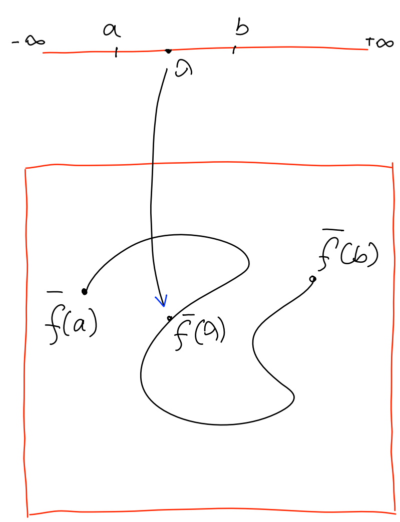
Topic 5:

3D Objects

- General curves & surfaces in 3D
- Normal vectors, surface curves & tangent planes
- Implicit surface representations
- Example surfaces:
  surfaces of revolution, bilinear patches, quadrics

Reminder: Curves in 2D

Space of the curve parameter (1D)



$\lambda \in (a,b) \subseteq \mathbb{R}$

$\bar{f}(\lambda) \in \mathbb{R}^2$

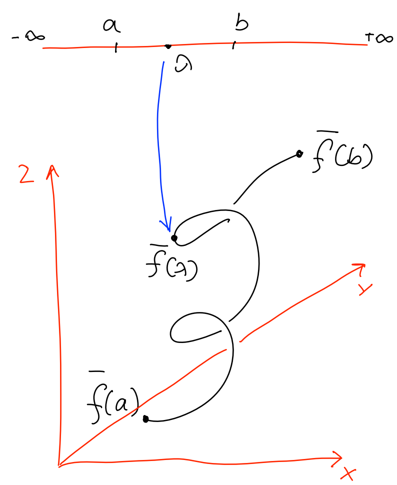
where

$\bar{f}(\lambda) = \left(x(\lambda), y(\lambda)\right)$

Space of the curve (2D)

Curves in 3D

Space of the curve parameter (1D)



$\lambda \in (a, b) \subseteq \mathbb{R}$

$\bar{f}(\lambda) \in \mathbb{R}^3$

where

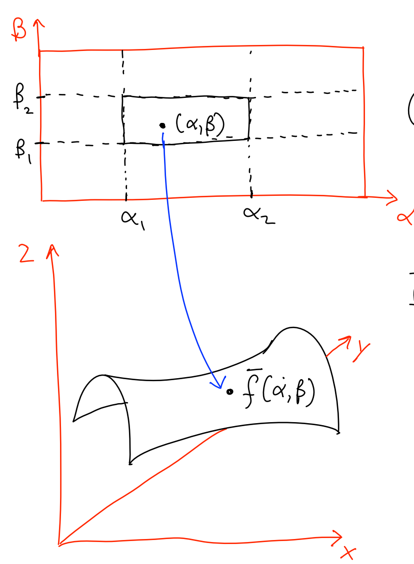
$\bar{f}(\lambda) =$

$\big(x(\lambda), y(\lambda), z(\lambda)\big)$

Space of the curve (3D)

Surfaces in 3D

Space of the surface parameters



$(\alpha, \beta) \in [\alpha_1, \alpha_2] \times [\beta_1, \beta_2]$

$\bar{f}(\alpha, \beta) \in \mathbb{R}^3$

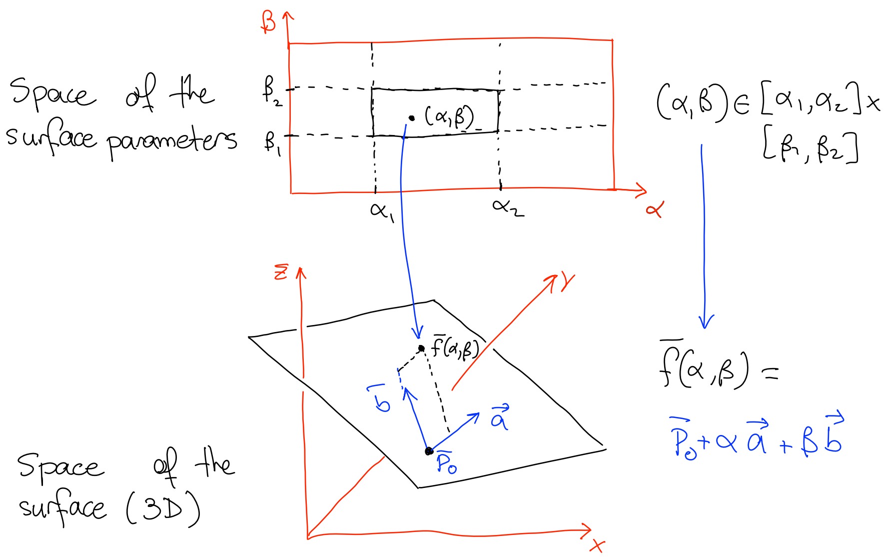
where

$\bar{f}(\alpha, \beta) =$

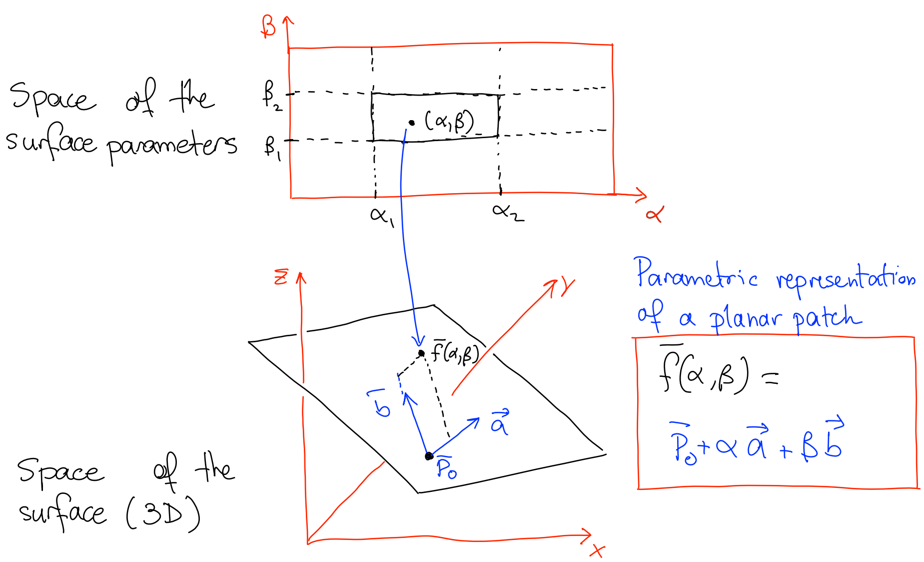
$\big(x(\alpha, \beta),$
$y(\alpha, \beta),$
$z(\alpha, \beta)\big)$

