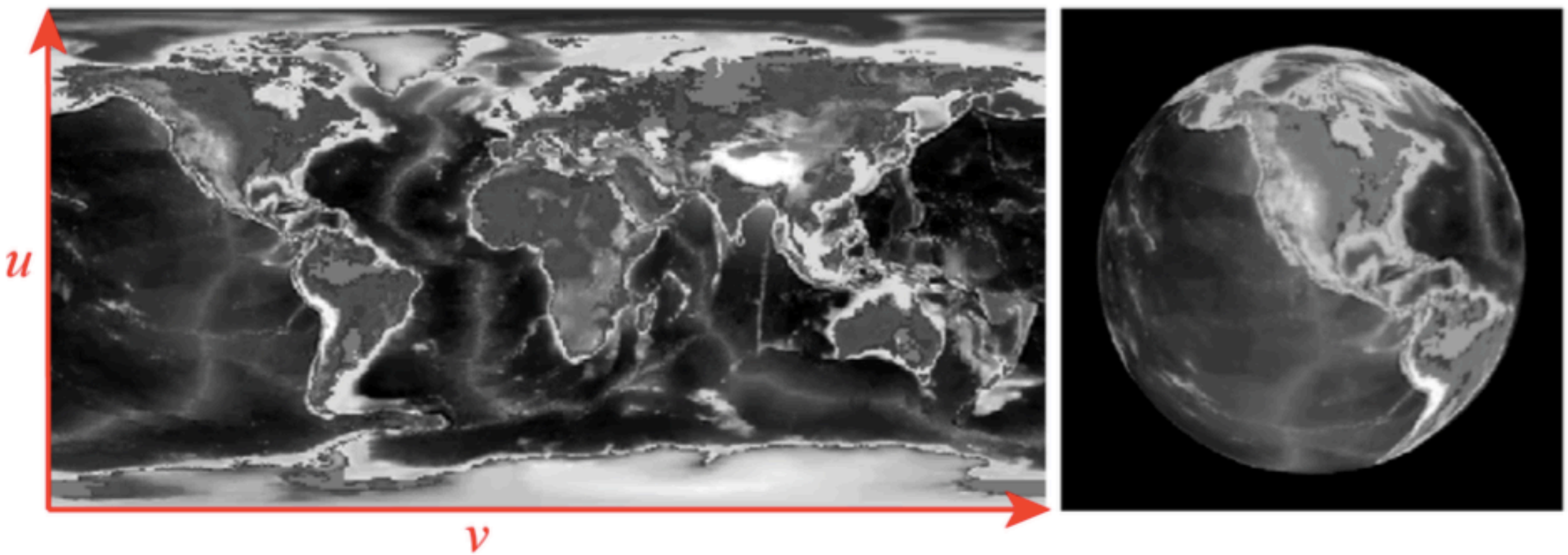
How do we map a rectangular image onto a sphere?





Texture coordinates

Example: use world map and sphere to create a globe



Per conventions we usually assume $u, v \in [0, 1]$.

Texture coordinates

We have seen the parametric equation of a sphere with radius r and center c:

$$\begin{aligned} x &= x_c + r \cos\phi \sin\theta \\ y &= y_c + r \sin\phi \sin\theta \\ z &= z_c + r \cos\theta \end{aligned}$$

Given a point (x, y, z) on the surface of the sphere, we can find θ and ϕ by

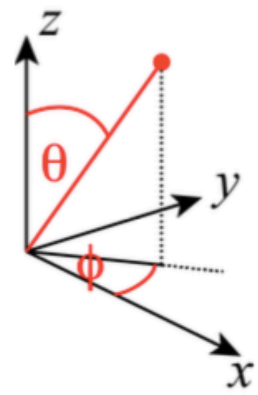
$$\begin{aligned} \theta &= \arccos \tfrac{z - z_c}{r} \qquad \text{(cf. longitude)} \\ \phi &= \arctan \tfrac{y - y_c}{x - x_c} \qquad \text{(cf. latitude)} \end{aligned}$$

(Note: $\arccos$ is the inverse of $\cos$, $\arctan$ is the inverse of $\tan = \frac{\sin}{\cos}$)

Texture coordinates

For a point (x, y, z) we have

$$\theta = \arccos \frac{z - z_c}{r}$$
$$\phi = \arctan \frac{y - y_c}{x - x_c}$$

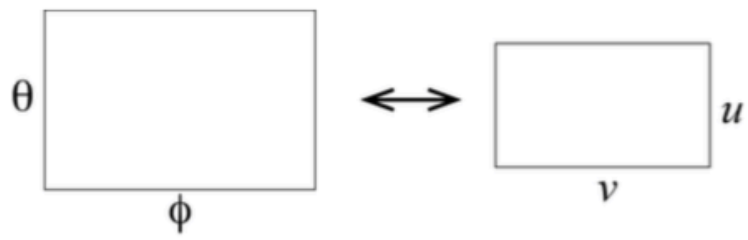


$(\theta, \phi) \in [0, \pi] \times [-\pi, \pi]$, and u, v must range from $[0, 1]$.

Hence, we get:

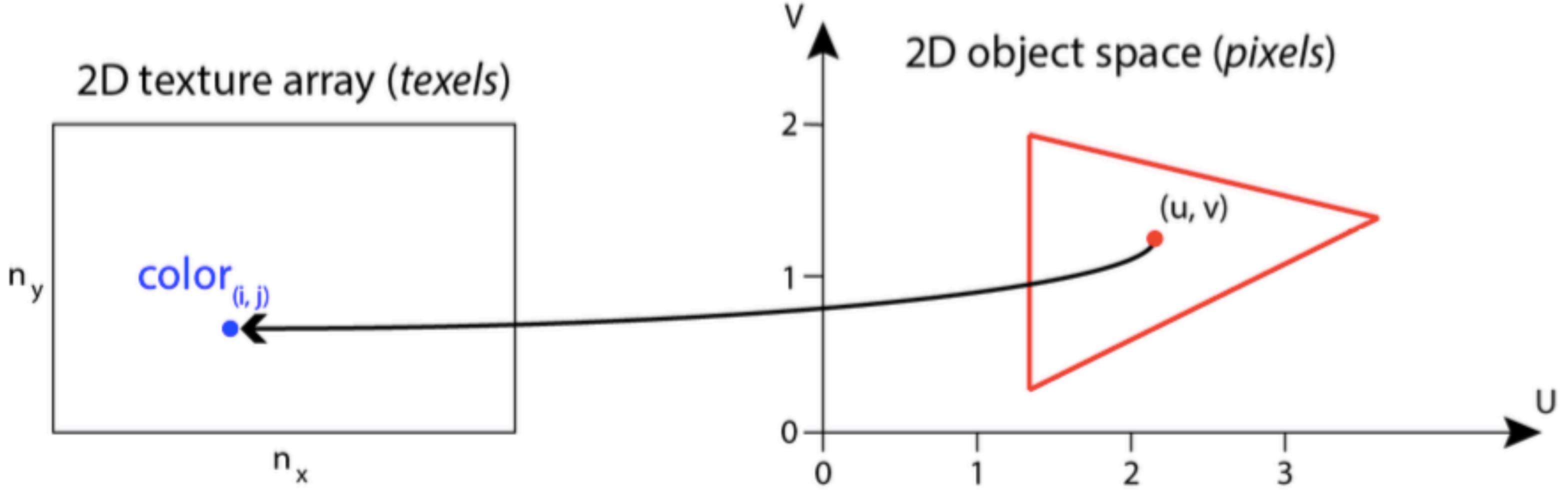
$$u = \frac{\phi \mod 2\pi}{2\pi}$$
$$v = \frac{\pi - \theta}{\pi}$$

(Note that this is a simple scaling transformation in 2D)



Texture coordinates

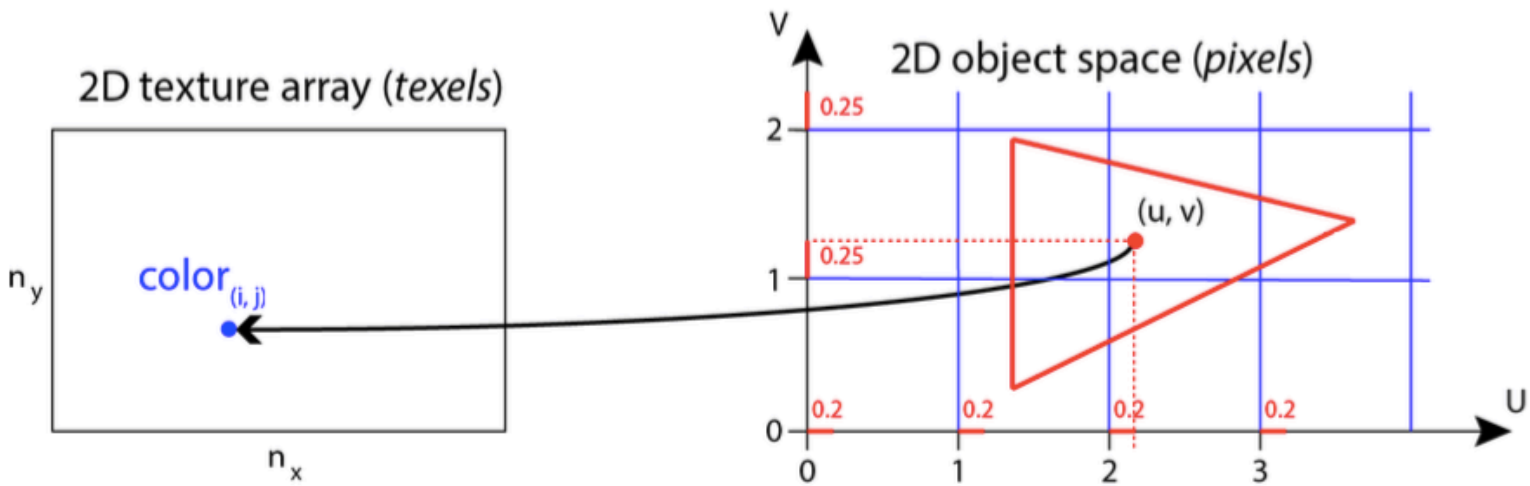
Example: "Tiling" of 2D textures into a UV-object space



We'll call the two dimensions to be mapped u and v, and assume an $n_x \times n_y$ image as texture.

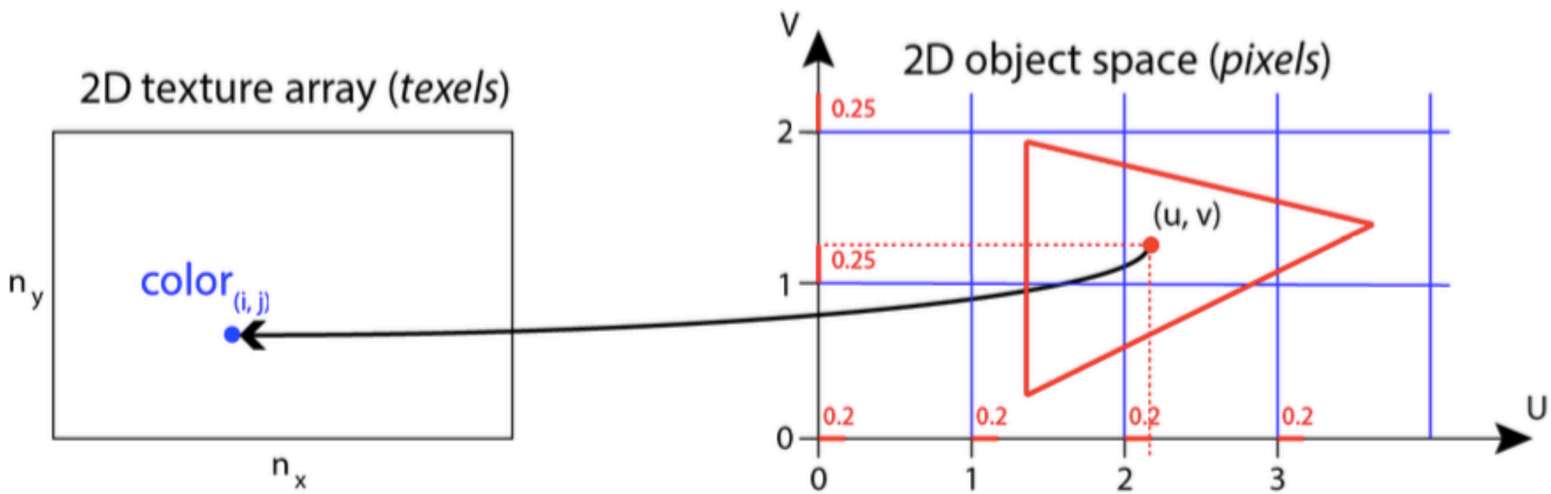
Then every (u, v) needs to be mapped to a color in the image, i.e. we need a mapping from pixels to texels.

Texture coordinates



A standard way is to first
remove the integer portion of u and v,
so that (u, v) lies in the unit square.

Texture coordinates

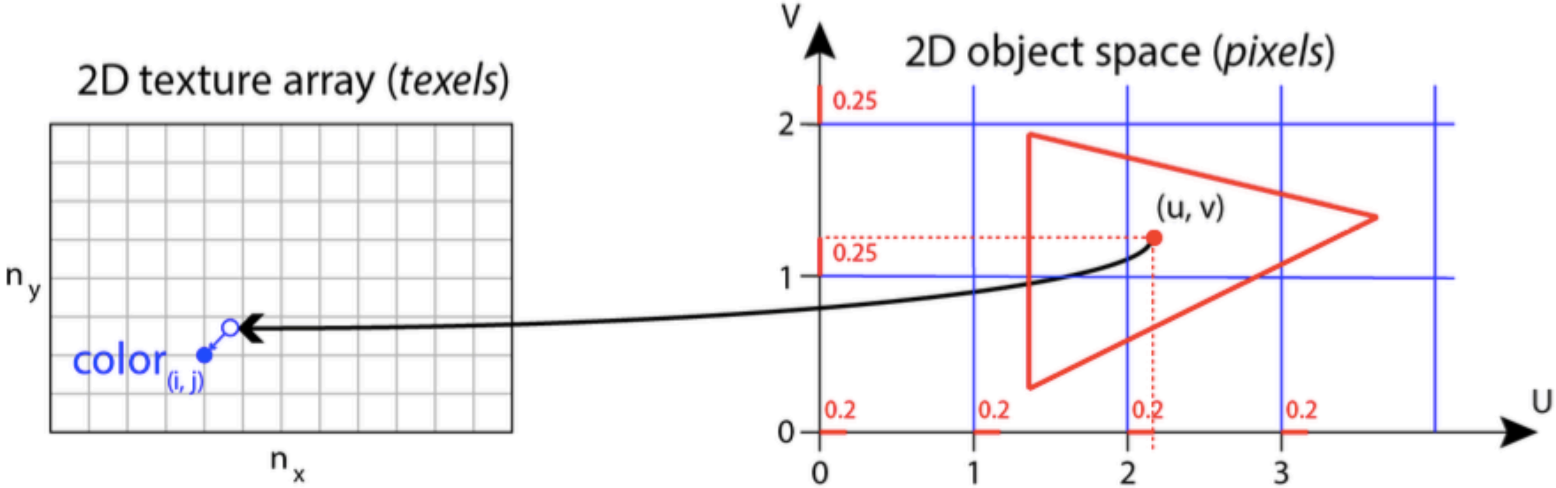


This results in a simple mapping from $0 \leq u, v \leq 1$ to the size of the texture array, i.e. $n_x \times n_y$.

$$i = un_x \text{ and } j = vn_y$$

Yet, for the array lookup, we need integer values.

