## CSC 258 midterm solutions

27 February 2002, 11:00

1. a) What function does the following logic gate diagram compute? [not shown here; see the test paper]

Answer: $x(x y \oplus x y z)$
b) Simplify this formula (using any appropriate technique).

Answer:

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    \(x(x y \oplus x y z)\)
\(=x(\overline{(x y)} x y z+x y \overline{(x y z)})\)
\(=x((\bar{x}+\bar{y}) x y z+x y(\bar{x}+\bar{y}+\bar{z}))\)
\(=x \bar{x} x y z+x \bar{y} x y z+x x y \bar{x}+x x y \bar{y}+x x y \bar{z}\)
    (you may prefer to expand to that in two or three steps, but it does all multiply through)
\(=0+0+0+0+x y \bar{z}\)
\(=x y \bar{z}\)
```

c) Draw a logic gate diagram for your simplified formula.
[answer not shown here]
2. Here is a three-bit ripple counter.
[standard ripple counter picture went here, similar to the answer below]
Draw a three-bit counter with two inputs: an "add one" input such as above, and a "complement" input which toggles all three bits (e.g. 010 will become 101).

You can assume that these two inputs will never be 1 at the same time.

(over)
3. What is the output sequence of the following "counter", after it gets established in its cycle?


Answer: either $0,0,1,0,0,1,0,0,1, \ldots$ or staying at 1 , depending upon initial state.
4. a) Here are some four-bit addition problems, some of which will overflow. Complete the addition operation and state the base-ten value of the operands and the result.

Question and answers combined:

| 0001 | 0111 | 0100 | 0101 | 1000 | 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| +0010 | +0010 | +0011 | +0011 | +1000 | +1011 |
| ----- | ----- | ----- | ---- | ----- | ----- |
| 0011 | 1001 | 0111 | 1000 | 0000 | 1100 |
| $1+2=3$ | $7+2=-7$ | $4+3=7$ | $5+3=-8$ | $-8+-8=0$ | $1+-5=-4$ |

b) Show that overflow is not the same as carry out (from the entire addition, i.e. $c_{n}$ ) by using one of the examples above.

Answer: Either $7+2=-7$ or $5+3=-8$ would do; in both of these examples, overflow $=1$ and carry-out $=0$.