Space of the surface (3D)

Surface Example: Planes in 3D



Surface Example: Planes in 3D



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Normal Vector of a Plane

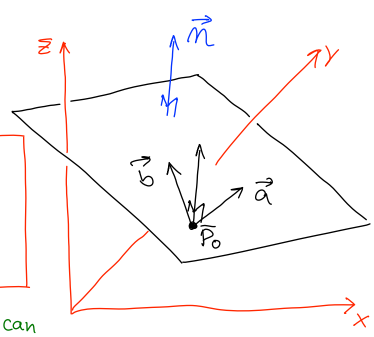
Definition

The normal vector of a plane is any vector that is perpendicular to it (i.e. normal $\vec{n}$ is such that $\vec{n} \cdot \vec{a} = \vec{n} \cdot \vec{b} = 0$)

Normal vector of a plane

$$\boxed{\vec{n} = \vec{a} \times \vec{b}}$$

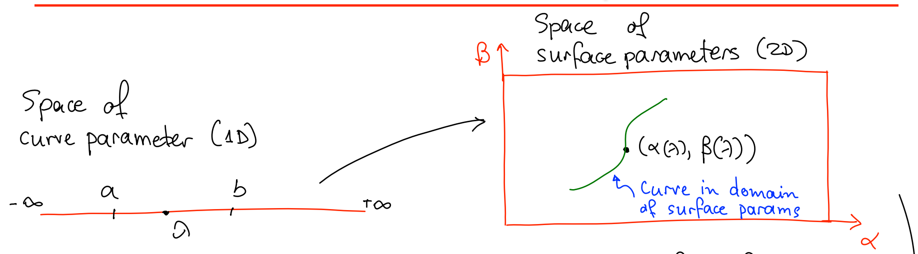
reminder: cross product can be implemented as a matrix-vector multiplication

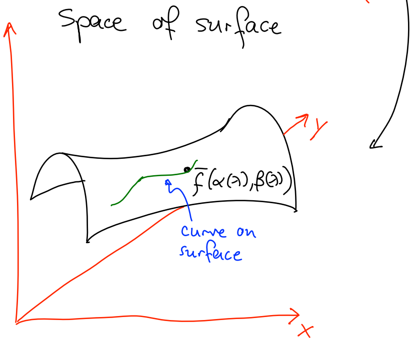


Parametric representation of a planar patch

$$\boxed{f(\alpha, \beta) = P_0 + \alpha \vec{a} + \beta \vec{b}}$$

Surface Curves: Parametric Representation





Parametric expressions

surface

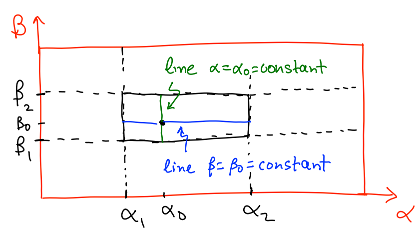
$$\bar{f}(\alpha,\beta) = \big(x(\alpha,\beta),\, y(\alpha,\beta),\, z(\alpha,\beta)\big)$$

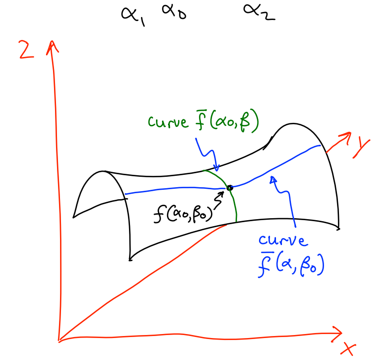
$$\bar{\gamma}(\lambda) = \Big(x\big(\alpha(\lambda),\beta(\lambda)\big),\; y\big(\alpha(\lambda),\beta(\lambda)\big),\; z\big(\alpha(\lambda),\beta(\lambda)\big)\Big)$$

curve on
surface

The Coordinate Curves of a Surface

Space of the
surface parameters





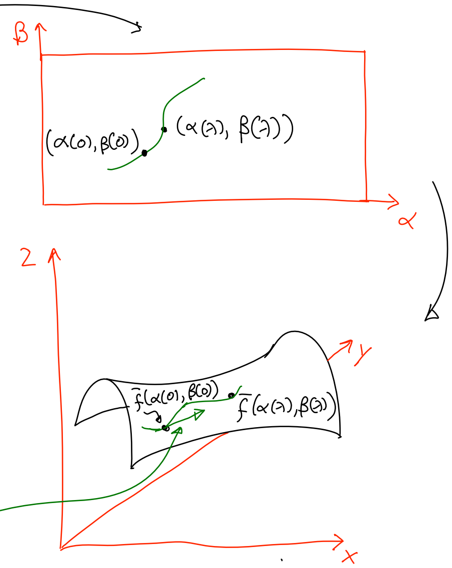
Space of the
surface (3D)

The coordinate curves
through $\bar{f}(\alpha_0,\beta_0)$
are the curves

$\bar{f}(\alpha_0,\beta),\ \beta\in[\beta_1,\beta_2]$

$\bar{f}(\alpha,\beta_0),\ \alpha\in[\alpha_1,\alpha_2]$

The Tangent Vector of a Surface Curve



Parametric representation of surface curve

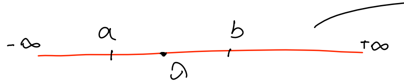
$$\bar{\gamma}(\lambda) = \Big(x(\alpha(\lambda), \beta(\lambda)),\ y(\alpha(\lambda), \beta(\lambda)),\ z(\alpha(\lambda), \beta(\lambda)) \Big)$$

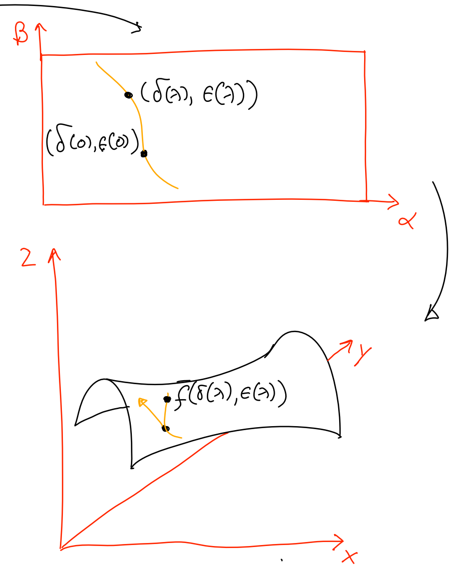
1st order Taylor series approx.

$$\bar{\gamma}(\lambda) \simeq \bar{\gamma}(0) + \lambda \frac{d\bar{\gamma}}{d\lambda}(0)$$

3D tangent vector at $\bar{\gamma}(0) = \bar{f}(\alpha(0), \beta(0))$

The Tangent Vector of a Surface Curve





Parametric representation of a second surface curve

$$\bar{\rho}(\lambda) = \Big(x(\delta(\lambda), \epsilon(\lambda)),\ y(\delta(\lambda), \epsilon(\lambda)),\ z(\delta(\lambda), \epsilon(\lambda)) \Big)$$

1st order Taylor series approx.