Texture coordinates



The texel (i, j) in the $n_x \times n_y$ image for (u, v) can be determined using the floor function $\lfloor x \rfloor$ which returns the highest integer value $\leq x$.

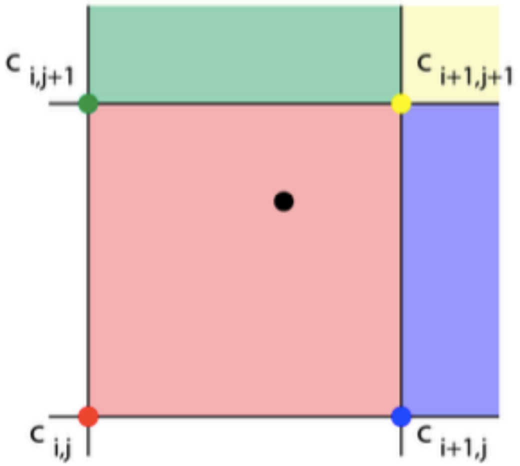
$$i = \lfloor u n_x \rfloor \text{ and } j = \lfloor v n_y \rfloor$$

Texture coordinates

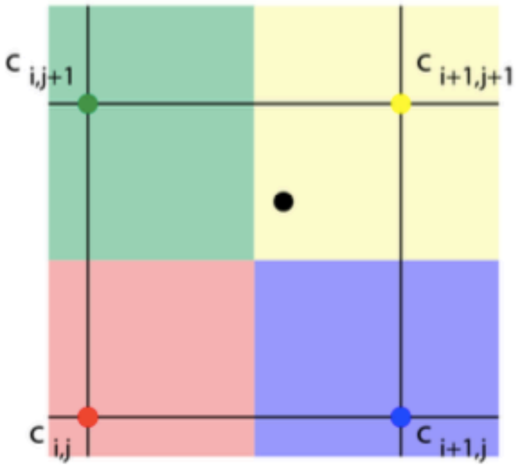
$$c(u,v) = c_{i,j} \text{ with } i = \lfloor u n_x \rfloor \text{ and } j = \lfloor v n_y \rfloor$$

This is a version of nearest-neighbor interpolation, where we take the color of the nearest neighbor.

Floor function



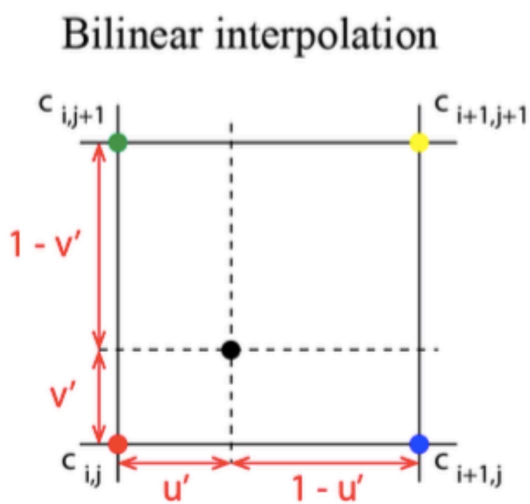
Nearest neighbor mapping



Texture coordinates

For smoother effects we may use bilinear interpolation:

$$c(u,v) = (1-u')(1-v')c_{ij} + u'(1-v')c_{(i+1)j} + (1-u')v'c_{i(j+1)} + u'v'c_{(i+1)(j+1)}$$



with

$u' = un_x - \lfloor un_x \rfloor$ and

$v' = vn_y - \lfloor vn_y \rfloor$

Notice that all weights are between 0 and 1 and add up to 1:

$$(1-u')(1-v') + u'(1-v') + (1-u')v' + u'v' = 1$$

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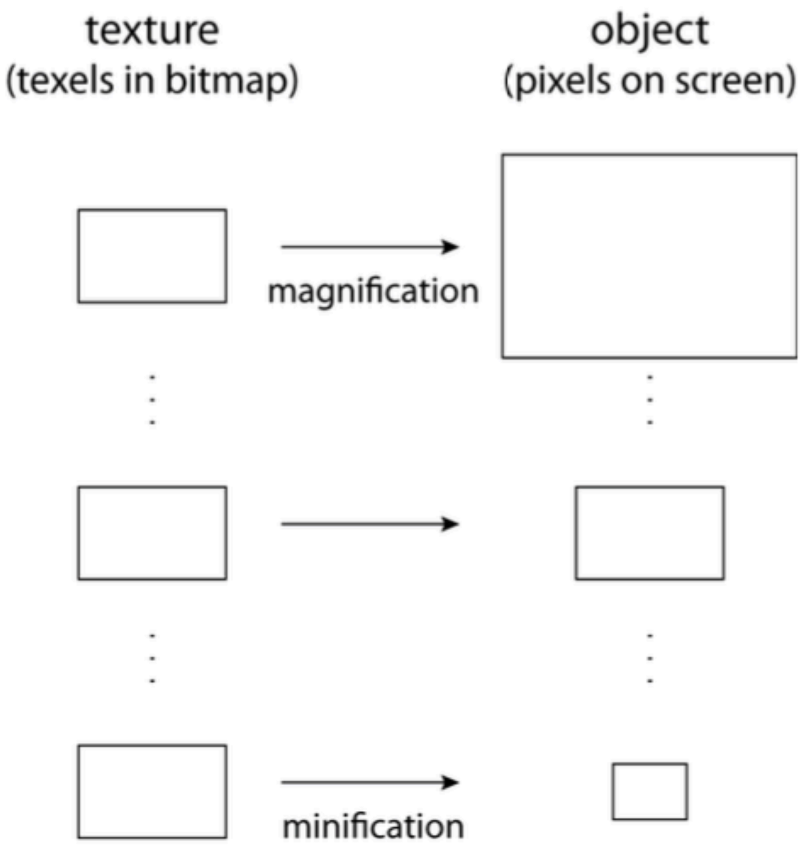
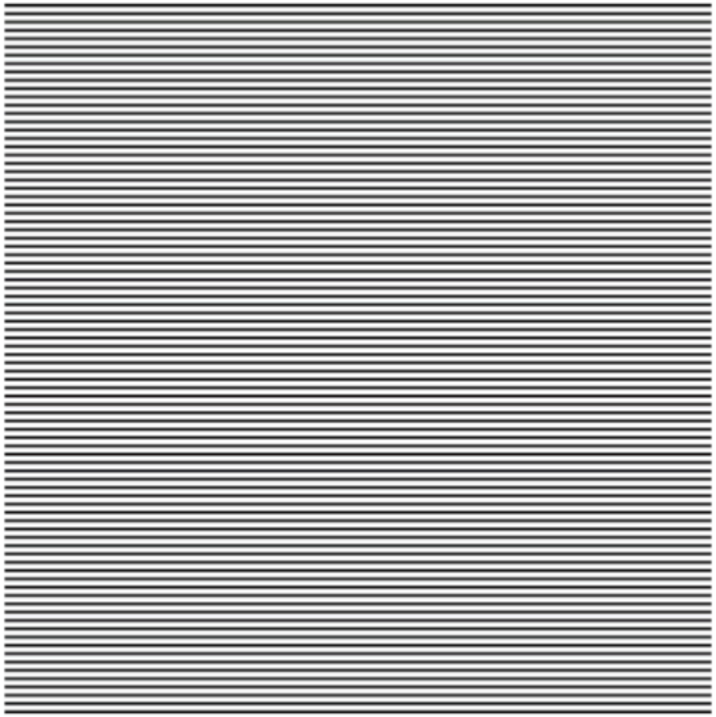
Mipmapping

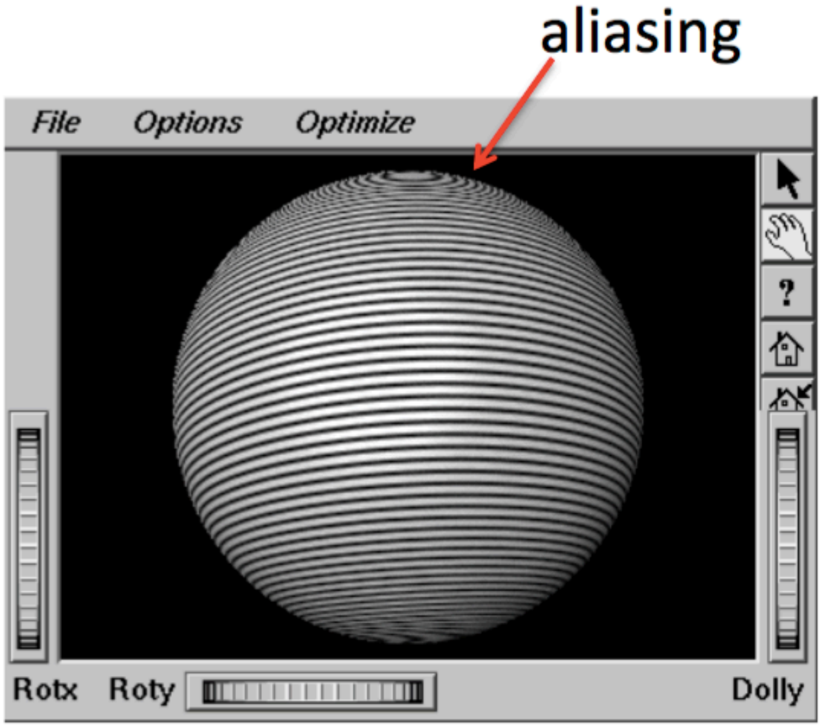


- If viewer is close:
  Object gets larger
  → Magnify texture
- "Perfect" distance:
  Not always "perfect" match
  (misalignment, etc.)
- If viewer is further away:
  Object gets smaller
  → Minify texture

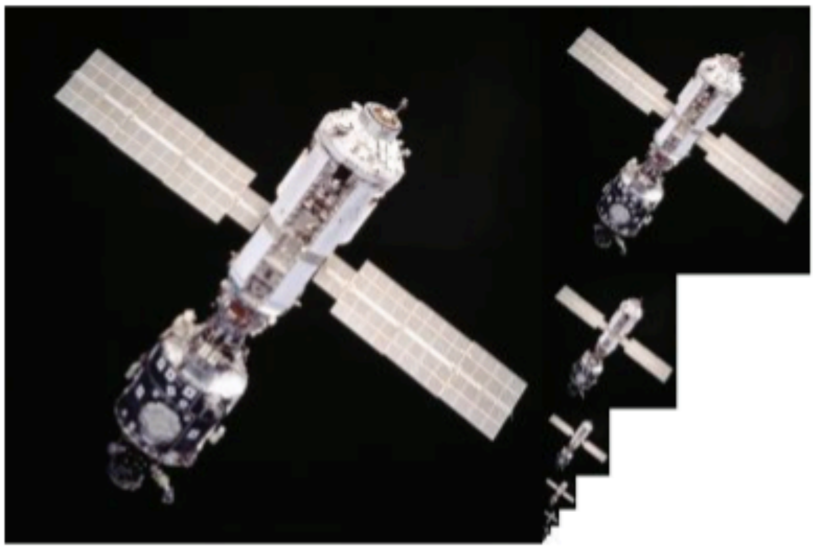
Problem with minification:
efficiency (esp. when whole
texture is mapped onto one pixel!)

Mipmapping





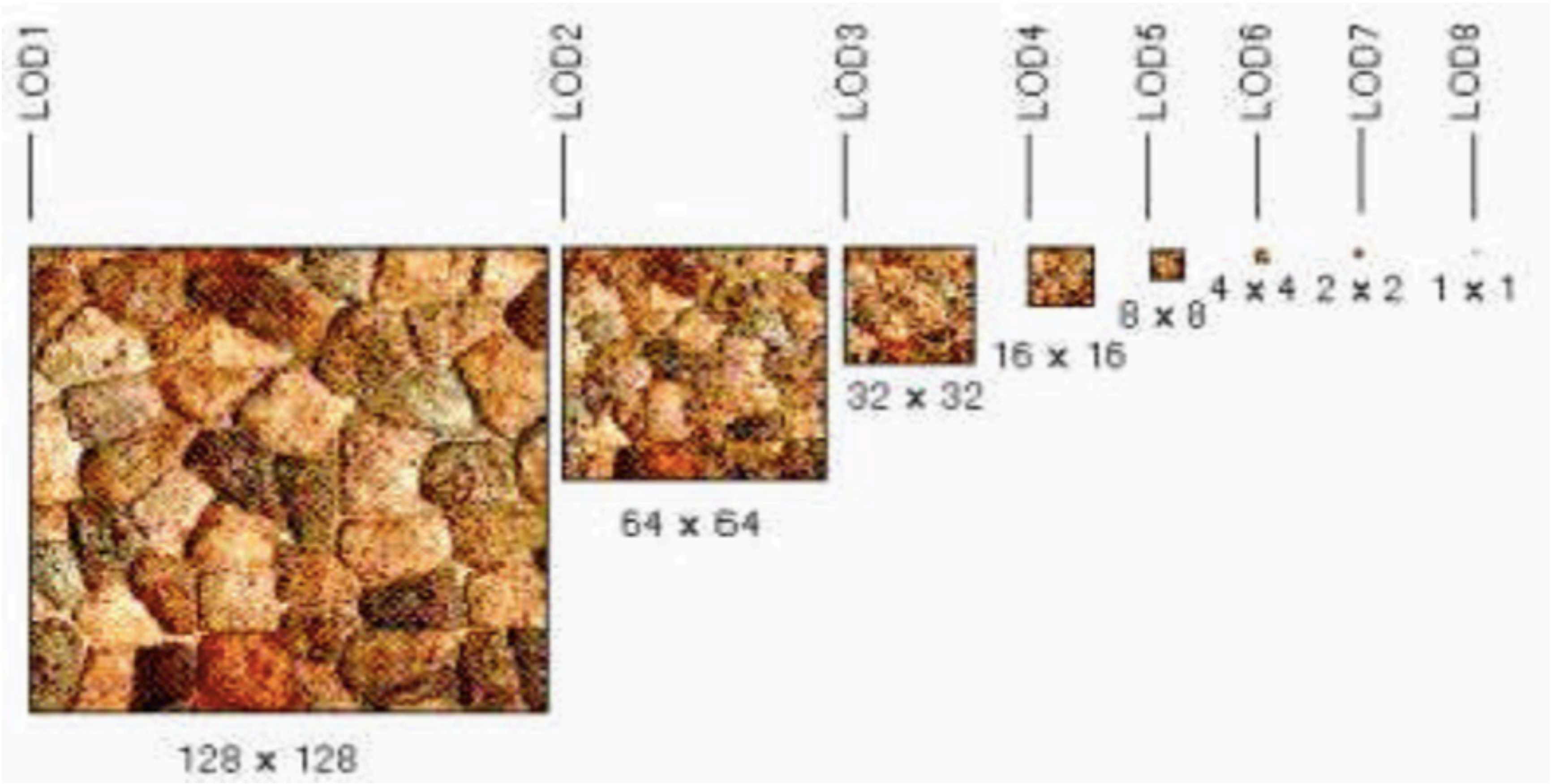
Mipmapping



Solutions: MIP maps

- Pre-calculated, optimized collections of images based on the original texture
- Dynamically chosen based on depth of object (relative to viewer)
- Supported by todays hardware and APIs

Mipmapping



LOD1
LOD2
LOD3
LOD4
LOD5
LOD6
LOD7
LOD8
128 x 128
64 x 64
32 x 32
16 x 16
8 x 8
4 x 4
2 x 2
1 x 1

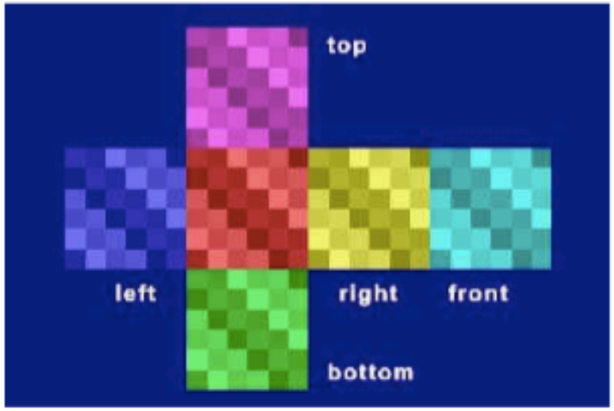
Environment mapping

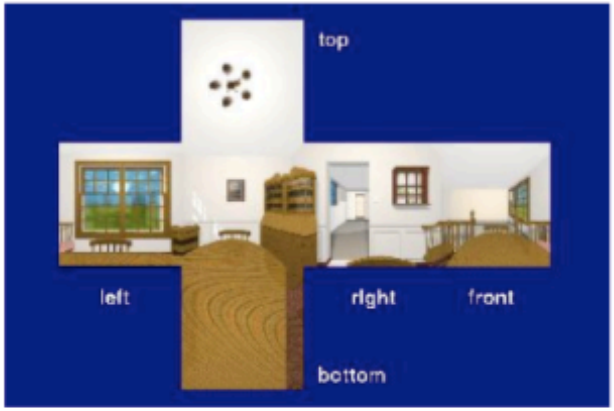
... why not use this to make objects appear to reflect their surroundings specularly?

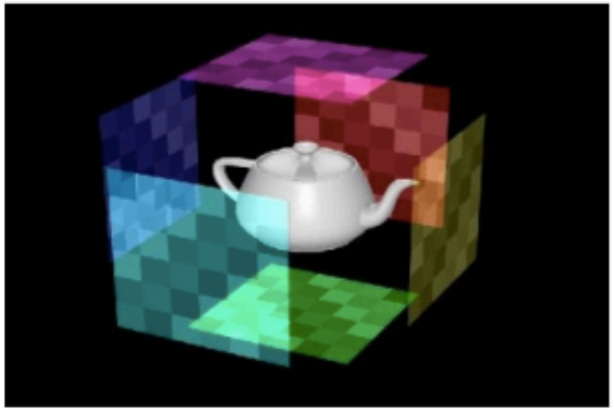
Idea: place a cube around the object, and project the environment of the object onto the planes of the cube in a preprocessing stage; this is our texture map.

During rendering, we compute a reflection vector, and use that to look-up texture values from the cubic texture map.

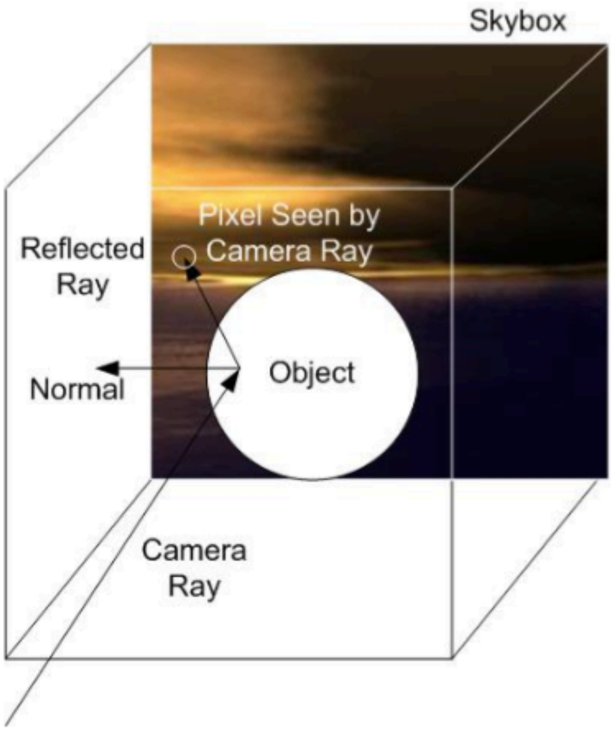
Environment mapping







Environment mapping



Remember Phong shading: "perfect" reflection if

angle between eye vector $\vec{e}$ and $\vec{n}$ $=$ angle between $\vec{n}$ and reflection vector $\vec{r}$

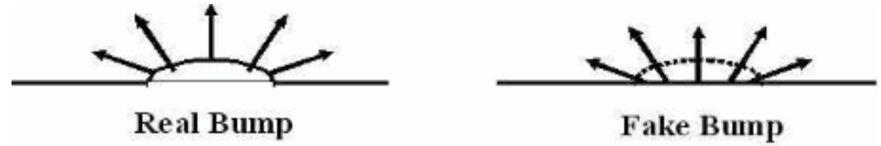
Environment mapping

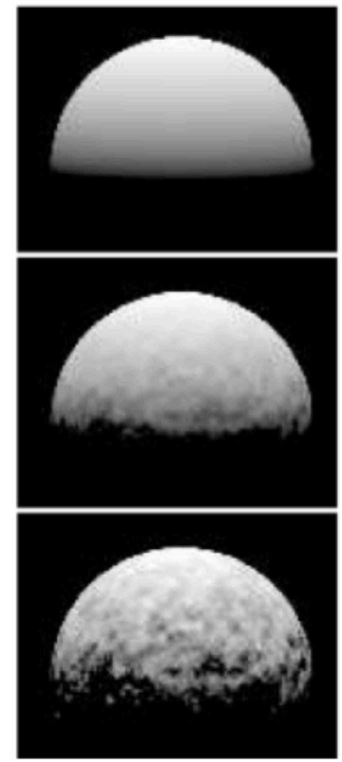
Image from slides by

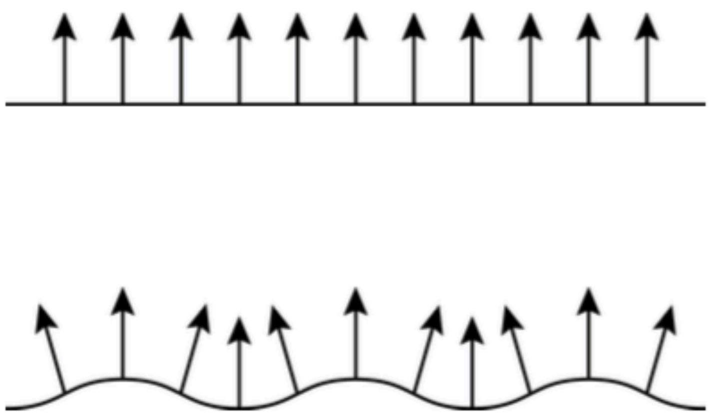
Bump mapping

One of the reasons why we apply texture mapping:

Real surfaces are hardly flat but often rough and bumpy. These bumps cause (slightly) different reflections of the light.

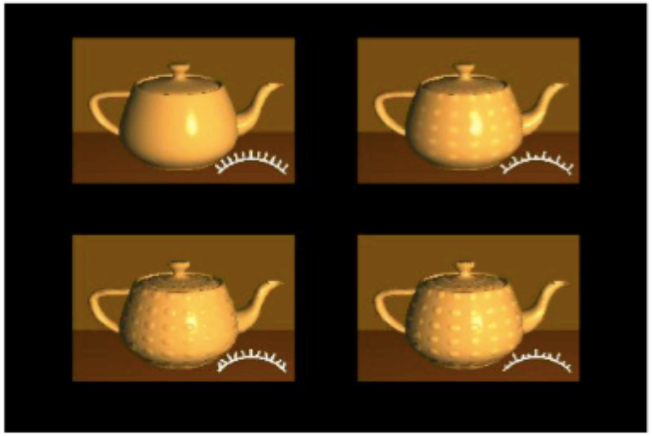






Bump mapping

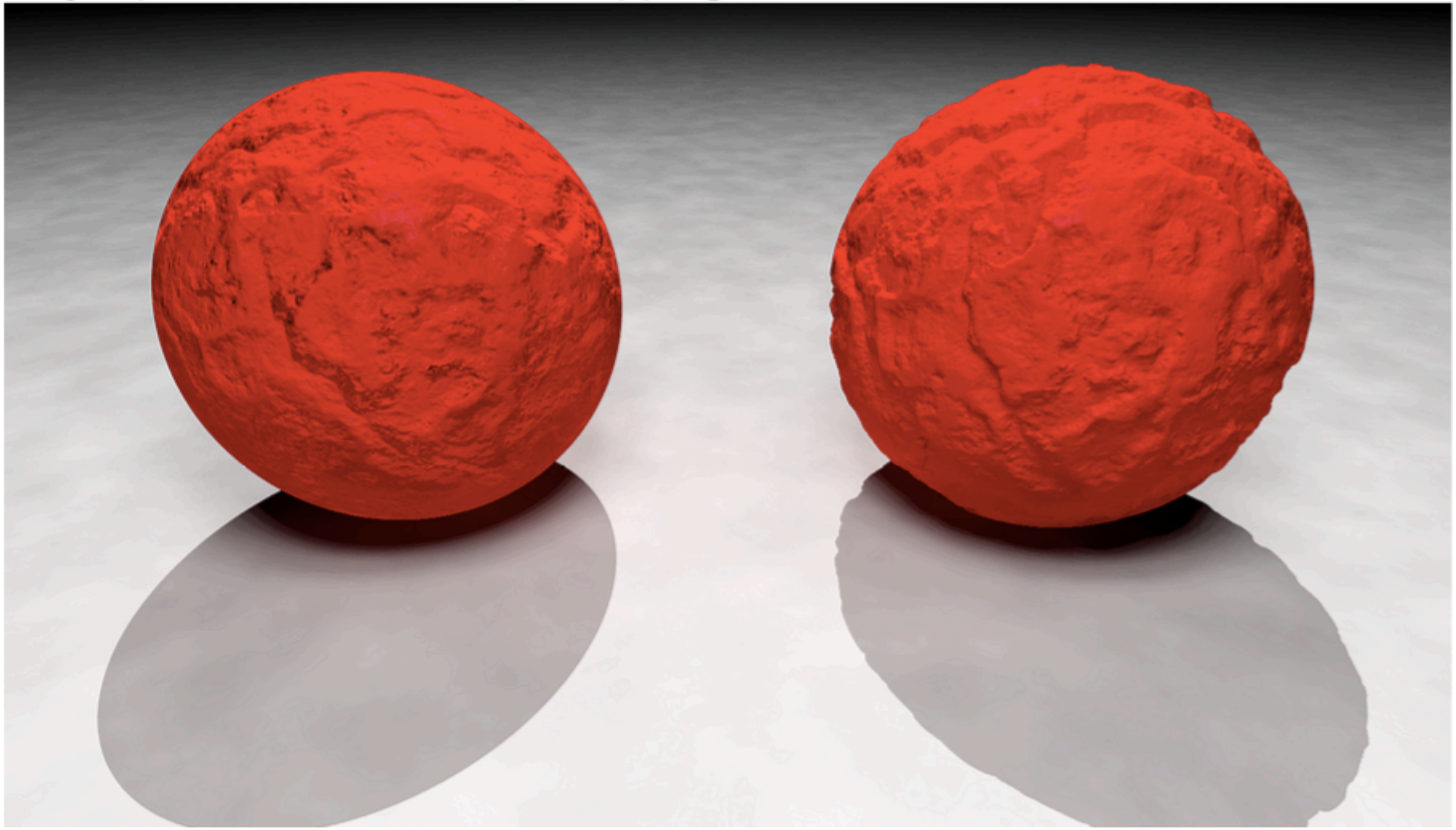
Instead of mapping an image or noise onto an object, we can also apply a bump map, which is a 2D or 3D array of vectors. These vectors are added to the normals at the points for which we do shading calculations.



The effect of bump mapping is an apparent change of the geometry of the object.

Bump mapping

Major problems with bump mapping: silhouettes and shadows

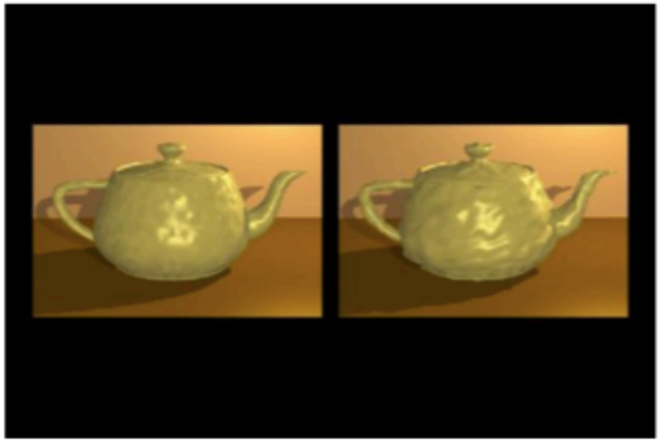


We can use textures that perturb <u>normals</u> instead of colors or reflectances

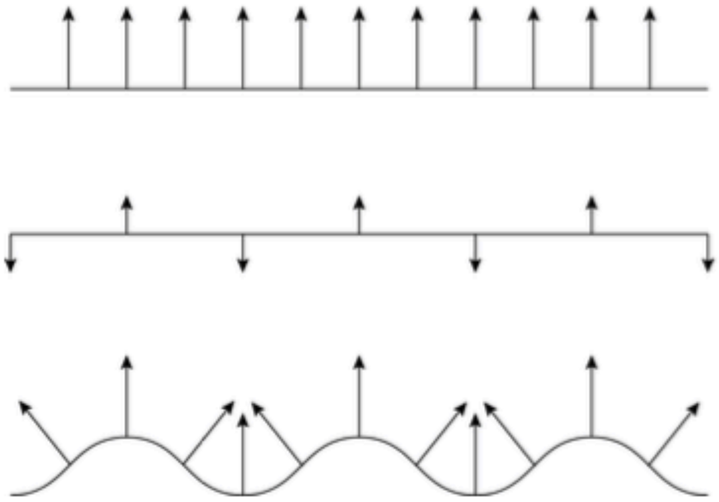
Displacement mapping

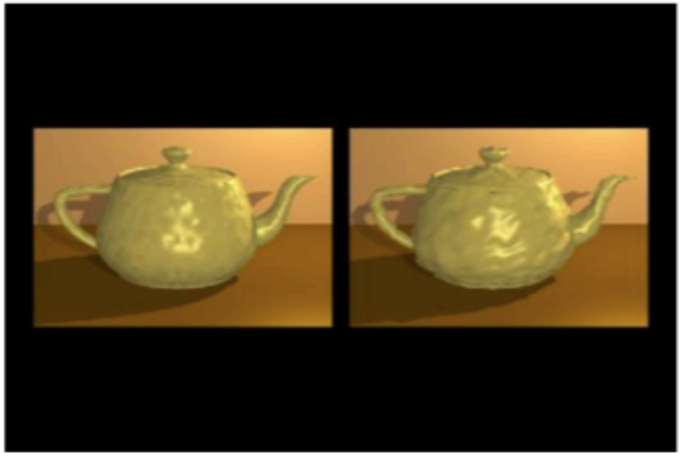
To overcome this shortcoming, we can use a displacement map. This is also a 2D or 3D array of vectors, but here the points to be shaded are actually displaced.