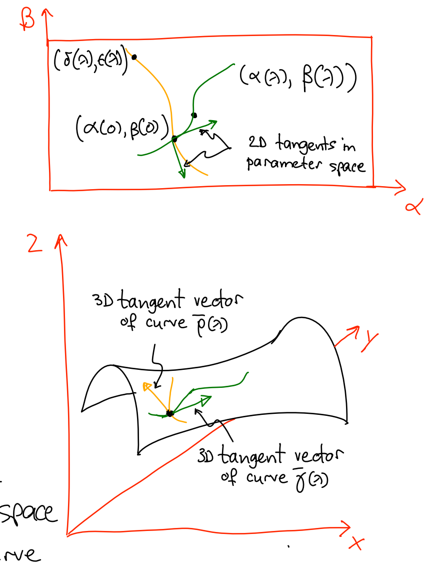
$$\bar{\rho}(\lambda) \simeq \bar{\rho}(0) + \lambda \frac{d\bar{\rho}}{d\lambda}(0)$$

3D tangent vector at $\bar{\rho}(0) = \bar{f}(\delta(0), \epsilon(0))$

The Set of All Curve Tangents at a Point

Question: What is the relation between

- the 3D tangent of a surface curve
- the corresponding 2D curve in the parameter space
- the 3D surface on which the curve lies?



Ans (intuitive)

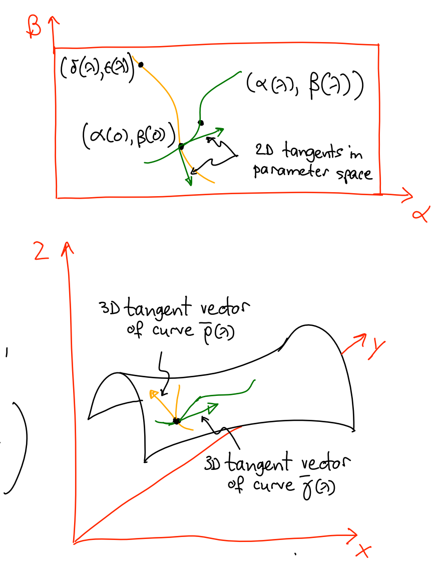
- 3D tangent depends on 2D tangent in parameter space
- The set of all possible curve tangents spans a plane

The Set of All Curve Tangents at a Point

Surface: $\bar{f}(\alpha,\beta) = (x(\alpha,\beta), y(\alpha,\beta), z(\alpha,\beta))$

Curve: $\bar{\gamma}(\lambda) = \big(x(\alpha(\lambda), \beta(\lambda)),\ y(\alpha(\lambda), \beta(\lambda)),\ z(\alpha(\lambda), \beta(\lambda))\big)$

3D tangent: $\frac{d\bar{\gamma}}{d\lambda}$



applying the chain rule, we get

$$\frac{d\bar{\gamma}}{d\lambda} = \left(\frac{\partial x}{\partial \alpha}\cdot\frac{d\alpha}{d\lambda} + \frac{\partial x}{\partial \beta}\cdot\frac{d\beta}{d\lambda},\ \frac{\partial y}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial y}{\partial \beta}\cdot\frac{d\beta}{d\lambda},\ \frac{\partial z}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial z}{\partial \beta}\cdot\frac{d\beta}{d\lambda} \right)$$

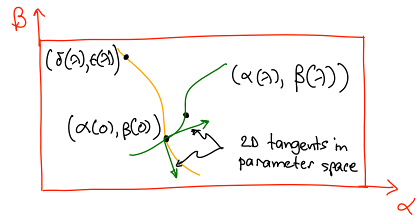
$$= \frac{\partial \bar{f}}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar{f}}{\partial \beta}\frac{d\beta}{d\lambda}$$

The Set of All Curve Tangents at a Point

Surface: $\bar{f}(\alpha,\beta) = (x(\alpha,\beta), y(\alpha,\beta), z(\alpha,\beta))$

Curve: $\bar{\gamma}(\lambda) = \big(x(\alpha(\lambda),\beta(\lambda)),\ y(\alpha(\lambda),\beta(\lambda)),\ z(\alpha(\lambda),\beta(\lambda))\big)$

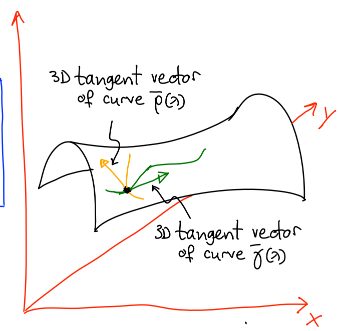
3D tangent: $\dfrac{d\bar{\gamma}}{d\lambda}$



3D vectors that depend only on the surface, NOT the curve itself!

scalars that depend on the curve but not the surface!