Normally, the objects are refined using the displacement map, giving an increase in storage requirements.



Displacement mapping





Topic 12:

Basic Ray Tracing

- Introduction to ray tracing
- Computing rays
- Computing intersections
  - ray-triangle
  - ray-polygon
  - ray-quadric
  - the scene signature
- Computing normals
- Evaluating shading model
- Spawning rays
- Incorporating transmission
  - refraction
  - ray-spawning & refraction

Ray tracing in the movies

Christensen et al, 2006

www.povray.org/community/hof

" Main steet (blue) "

" The Cool Cows "

www.povray.org/community/hof

"The Dark Side of Trees"

"Capriccio" G. Obukhov et al, 2003

Online Ray Tracing Competitions

www.irtc.org/stills

380K triangles, 104 lights, full global illumination in real time

SIGGRAPH'05 Course by Slusallek et al

15 cars, 240M quads, 80M rays

Render time : without optimizations > 4 days
with optimizations ~ 6 hrs

www.povray.org/community/hof

"The Dark Side of Trees"

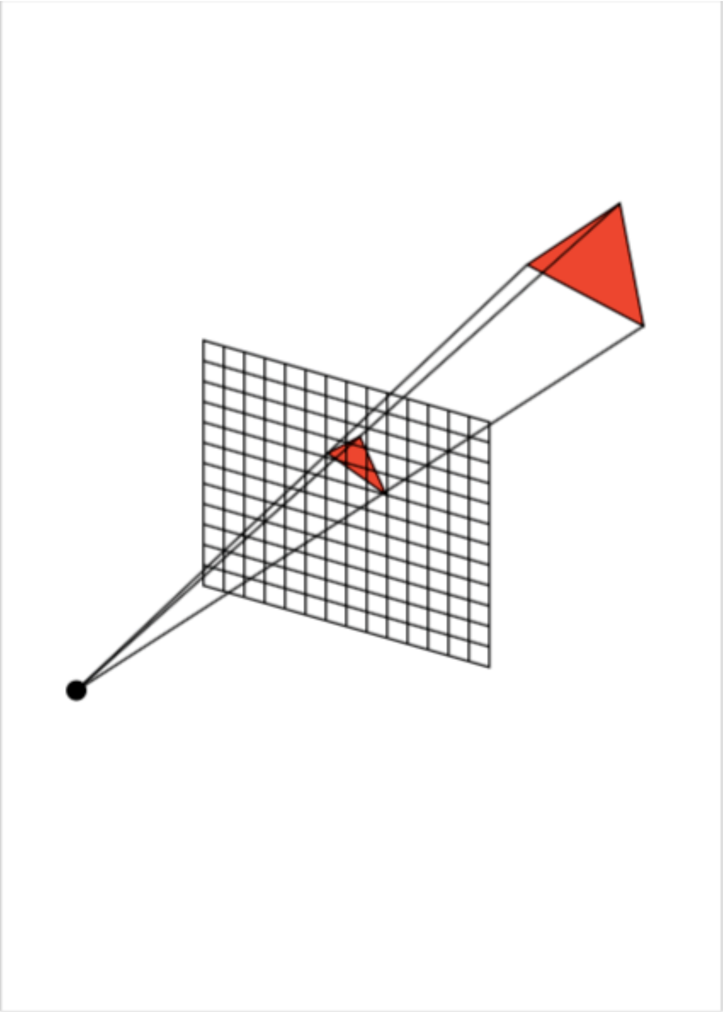
"Capriccio" G. Obukhov et al, 2003

Projective methods

A popular method for generating images from a 3D-model is projection, e.g.:

- 3D triangles project to 2D triangles
- Project vertices
- Fill/shade 2D triangle

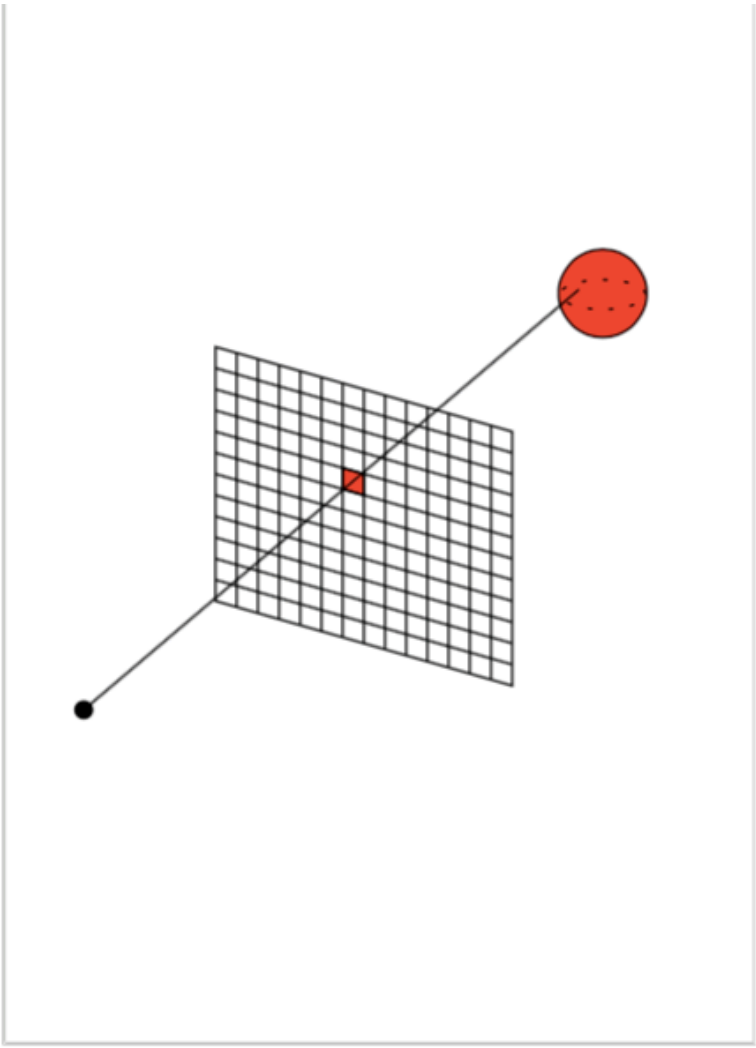
Notice:
Ray tracing = pixel-based,
proj. methods = object-based



Ray tracing / ray casting

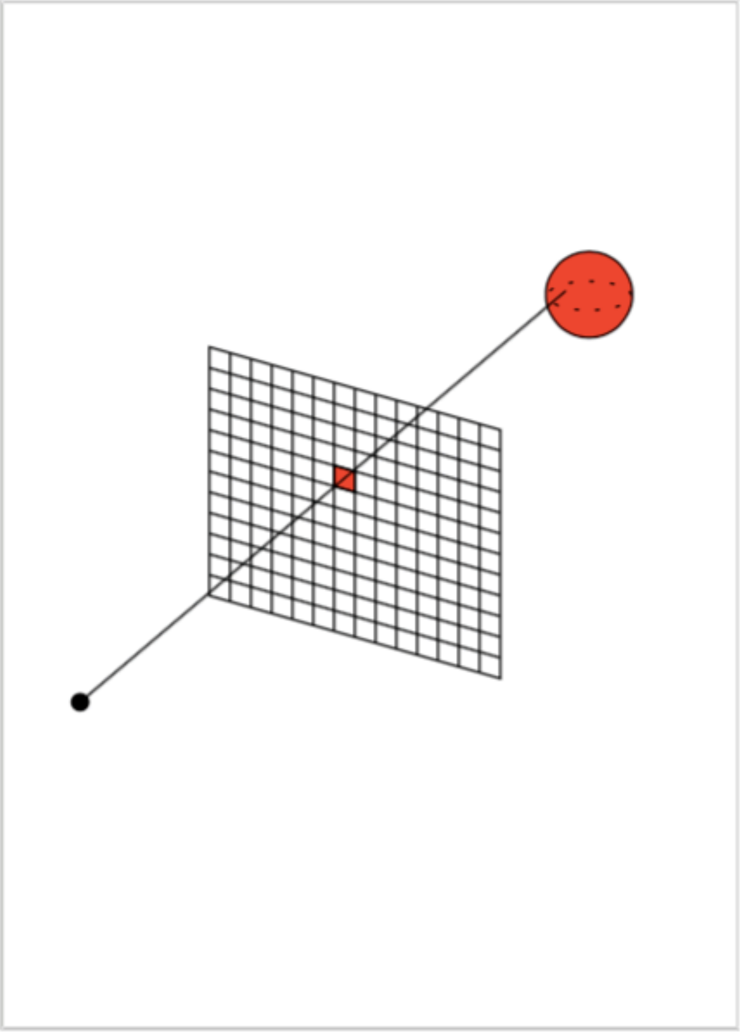
For photo-realistic rendering, usually ray tracing algorithms are used: for every pixel

- Compute ray from viewpoint through pixel center
- Determine intersection point with first object hit by ray
- Calculate shading for the pixel (possibly with recursion)



Ray tracing / ray casting

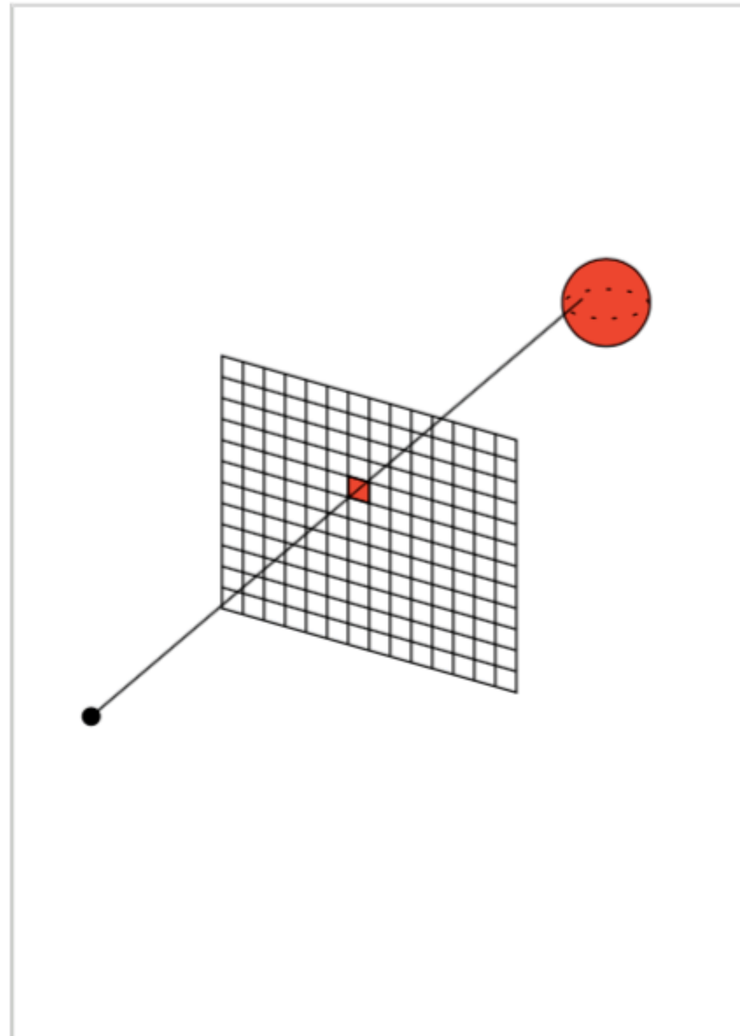
- Global Illumination
- Traditionally (very) slow
- Recent developments: real-time ray tracing



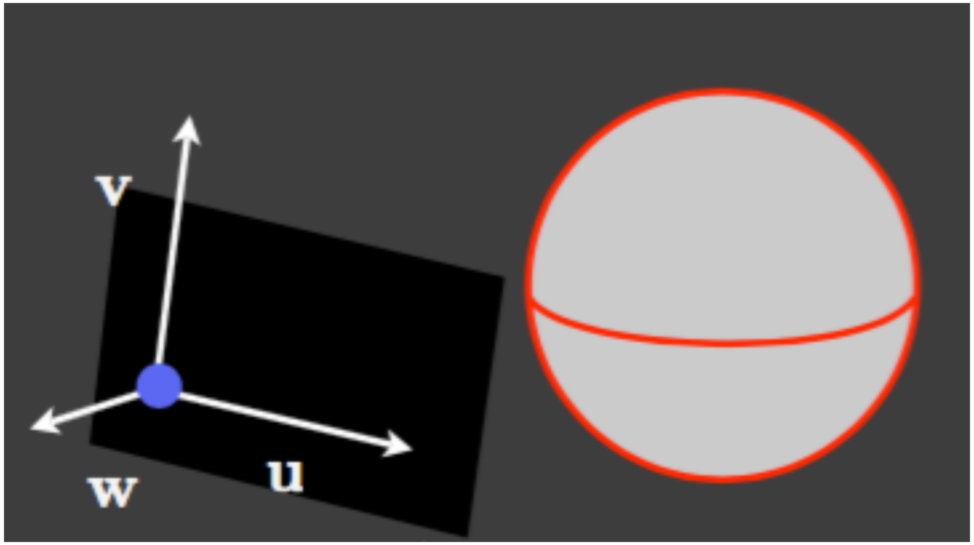
Ray tracing / ray casting

Why ray tracing is important (even if you are just interested in real-time rendering):

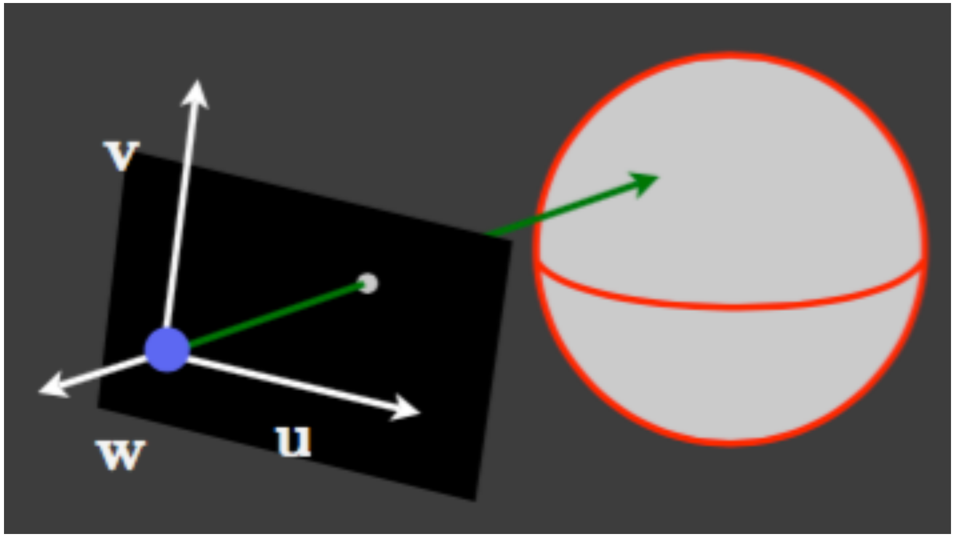
- Recent developments: real-time ray tracing, path tracing, etc.
- Important in games for interaction
- Important computer graphics technique (also: shares many techniques with other approaches)



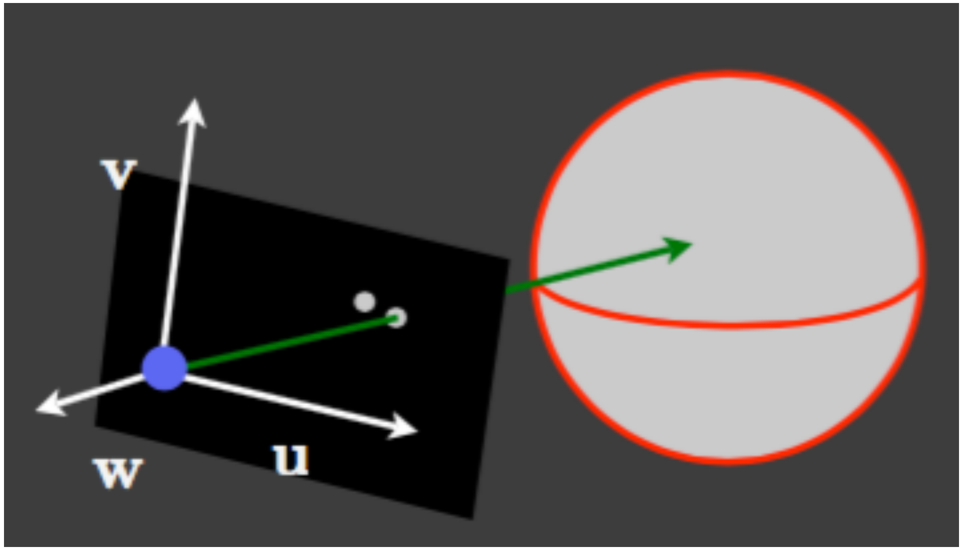
Ray tracing



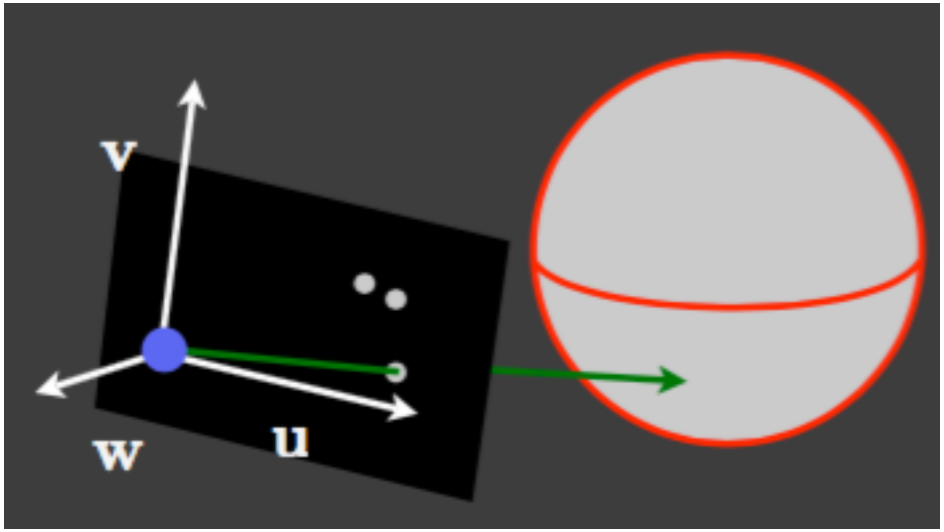
Ray tracing



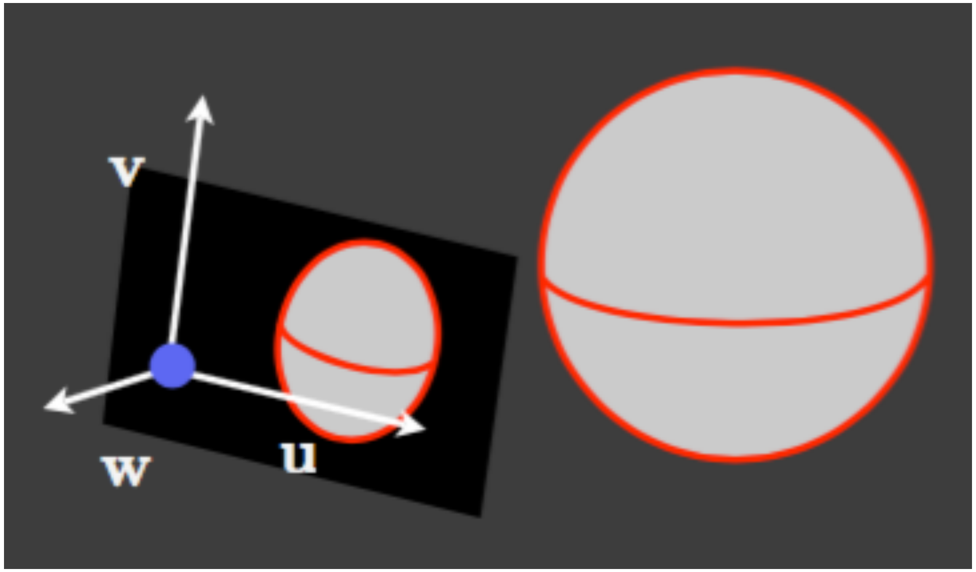
Ray tracing



Ray tracing



Ray tracing



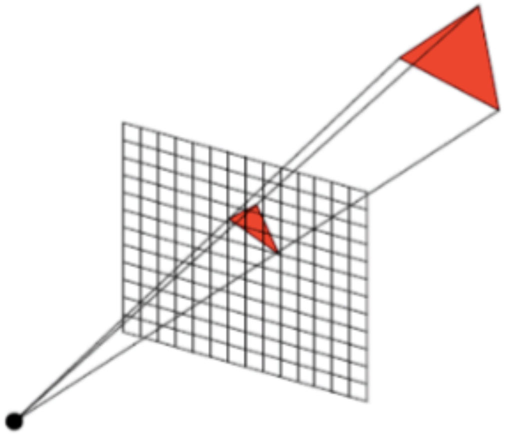
Projective methods vs. ray tracing

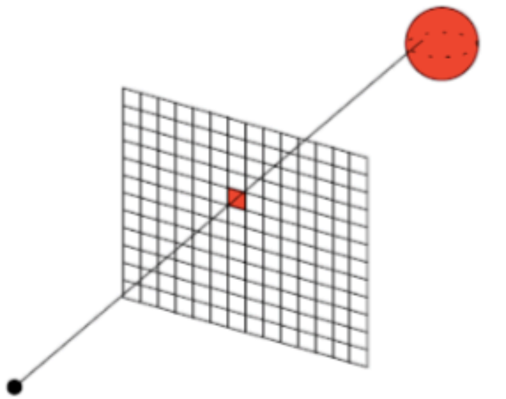
Projective methods & Ray tracing

. . . share lots of techniques,
e.g., shading models,
calculation of intersections, etc.

. . . but also have major differences,
e.g., projection and hidden
surface removal come “for free”
in ray tracing

And most importantly . . .



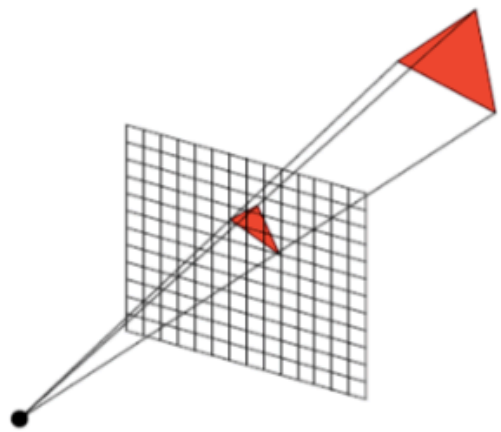


Projective methods vs. ray tracing

Projective methods:

Object-order rendering, i.e.

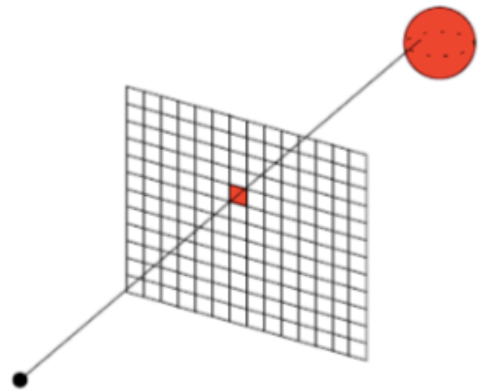
- For each object . . .
- . . . find and update all pixels that it influences and draw them accordingly



Ray tracing:

Image-order rendering, i.e.

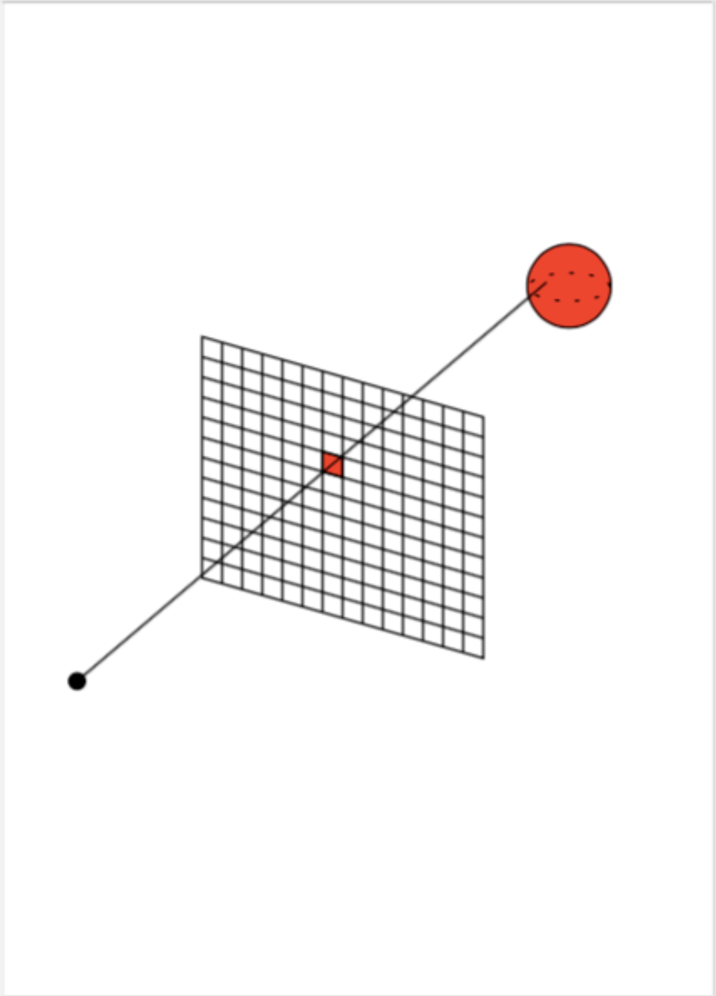
- For each pixel . . .
- . . . find all objects that influence it and update it accordingly



A basic ray tracing algorithm

FOR each pixel DO

- compute viewing ray
- find the 1st object hit by the ray and its surface normal $\vec{n}$
- set pixel color to value computed from hit point, light, and $\vec{n}$

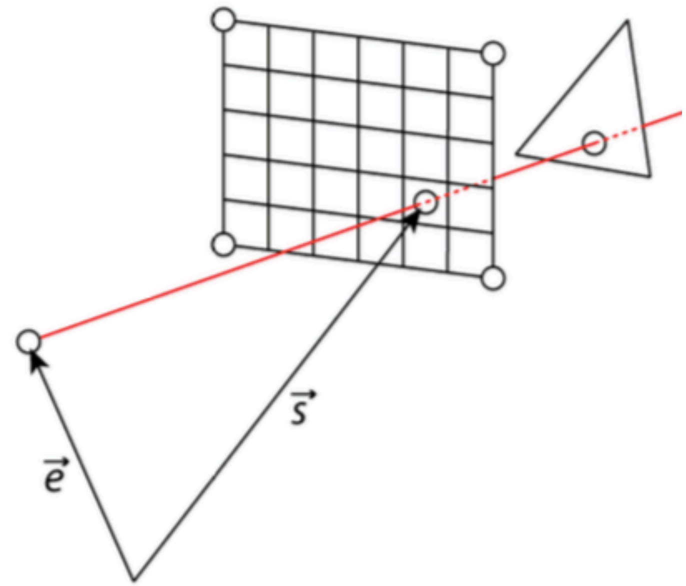


Lines and rays

We need to "shoot" a ray

- from the view point $\vec{e}$
- through a pixel $\vec{s}$ on the screen
- towards the scene/objects

Hmm, that should be easy with . . .



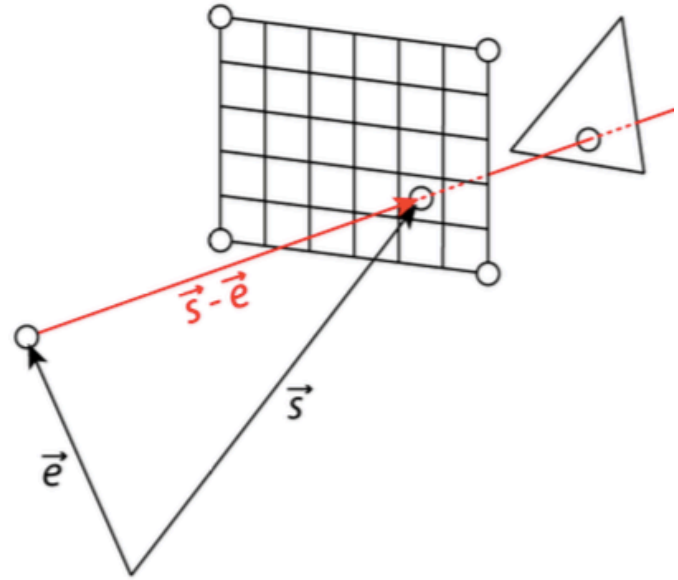
Lines and rays

. . . a parametric line equation:

$$\vec{p}(t) = \vec{e} + t(\vec{s} - \vec{e})$$

where

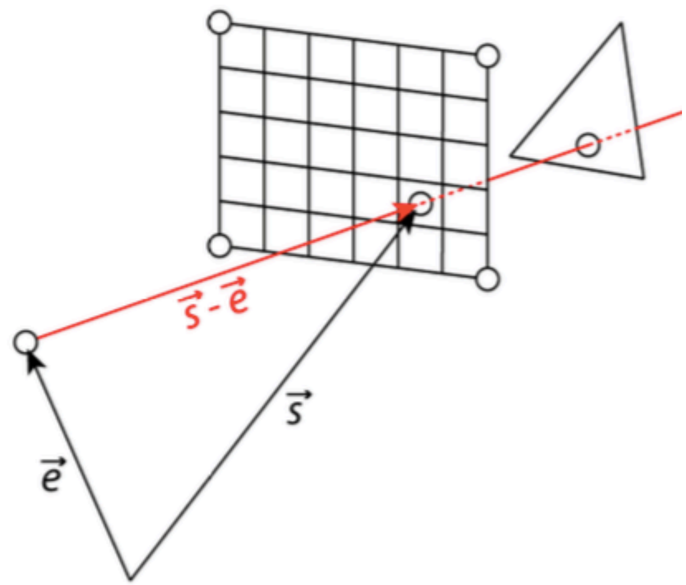
- $\vec{e}$ is a point on the line (aka its *support vector*)
- $\vec{s} - \vec{e}$ is a vector on the line (aka its *direction vector*)



Lines and rays

With this, our ray ...