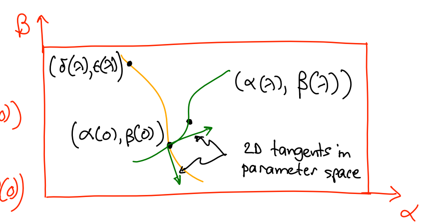
$$\frac{d\bar{\gamma}}{d\lambda} = \frac{\partial \bar{f}}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar{f}}{\partial \beta}\frac{d\beta}{d\lambda}$$



The Tangent Plane of the Surface at a Point

The tangent of every surface curve through a point $\bar{f}(\alpha(0),\beta(0))$ is a linear combination of the two 3D vectors $\frac{\partial \bar{f}}{\partial \alpha}(\alpha(0),\beta(0))$ and $\frac{\partial \bar{f}}{\partial \beta}(\alpha(0),\beta(0))$

$\Rightarrow$ they span a plane called the tangent plane at $\bar{f}(\alpha(0),\beta(0))$



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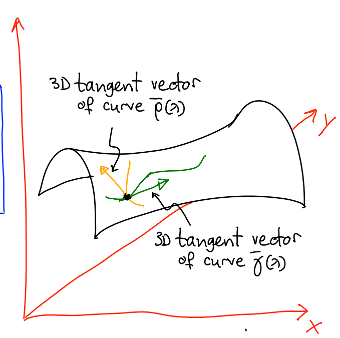
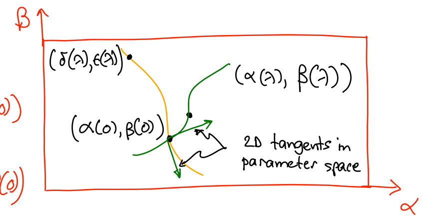


The Tangent Plane of the Surface at a Point

The tangent of every surface curve through a point $\bar f(\alpha(0), \beta(0))$ is a linear combination of the two 3D vectors $\frac{\partial \bar f}{\partial \alpha}(\alpha(0), \beta(0))$ and $\frac{\partial \bar f}{\partial \beta}(\alpha(0), \beta(0))$

$\Rightarrow$ they span a plane called the tangent plane at $\bar f(\alpha(0), \beta(0))$



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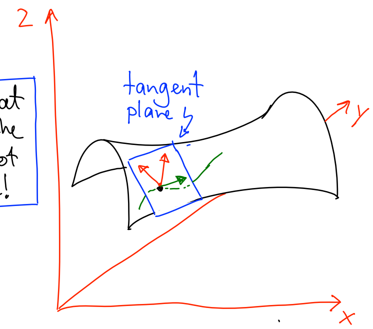
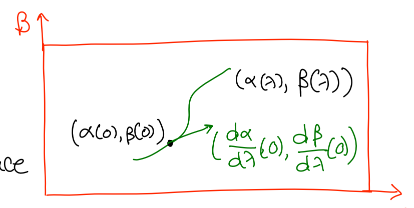


$$\frac{d\bar\gamma}{d\lambda} = \frac{\partial \bar f}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar f}{\partial \beta}\frac{d\beta}{d\lambda}$$

The Tangent of a Surface Curve: Geometry

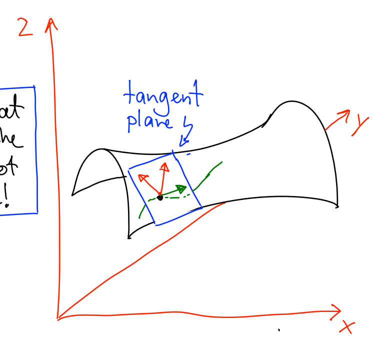
Question: What do the scalars $\frac{d\alpha}{d\lambda}, \frac{d\beta}{d\lambda}$ represent geometrically?

Ans: They define the 2D tangent in parameter space



3D vectors that depend only on the surface, NOT the curve itself!

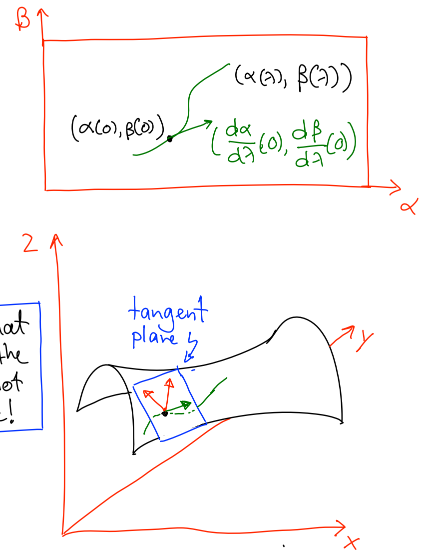
scalars that depend on the curve but not the surface!



$$\frac{d\bar\gamma}{d\lambda} = \frac{\partial \bar f}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar f}{\partial \beta}\frac{d\beta}{d\lambda}$$

The Tangent of a Surface Curve: Geometry

The tangent at parameter point $(\alpha(0), \beta(0))$ determines the linear combination of vectors $\frac{\partial \bar{f}}{\partial \alpha}$ and $\frac{\partial \bar{f}}{\partial \beta}$



3D vectors that depend only on the surface, NOT the curve itself!

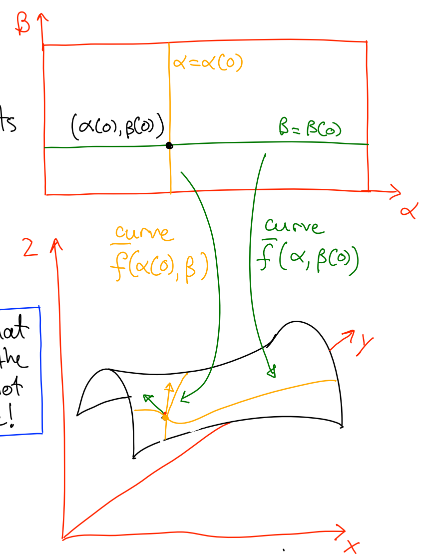
scalars that depend on the curve but not the surface!

$$\frac{d\bar{\gamma}}{d\lambda} = \frac{\partial \bar{f}}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar{f}}{\partial \beta}\frac{d\beta}{d\lambda}$$

The Tangent of a Surface Curve: Geometry

Question: What do the vectors $\frac{\partial \bar{f}}{\partial \alpha}, \frac{\partial \bar{f}}{\partial \beta}$ represent geometrically?

Ans: They are the 3D tangents of the coordinate curves at $(\alpha(0), \beta(0))$. (Exercise: prove this!)



3D vectors that depend only on the surface, NOT the curve itself!

scalars that depend on the curve but not the surface!

$$\frac{d\bar{\gamma}}{d\lambda} = \frac{\partial \bar{f}}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar{f}}{\partial \beta}\frac{d\beta}{d\lambda}$$

The Surface Normal at a Point

The tangent plane at $\bar{f}(\alpha,\beta)$ is the plane spanned by vectors

$$\frac{\partial \bar{f}}{\partial \alpha}(\alpha,\beta) \text{ and } \frac{\partial \bar{f}}{\partial \beta}(\alpha,\beta)$$

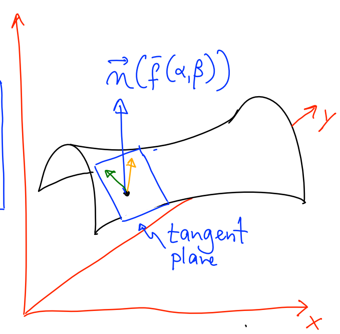
The surface normal at $\bar{f}(\alpha,\beta)$ is the normal to the tangent plane:

$$\vec{n}(\bar{f}(\alpha,\beta)) = \frac{\partial \bar{f}}{\partial \alpha}(\alpha,\beta) \times \frac{\partial \bar{f}}{\partial \beta}(\alpha,\beta)$$

3D vectors that depend only on the surface, NOT the curve itself!

scalars that depend on the curve but not the surface!

$$\frac{d\bar{\gamma}}{d\lambda} = \frac{\partial \bar{f}}{\partial \alpha}\frac{d\alpha}{d\lambda} + \frac{\partial \bar{f}}{\partial \beta}\frac{d\beta}{d\lambda}$$



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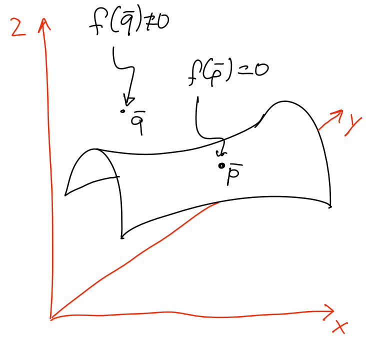
Representing Surfaces by an Implicit Function

- Representation consists of a scalar function $f: \mathbb{R}^3 \to \mathbb{R}$ (called the Implicit Function)
- Surface defined as the set
  $$\boxed{\alpha_0 = \{\bar{p} \in \mathbb{R}^3 \mid f(\bar{p}) = 0\}}$$
- Intuitively, f can be thought of as measuring a "distance" to the surface
- Typically, f does NOT measure Euclidean distance to the surface



Example: The Implicit Function of a Plane

- Representation consists of a scalar function $f: \mathbb{R}^3 \to \mathbb{R}$
- Surface defined as the set
  $$\boxed{\alpha_0 = \{\bar{p} \in \mathbb{R}^3 \mid f(\bar{p}) = 0\}}$$
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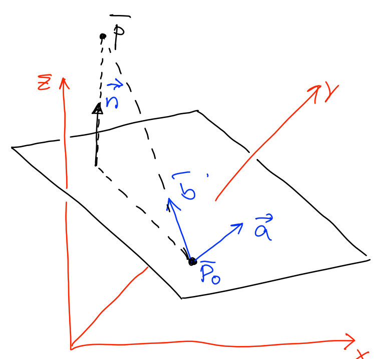


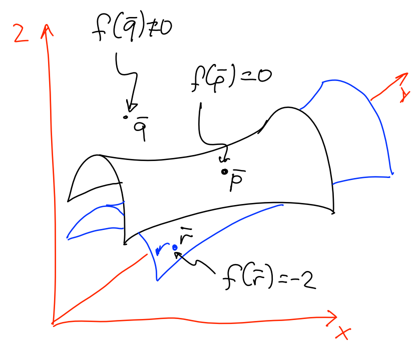
Example: Implicit function for a plane through $\bar{p}_0$ with normal $\vec{n}$:

$$f(\bar{p}) = (\bar{p} - \bar{p}_0) \cdot \vec{n}$$

The Level Sets of an Implicit Function

- Representation consists of a scalar function $f: \mathbb{R}^3 \to \mathbb{R}$
- Surface defined as the set $\alpha_0 = \{ \bar{p} \in \mathbb{R}^3 \mid f(\bar{p}) = 0 \}$
- Intuitively, f can be thought of as measuring a "distance" to the surface

$$\boxed{\text{Given a value } c \in \mathbb{R}, \text{ the set } \alpha_c = \{ \bar{p} \in \mathbb{R}^3 \mid f(\bar{p}) = c \} \text{ also defines a surface called the } c\text{-level set of } f}$$



Example: Point $\bar{r}$ belongs to the -2-level set of f.

Surface Normals from the Implicit Function

If $\bar{p}$ is a surface point, the normal at $\bar{p}$ is given by

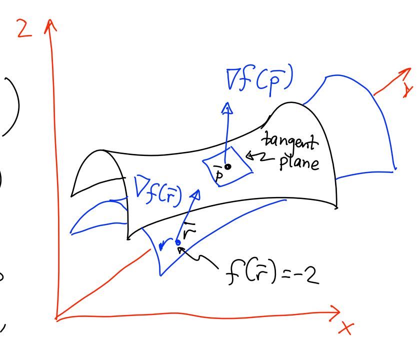
$$\boxed{\vec{n}(\bar{p}) = \nabla f(\bar{p})}$$

where

$$\nabla f(\bar{p}) = \left(\frac{\partial f}{\partial x}(\bar{p}), \frac{\partial f}{\partial y}(\bar{p}), \frac{\partial f}{\partial z}(\bar{p}) \right)$$

(a.k.a. <u>the gradient of f</u>)

Note: the above definition works for any level set: if $\bar{r} \in \alpha_c$, the normal of the c-level-set at point $\bar{r}$ is given by $\nabla f(\bar{r})$



Surface Normals from the Implicit Function

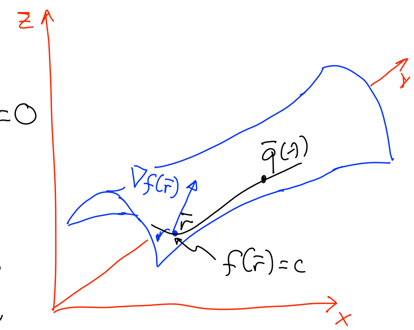
Proof: Let $\bar{q}(\lambda) = (x(\lambda), y(\lambda), z(\lambda))$ be a curve on the surface α_c with $\bar{q}(0) = \bar{r}$.

$$\Leftrightarrow \quad \forall \lambda \quad f(\bar{q}(\lambda)) = c$$

$$\Leftrightarrow \quad \frac{df}{d\lambda}(\bar{q}(0)) = 0$$

$$\Leftrightarrow \quad \frac{\partial f}{\partial x} \frac{dx}{d\lambda} + \frac{\partial f}{\partial y} \cdot \frac{dy}{d\lambda} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial \lambda} = 0$$

$$\Leftrightarrow \quad \nabla f(\bar{q}(0)) \cdot \frac{d\bar{q}}{d\lambda}(0) = 0$$