- starts at $\vec{e}$ $(t = 0)$,
- goes throught $\vec{s}$ $(t = 1)$,
- and "shoots" towards the scene/objects $(t > 1)$



Hmm, calculation would become much easier if we would have ...

Coordinate system

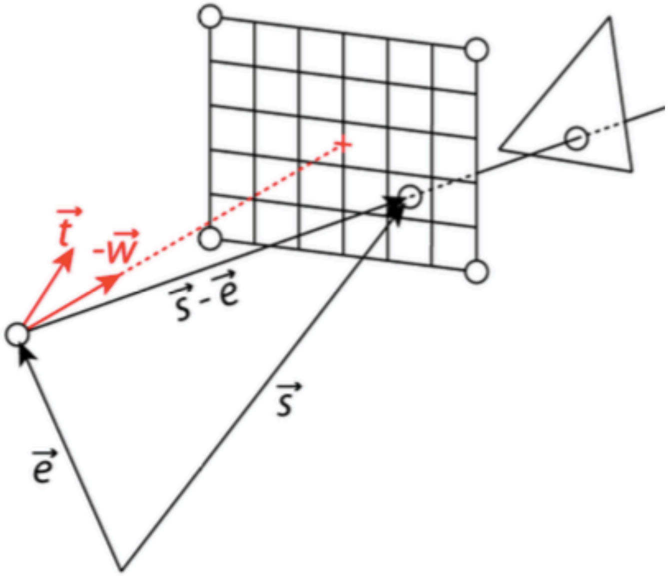
. . . a camera coordinate system:

That's easy! Using

- our camera position $\vec{e}$
- our viewing direction $-\vec{w}$
- and a view up vector $\vec{t}$

we get

- $\vec{u} = -\vec{w} \times \vec{t}$
- $\vec{v} = -\vec{w} \times \vec{u}$



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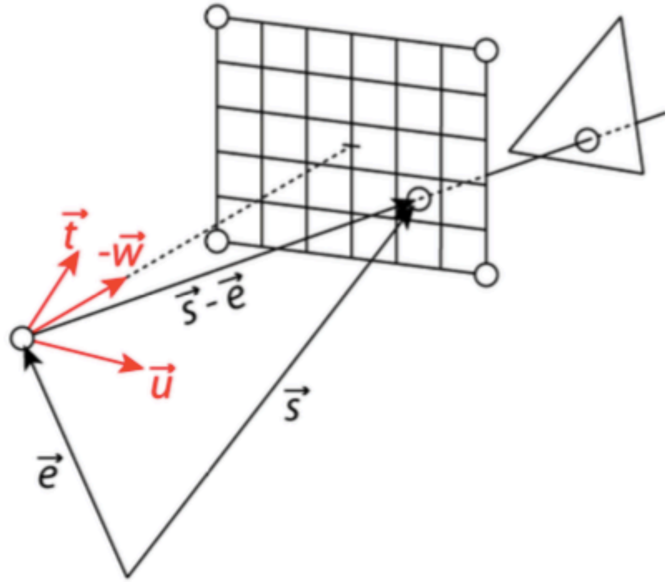

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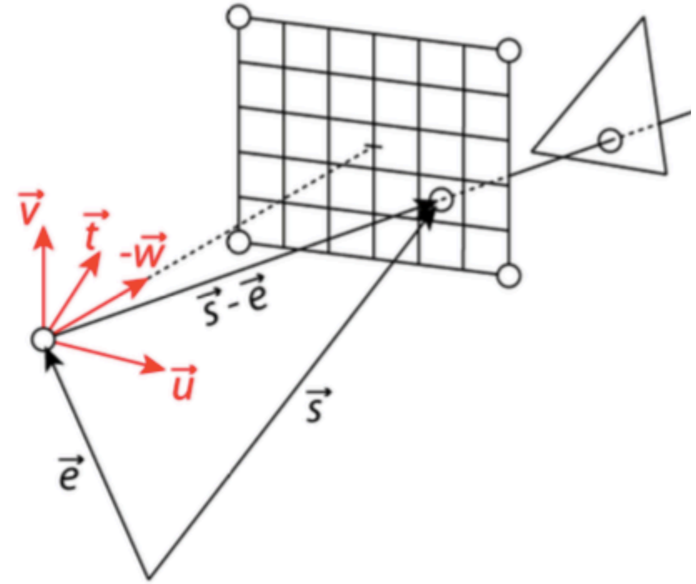
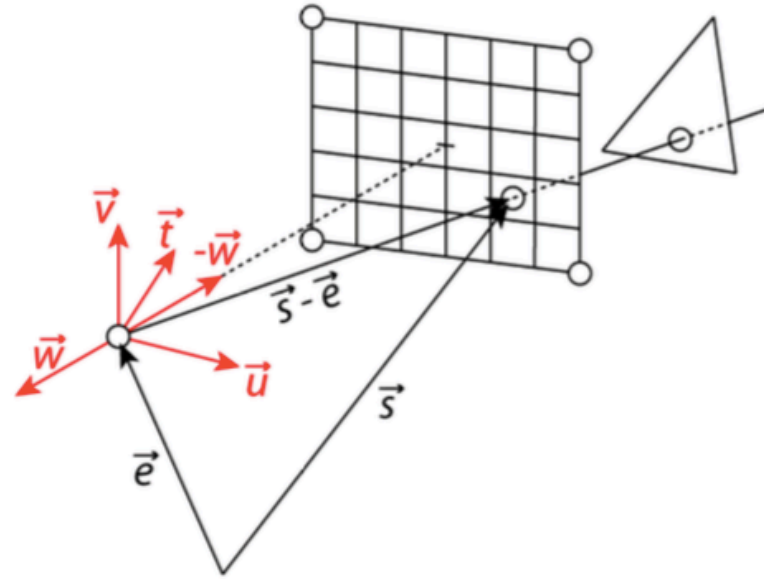


Coordinate system

Notice that we chose $-\vec{w}$ as viewing direction and not $\vec{w}$, in order to get a right handed coordinate system.

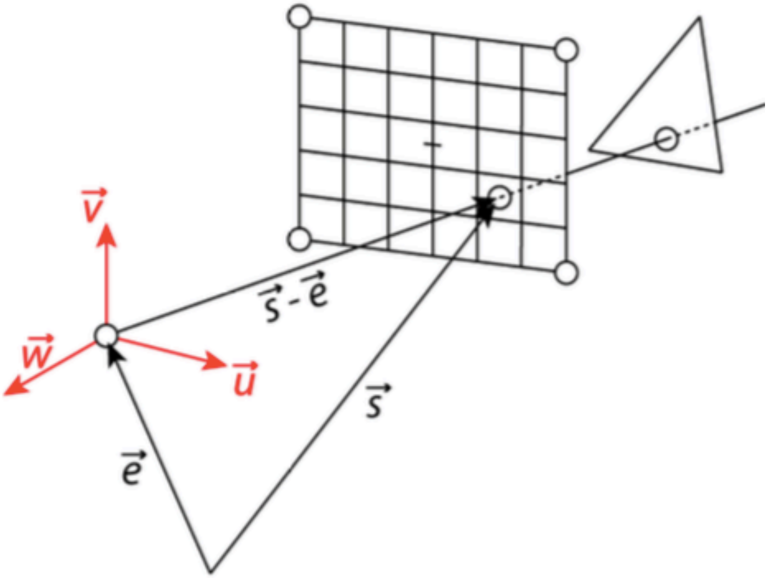


Coordinate system

Normalizing, i.e.

- $\vec{w}/||\vec{w}||$
- $\vec{u}/||\vec{u}||$
- $\vec{v}/||\vec{v}||$

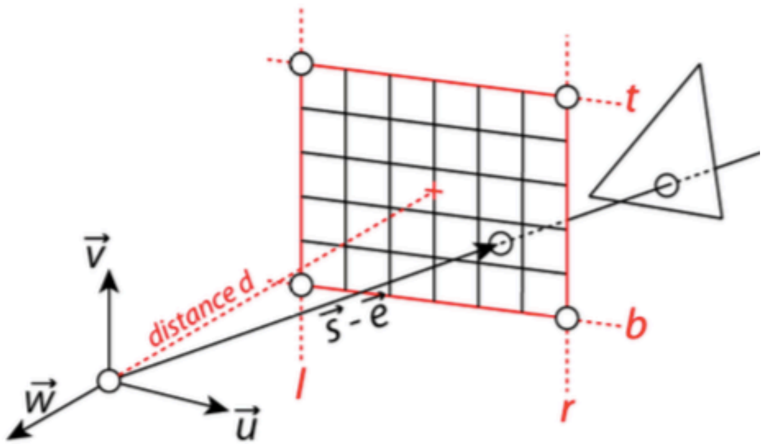
gives us our coordinate system.



Viewing window

With this new coordinate system we can easily define our viewing window:

- left side: $u = l$
- right side: $u = r$
- top: $v = t$
- bottom: $v = b$



Plus the viewing plane at a distance d from the eye/camera:

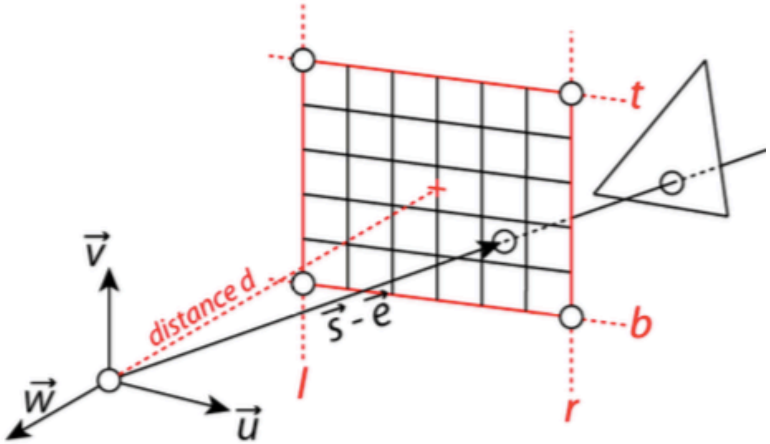
- distance: $-w = d$

Viewing window

Assuming our window has $n_x \times n_y$ pixels, expressing a pixel position (i, j) on the viewing window in our new coordinate system (u, v) can be done with a simple window transformation from $n_x \times n_y$ to $(r - l) \times (t - b)$:

$$u = l + (r - l)(i + 0.5)/n_x$$

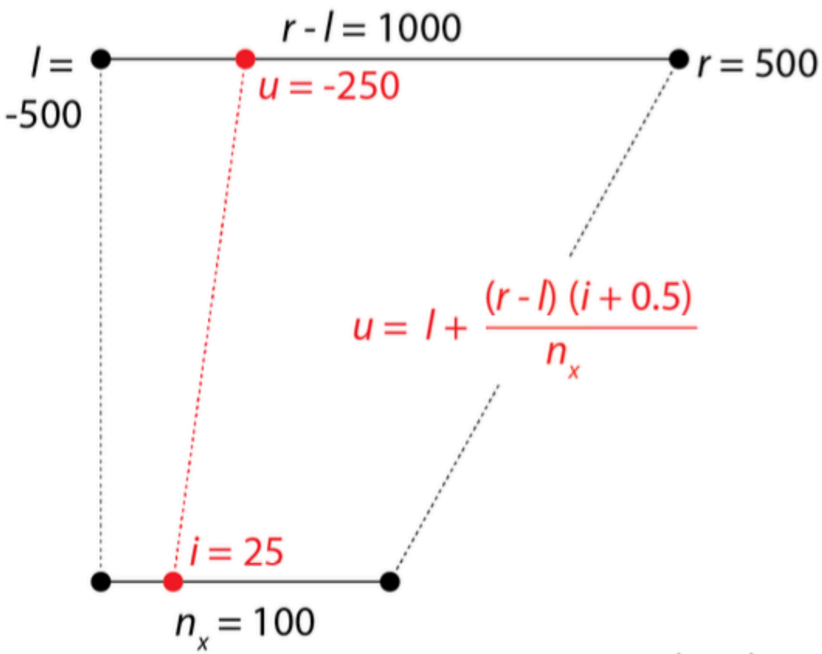
$$v = b + (t - b)(j + 0.5)/n_y$$



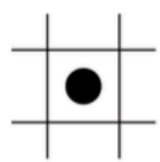
Viewing window

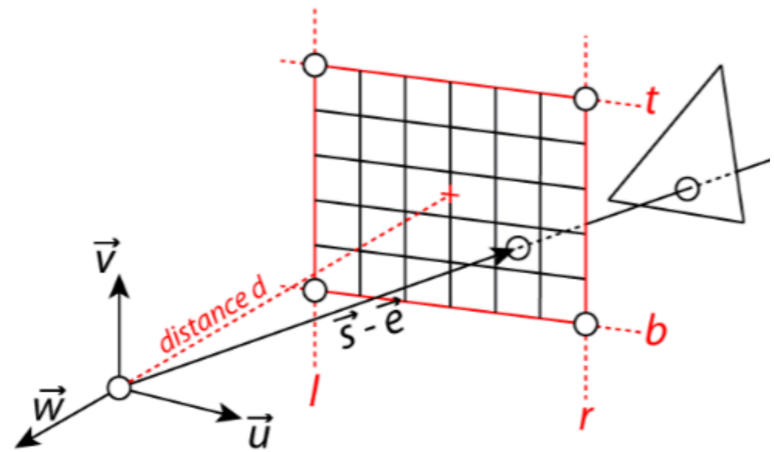
Example for u: Transformation from

$l = -500$, $r = 500$ to $n_x = 100$



Note: we add +0.5 to *i* because we are dealing with pixel centers.





Viewing rays

For perspective views, viewing rays

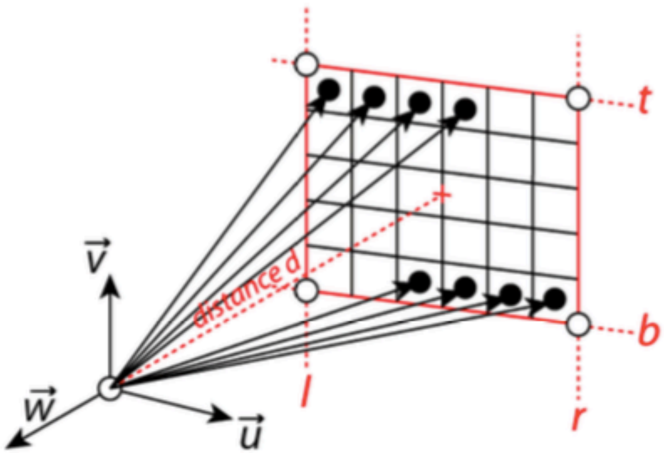
- have the same origin $\vec{e}$
- but different direction

If d denotes the origin's distance to the plane, and u, v are calculated as before, we can write the direction as

- $u\vec{u} + v\vec{v} - d\vec{w}$.