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Surface Normals from the Implicit Function

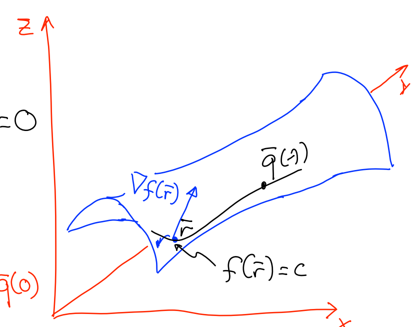
Proof: Let $\bar{q}(\lambda) = (x(\lambda), y(\lambda), z(\lambda))$ be a curve on the surface α_c with $\bar{q}(0) = \bar{r}$.

$$\Leftrightarrow \quad \forall \lambda \quad f(\bar{q}(\lambda)) = c$$

$$\Leftrightarrow \quad \frac{df}{d\lambda}(\bar{q}(0)) = 0$$

$$\Leftrightarrow \quad \frac{\partial f}{\partial x} \frac{dx}{d\lambda} + \frac{\partial f}{\partial y} \cdot \frac{dy}{d\lambda} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial \lambda} = 0$$

$$\Leftrightarrow \quad \underbrace{\nabla f(\bar{q}(0))}_{\text{gradient at } \bar{r}} \cdot \underbrace{\frac{d\bar{q}}{d\lambda}(0)}_{\text{3D tangent at } \bar{q}(0)} = 0$$



since the above orthogonality holds for any curve in α_c through $\bar{r}$, the gradient must be perpendicular to the tangent plane

(QED).

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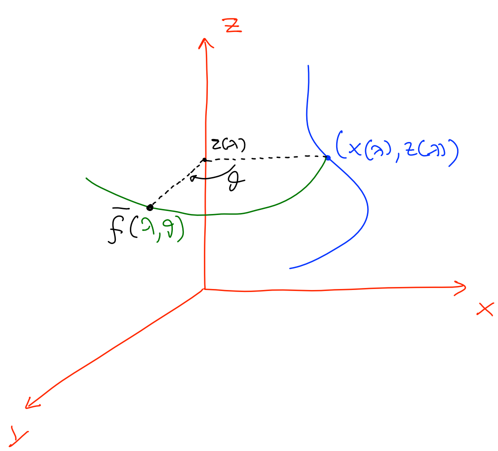
Surfaces of Revolution: Basic Construction



Conceptual steps:

1. Define a 2D curve on the xz-plane:
2. Rotate it about the z-axis

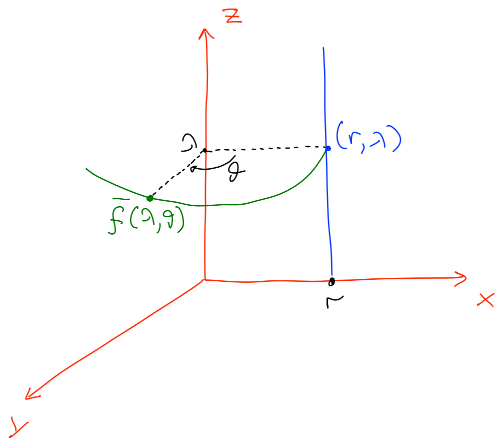
Another equivalent view:

- Point $(x(\lambda), y(\lambda))$ will trace a circle in xy-plane
- The circle's radius is equal to $x(\lambda)$
- θ ranges in $[0, 2\pi)$

Parametric representation

$$\bar{f}(\lambda,\theta) = \left(x(\lambda)\cos\theta,\ x(\lambda)\sin\theta,\ z(\lambda) \right)$$

Example: The Cylinder



Question: how do we express the cylinder of radius r?

Ans:

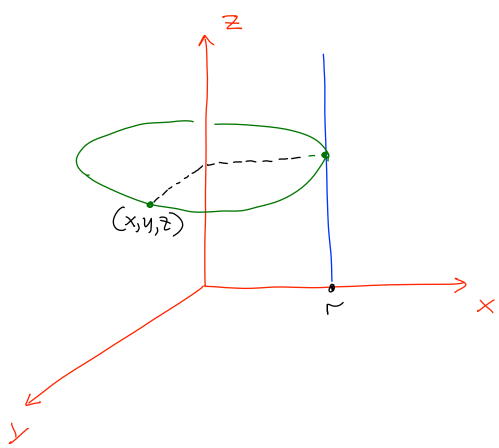
$$(x(\lambda), z(\lambda)) = (r, \lambda)$$

So

$$\bar{f}(\lambda,\theta) = (r\cos\theta, r\sin\theta, \lambda)$$

Parametric representation
$$\bar{f}(\lambda,\theta) = (x(\lambda)\cos\theta, x(\lambda)\sin\theta, z(\lambda))$$

Example: Implicit Function of the Cylinder



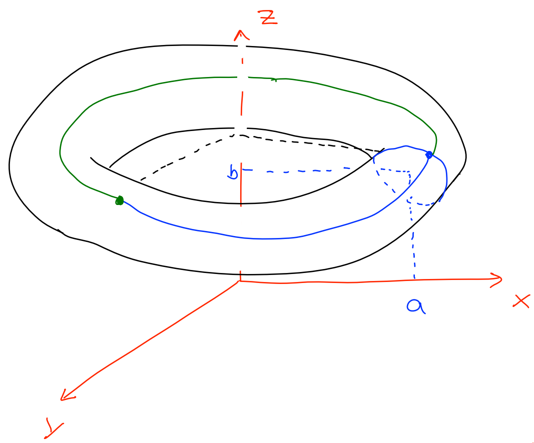
Question: how do we express the cylinder of radius r?

Ans:

The points (x,y,z) on the cylinder have constant distance r from z-axis

Implicit equation
$$f(x,y,z) = x^2 + y^2 - r^2 = 0$$

Example: The Torus as a Surface of Revolution



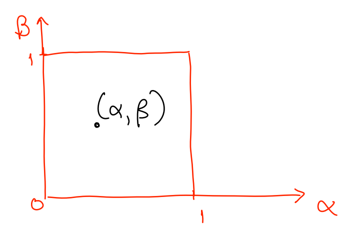
Question: how do we express the torus as a surface of revolution?

Ans: torus is formed by rotating a circle about the z-axis

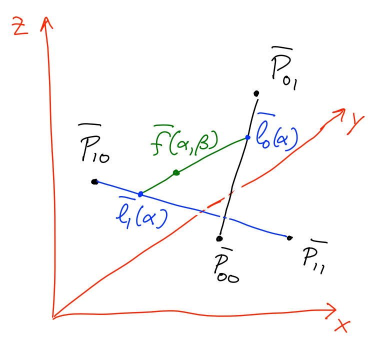
$(x(\lambda), z(\lambda)) =$
$(r\cos\lambda + a, r\sin\lambda + b)$

Parametric representation
$\bar{f}(\lambda,\theta) = (x(\lambda)\cos\theta, x(\lambda)\sin\theta, z(\lambda))$

Bilinear Patches: Basic Construction



Space of parameters
$(\alpha,\beta) \in [0,1] \times [0,1]$



Given 4 3D points $\bar{P}_{00}, \bar{P}_{01}, \bar{P}_{10}, \bar{P}_{11}$ the pair (α,β) uniquely determines $\bar{f}(\alpha,\beta)$:

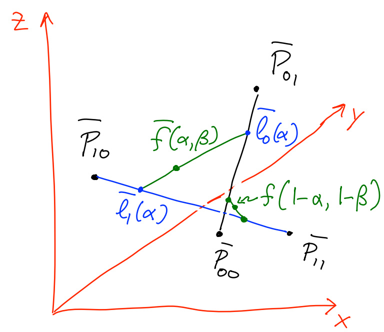
$\bar{l}_0(\alpha) = (1-\alpha)\bar{P}_{00} + \alpha\bar{P}_{01}$
$\bar{l}_1(\alpha) = (1-\alpha)\bar{P}_{10} + \alpha\bar{P}_{11}$
$\bar{f}(\alpha,\beta) = (1-\beta)\bar{l}_0(\alpha) + \beta\,\bar{l}_1(\alpha)$

Bilinear Patches: Basic Construction

Question: What kind of surface do we get?

Is it a plane?

Ans: It is a plane only if $\bar{P}_{00}, \bar{P}_{01}, \bar{P}_{10}, \bar{P}_{11}$ are coplanar!



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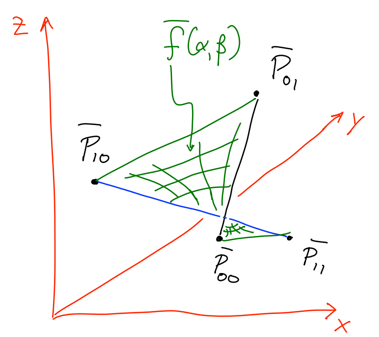

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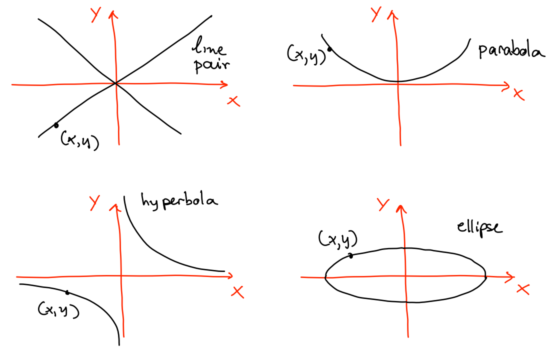
Refresher on (2D) Conic Sections



General implicit equation

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

shape controlled by params A, B, C, D, E, F (one of A, B, C must be $\neq 0$)

or

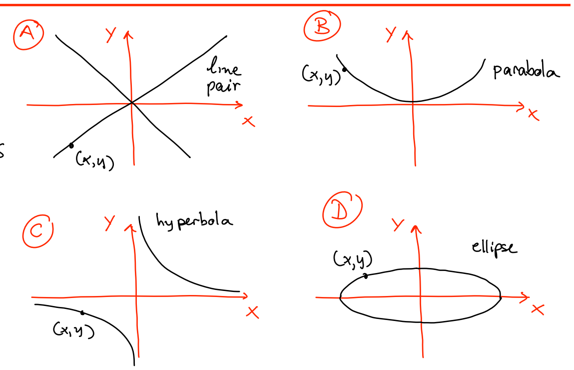
$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} A & C/2 & D/2 \\ C/2 & B & E/2 \\ D/2 & E/2 & F \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

same eq but in homogeneous/matrix notation

Quadric Surfaces: Basic Construction

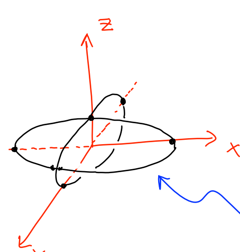
Definition:

Surfaces whose planar cross-sections are conics



Intuition:

To visualize all possible shapes, consider their cross-sections with planes parallel to xz and yz-planes

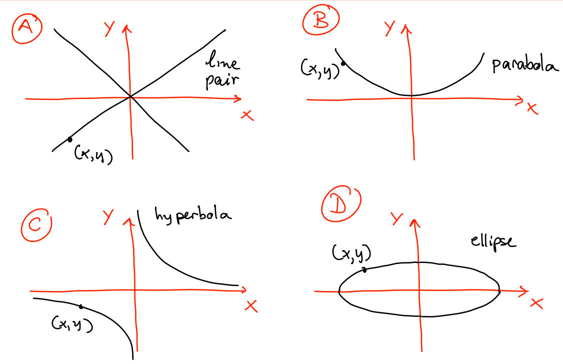


Example: two elliptical cross-sections

Quadric Surfaces: Basic Construction

Case D + D:

Ellipsoid



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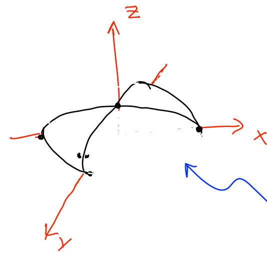
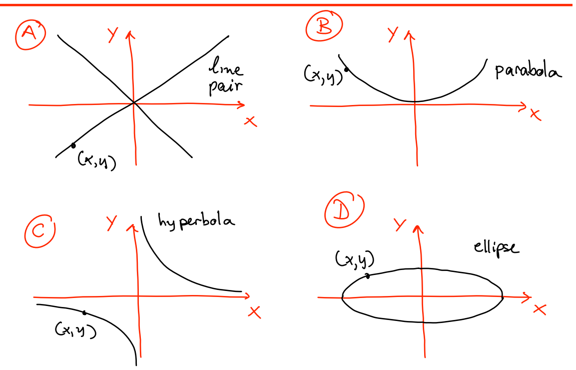