Our viewing ray becomes

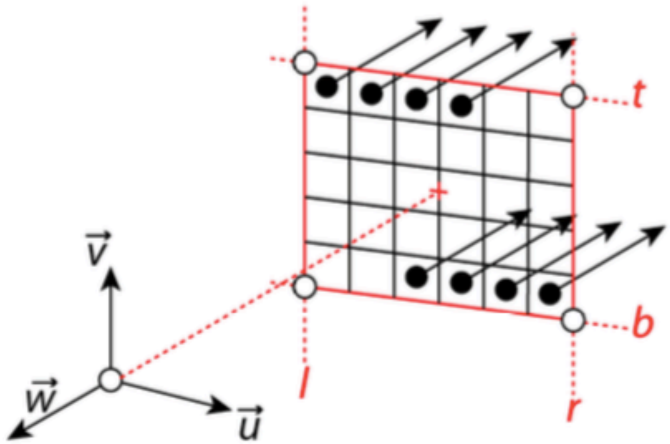
- $\vec{p}(\alpha) = \vec{e} + \alpha(u\vec{u} + v\vec{v} - d\vec{w})$



Viewing rays

For orthographic views, viewing rays

- have the same direction $-\vec{w}$
- but different origin



We get the origin with the previously introduced mapping from (i, j) to (u, v):

$$u = l + (r - l)(i + 0.5)/n_x$$
$$v = b + (t - b)(j + 0.5)/n_y$$

and can write it as $\vec{e} + u\vec{u} + v\vec{v}$.

Our viewing ray becomes

- $\vec{p}(\alpha) = \vec{e} + u\vec{u} + v\vec{v} - \alpha\vec{w}$

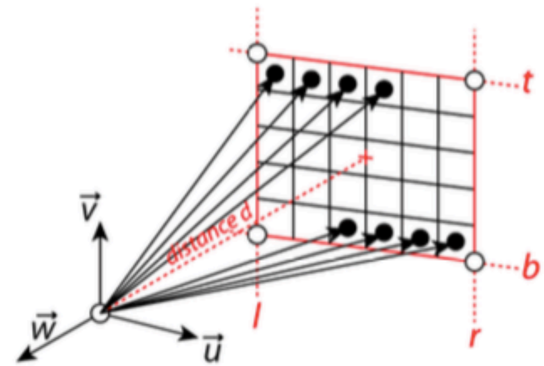
Viewing rays compared

Viewing rays for perspective views

- $\vec{p}(\alpha) = \vec{e} + \alpha(u\vec{u} + v\vec{v} - d\vec{w})$

with

- support vector $\vec{e}$
- direction vector $u\vec{u} + v\vec{v} - d\vec{w}$

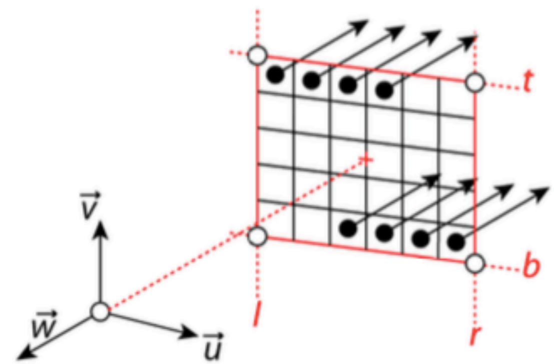


Viewing rays for orthographic views

- $\vec{p}(\alpha) = \vec{e} + u\vec{u} + v\vec{v} - \alpha\vec{w}$

with

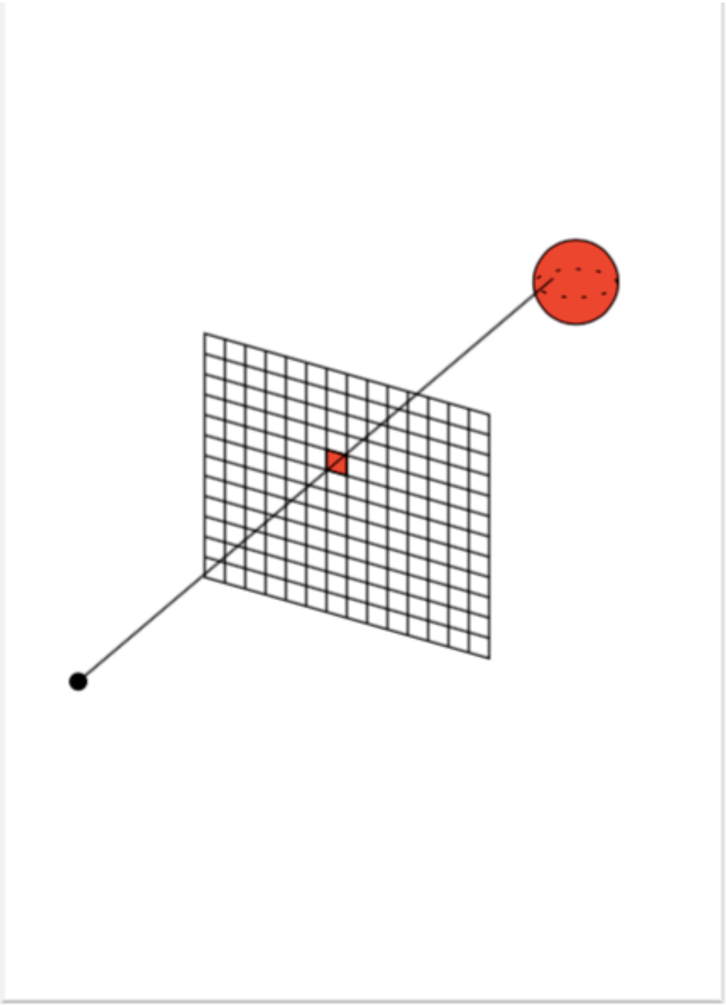
- support vector $\vec{e} + u\vec{u} + v\vec{v}$
- direction vector $-\vec{w}$



A basic ray tracing algorithm

FOR each pixel DO

- compute viewing ray
- find the 1st object hit by the ray and its surface normal $\vec{n}$
- set pixel color to value computed from hit point, light, and $\vec{n}$



Ray-object intersection (implicit surface)

In general, the intersection points of

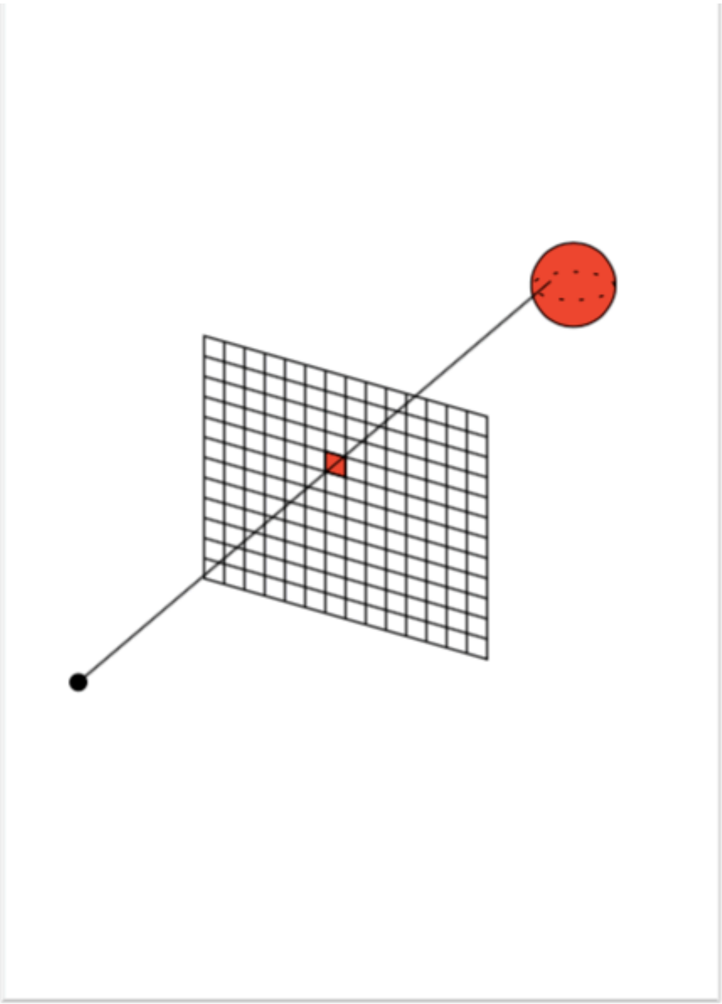
- a ray $\vec{p}(t) = \vec{e} + t\vec{d}$ and
- an implicit surface $f(\vec{p}) = 0$

can be calculated by

$$f(\vec{p}(t)) = 0$$

or

$$f(\vec{e} + t\vec{d}) = 0$$



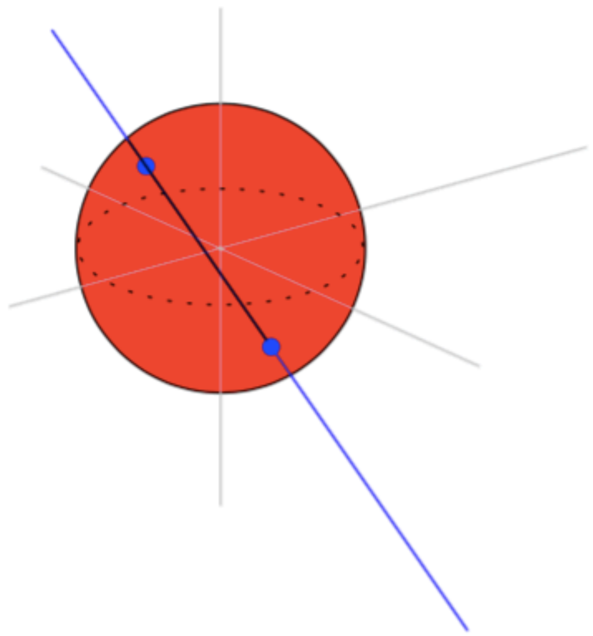
Spheres

The implicit equation for a sphere with center $\vec{c} = (x_c, y_c, z_c)$ and radius R is

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - R^2 = 0$$

or in vector form

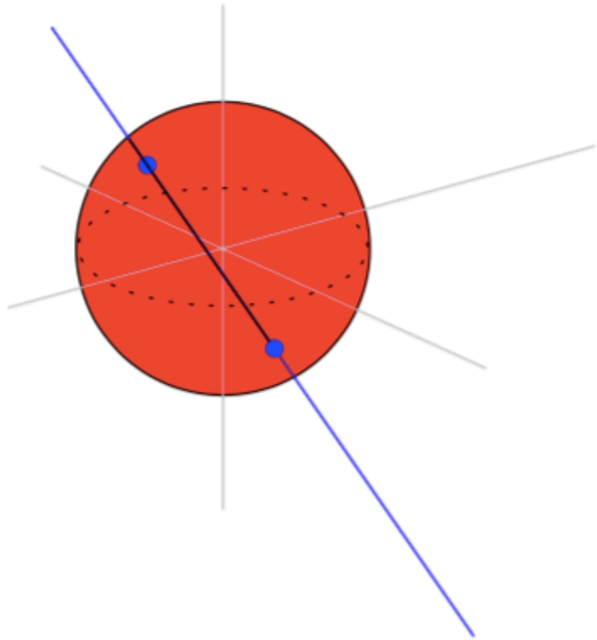
$$(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - R^2 = 0$$



Intersections between rays and spheres

Intersection points have to fullfil

- the ray equation
  $\vec{p}(t) = \vec{e} + t\vec{d}$
- the sphere equation
  $(x - x_c)^2 + (y - y_c)^2$
  $+(z - z_c)^2 - R^2 = 0$



Hence, we get
$(\vec{e} + t\vec{d} - \vec{c}) \cdot (\vec{e} + t\vec{d} - \vec{c}) - R^2 = 0$

which is the same as
$(\vec{d} \cdot \vec{d})t^2 + 2\vec{d} \cdot (\vec{e} - \vec{c})t + (\vec{e} - \vec{c}) \cdot (\vec{e} - \vec{c}) - R^2 = 0$

Intersections between rays and spheres

$$(\vec{d}\cdot\vec{d})t^2+2\vec{d}\cdot(\vec{e}-\vec{c})t+(\vec{e}-\vec{c})\cdot(\vec{e}-\vec{c})-R^2=0$$

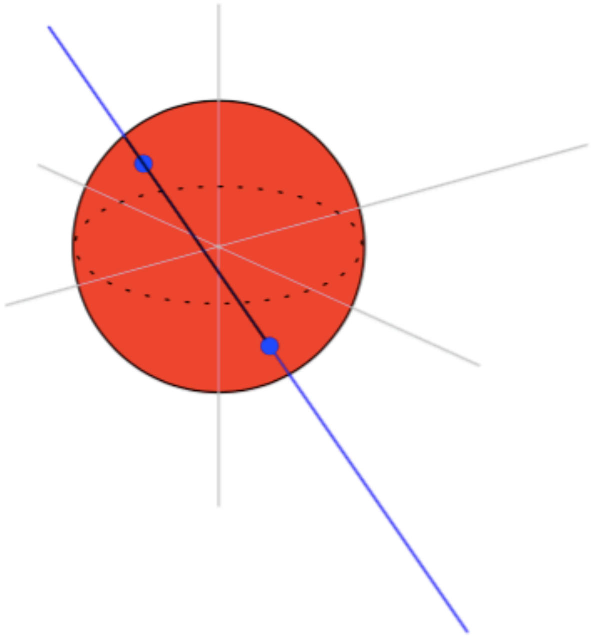
is a quadratic equation in t, i.e.

$$At^2+Bt+C=0$$

that can be solved by

$$t_{1,2}=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

and can have 0, 1, or 2 solutions.



Intersections between rays and planes

Given a ray in parametric form, i.e.

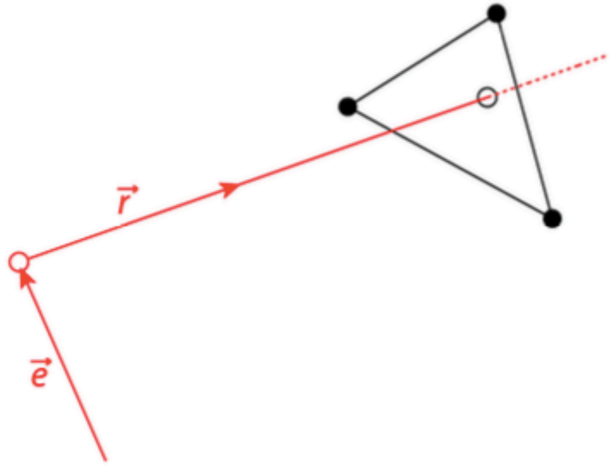
$$\vec{p}(t) = \vec{e} + t\vec{d}$$

and a plane in its implicit form, i.e.

$$(\vec{p} - \vec{p_1}) \cdot \vec{n} = 0$$

we can calculate the intersection point by putting the ray equation into the plane equation and solving for t, i.e.

$$t = \frac{(\vec{p_1} - e) \cdot \vec{n}}{\vec{d} \cdot \vec{n}}$$



Ray-object intersection (parametric surface)

Given a ray in parametric form, i.e.