Example: two elliptical cross-sections

Quadric Surfaces: Basic Construction

Case B + D

Elliptical paraboloid



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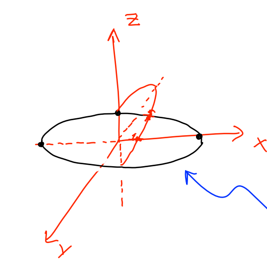
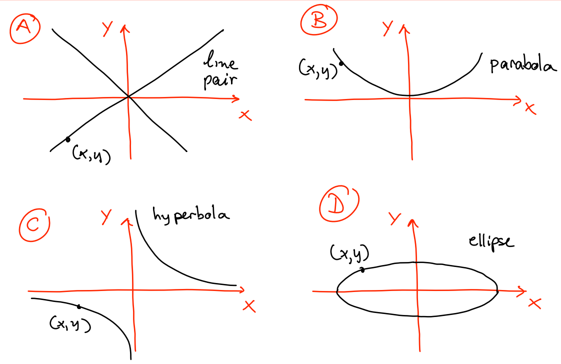


Example: an elliptical & a parabolic section

Quadric Surfaces: Basic Construction

Case (B)+(D)

Elliptical paraboloid



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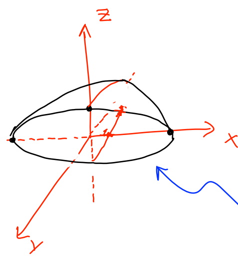
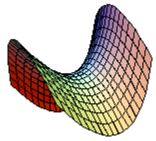


Example: an elliptical & a parabolic section

Quadric Surfaces: Basic Construction

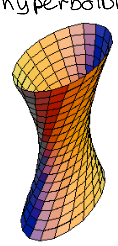
(B)+(C)

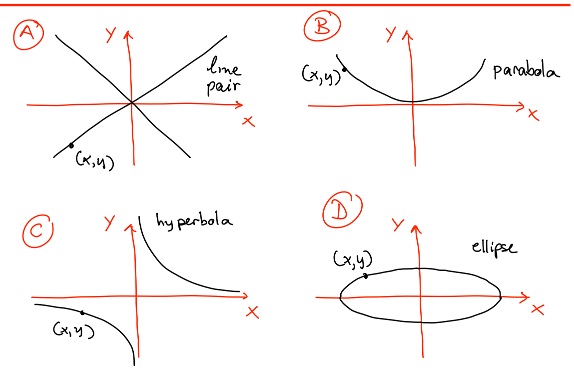
Hyperbolic paraboloid



(C)+(D)

Elliptic hyperboloid





Quadric Surfaces: Implicit Equation

General implicit equation

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \underbrace{\begin{bmatrix} A & D & E & G \\ D & B & F & H \\ E & F & C & I \\ G & H & I & J \end{bmatrix}}_{\text{defined up to a scale factor}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

- 9 total degrees of freedom
- Planar cross-sections produce curves with an implicit eq of a conic section
- Only 5 parameters used (those circled) when cross-sections are "centered" on xy-, yz-, xz-planes

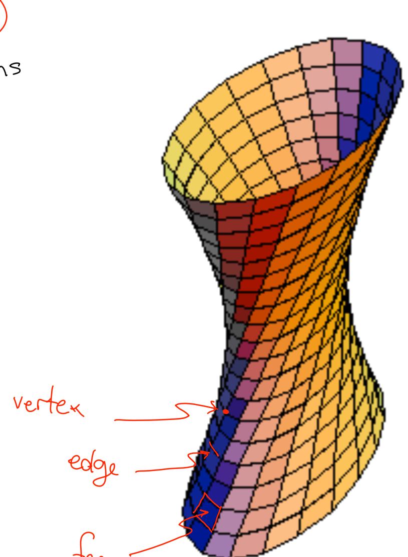
Polygonal Meshes

Definition (polygonal mesh)

A collection of polygons

- vertices
- edges
- faces

- Polyhedron: a closed, connected, polygonal mesh
- Face: Planar polygonal patch inside a mesh
- A mesh is simple when it has no holes (i.e. topologically equiv to a sphere)



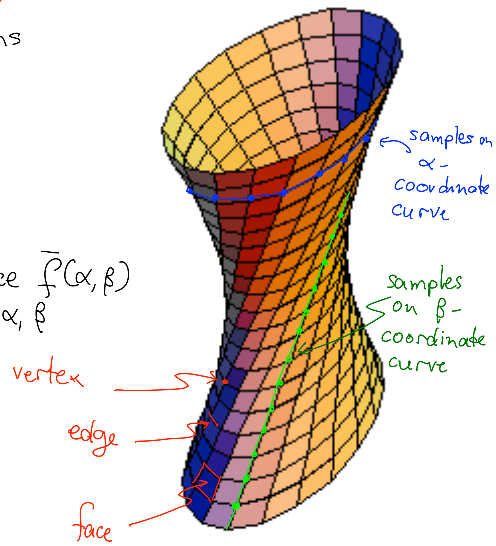
Polygonal Meshes

Definition (polygonal mesh)

A collection of polygons

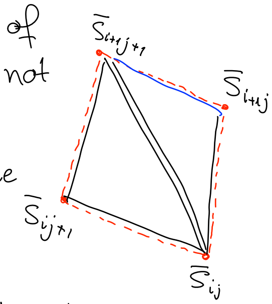
- vertices
- edges
- faces

Given a parametric surface $\bar{f}(\alpha, \beta)$ we can sample values of α, β to define a mesh that approxmates it



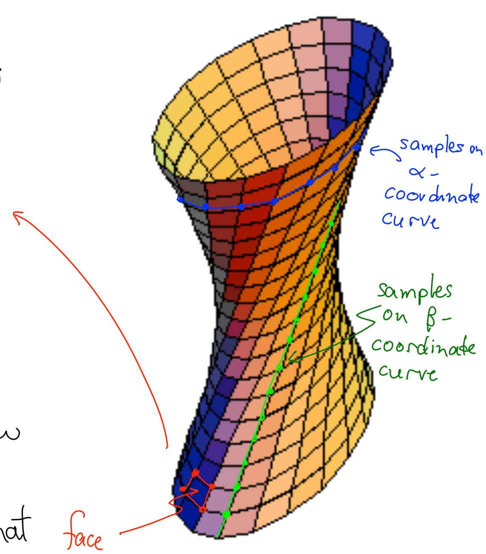
Generating Triangle Meshes

A quadruple of points may not define a planar face



$\Downarrow$

Solution: break up quad into 2 triangles



* See online notes on how to do this properly

* See online notes about what data structure to use to store a mesh.