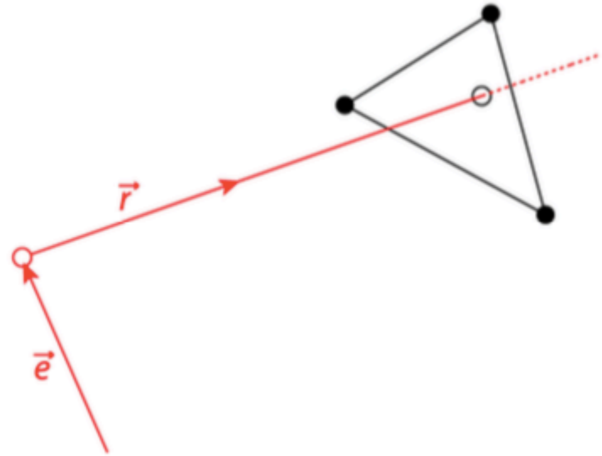
$$\vec{p}(t) = \vec{e} + t\vec{d}$$

and a surface in its parametric form, i.e.

$$f(u, v)$$

we can calculate the intersection point(s) by

$$\vec{e} + t\vec{d} = \vec{f}(u, v)$$



Ray-object intersection (parametric surface)

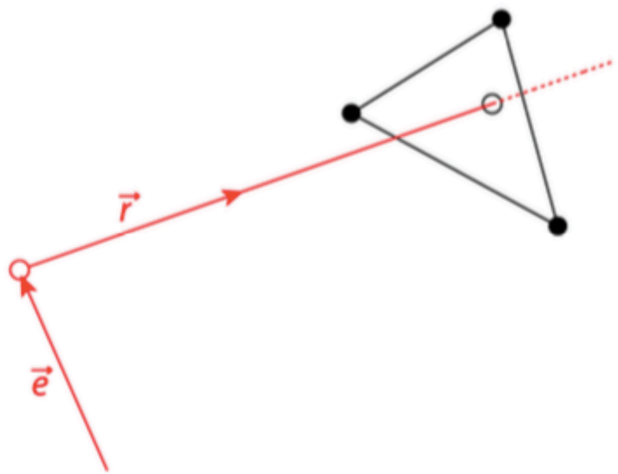
Notice that

$$\vec{e} + t\vec{d} = \vec{f}(u, v)$$

or

$$x_e + tx_d = f(u, v)$$
$$y_e + ty_d = f(u, v)$$
$$z_e + tz_d = f(u, v)$$

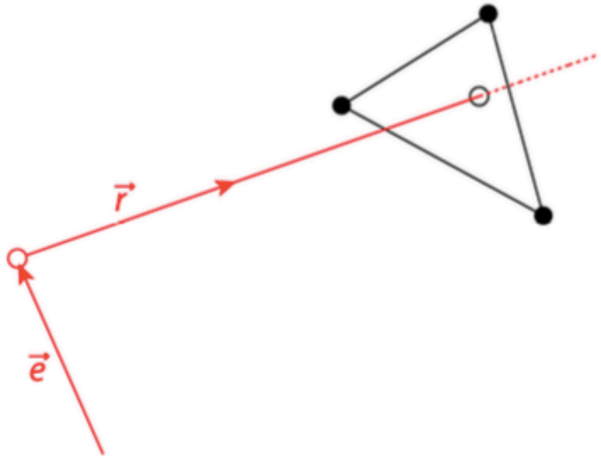
represents 3 equations
with 3 unknowns (t, u, v),
i.e. a linear equation system.



Ray-triangle intersection

This comes in very handy for ray-triangle intersections:

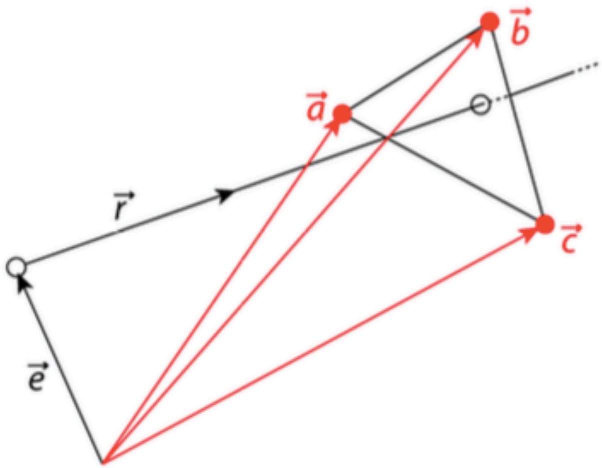
- We first calculate the intersection point of the ray with the plane defined by the triangle.
- Then we check if this point is within the triangle or not.



Plane specification

Recall that the plane V through the points $\vec{a}$, $\vec{b}$, and $\vec{c}$ can be written as

$$p(\vec{\beta}, \gamma) = \vec{a} + \beta(\vec{b} - \vec{a}) + \gamma(\vec{c} - \vec{a})$$



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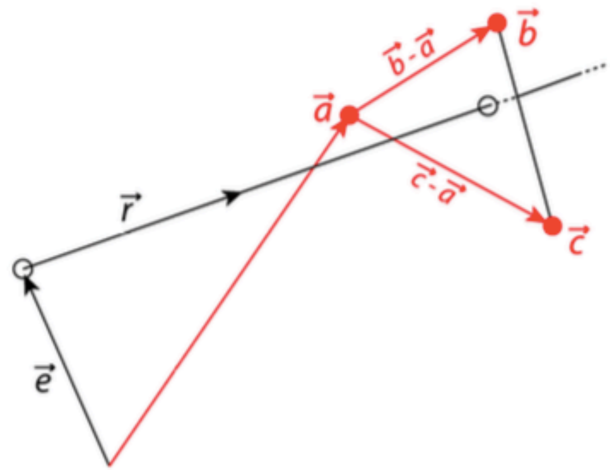

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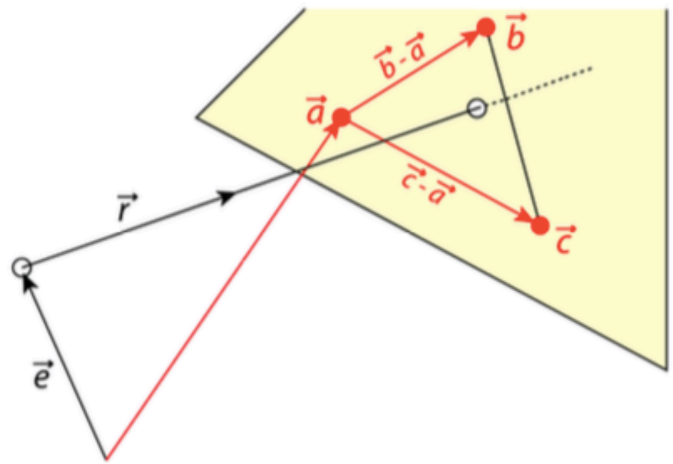


Notice that these are barycentric coordinates (if the direction vectors are chosen appropriately)

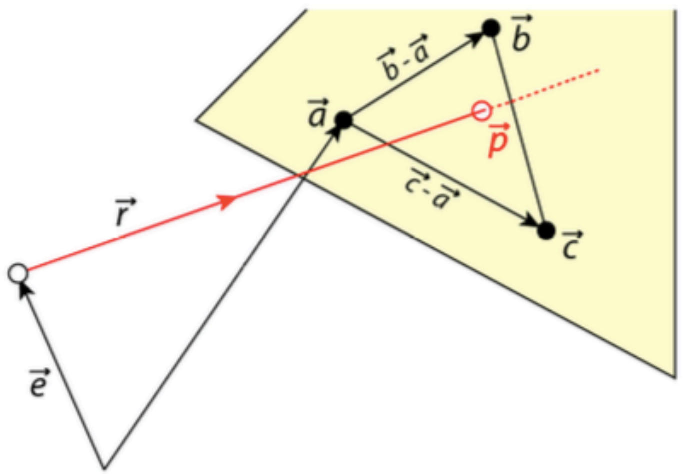
Ray-plane specification

Again, intersection points must fullfil the plane and the ray equation.

Hence, we get

$$\vec{e} + t\vec{d} = \vec{a} + \beta(\vec{b} - \vec{a}) + \gamma(\vec{c} - \vec{a})$$

That give us . . .



Ray-plane specification

…the following three equations

$$\begin{aligned} x_e + t x_d &= x_a + \beta(x_b - x_a) + \gamma(x_c - x_a) \\ y_e + t y_d &= y_a + \beta(y_b - y_a) + \gamma(y_c - y_a) \\ z_e + t z_d &= z_a + \beta(z_b - z_a) + \gamma(z_c - z_a) \end{aligned}$$

which can be rewritten as

$$\begin{aligned} (x_a - x_b)\beta + (x_a - x_c)\gamma + x_d t &= x_a - x_e \\ (y_a - y_b)\beta + (y_a - y_c)\gamma + y_d t &= y_a - y_e \\ (z_a - z_b)\beta + (z_a - z_c)\gamma + z_d t &= z_a - z_e \end{aligned}$$

or as

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

Ray-plane specification

If we write

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

as

$$A \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

then we see that

$$\begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = A^{-1} \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

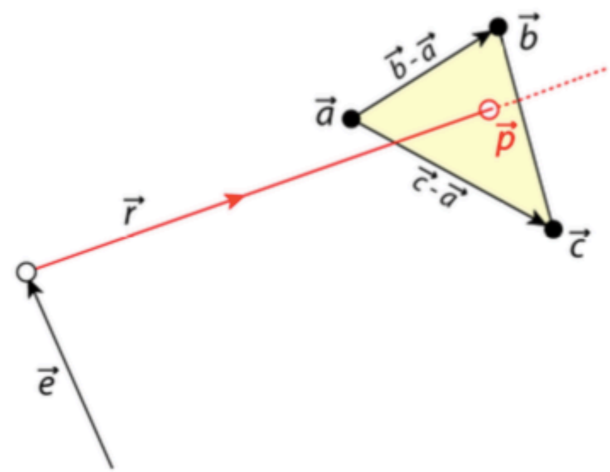
Rays: parametric representation

We can use t to calculate the intersection point $\vec{p}(t)$ (or β, γ to calculate $\vec{p}(\beta, \gamma)$).

But first, we can use β and γ to verify if it is inside of the triangle or not:

- $\beta > 0$
- $\gamma > 0$
- $\beta + \gamma < 1$

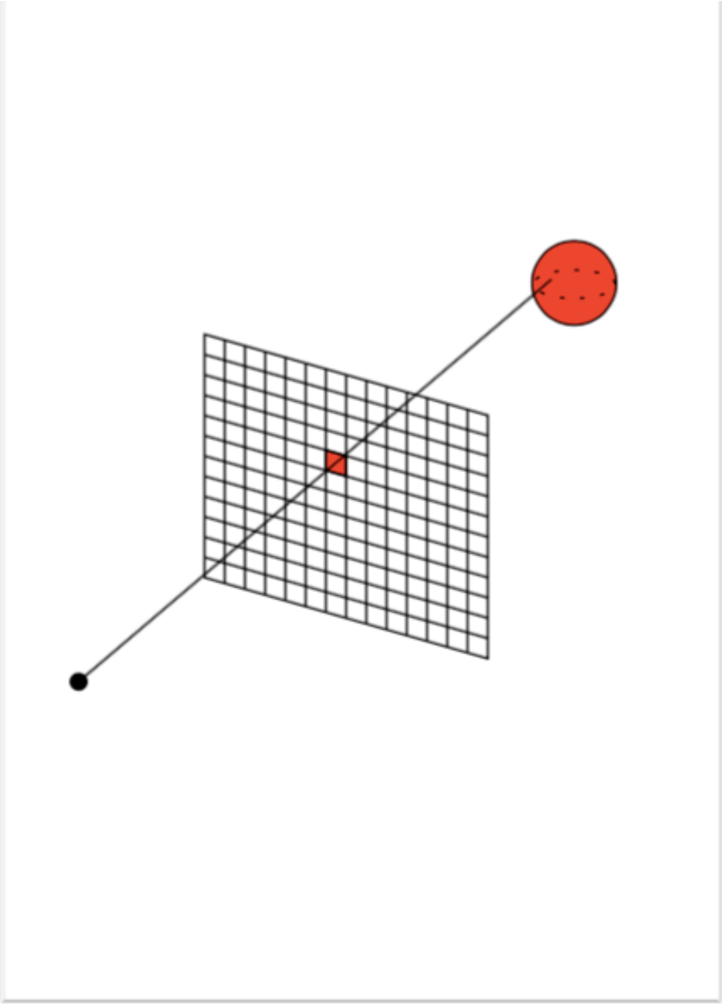
because we can interpret these as barycentric coordinates.



A basic ray tracing algorithm

FOR each pixel DO

- compute viewing ray
- find the 1st object hit by the ray and its surface normal $\vec{n}$
- set pixel color to value computed from hit point, light, and $\vec{n}$



Ideal specular or mirror reflection

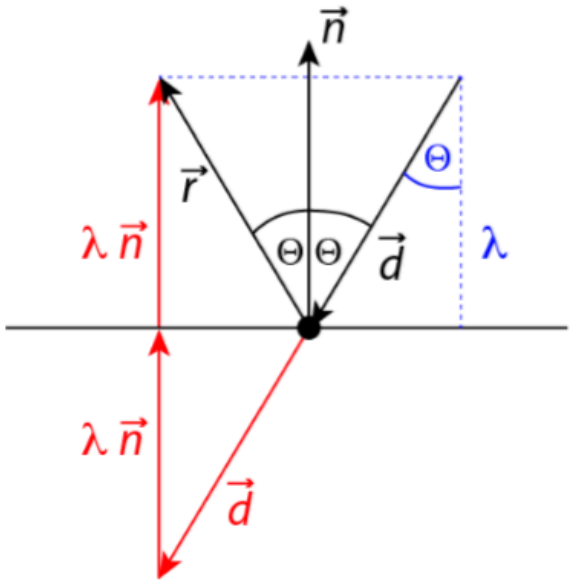
We know this from Phong reflection with light vector l:

$$\vec{r} = -\vec{l} + 2(\vec{l} \cdot \vec{n})\vec{n}$$

But be careful with the direction of $\vec{d}$ when calculating the reflection vector for mirroring:

$$\vec{r} = \vec{d} - 2(\vec{d} \cdot \vec{n})\vec{n}$$

Also: we need to include a max. recursion depth to avoid "infinite bouncing" of rays.

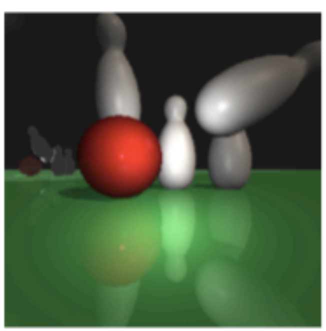


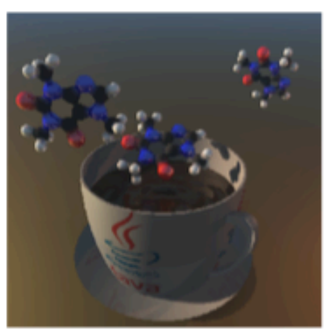
$$\lambda = \cos\theta \|\vec{n}\| \|\vec{d}\| = -\vec{d} \cdot \vec{n}$$

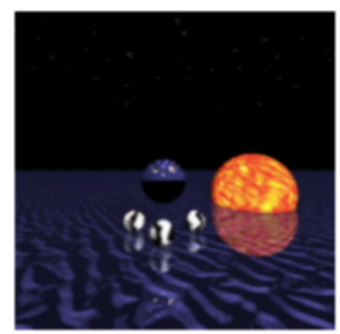
A basic ray tracing algorithm

FOR each pixel DO

- compute viewing ray
  - IF (ray hits an object with $t \in [0, \infty)$) THEN
    - Compute $\vec{n}$
    - Evaluate shading model and set pixel to that color
  - ELSE
    - set pixel color to background color







Shading model

Remember our shading model:

$$c = c_r(c_a + c_l \max(0, n \cdot l)) + c_l(\vec{h} \cdot \vec{n})^p$$

with

- Ambient shading
- Lambertian shading
- Phong shading

and Gouraud interpolation.