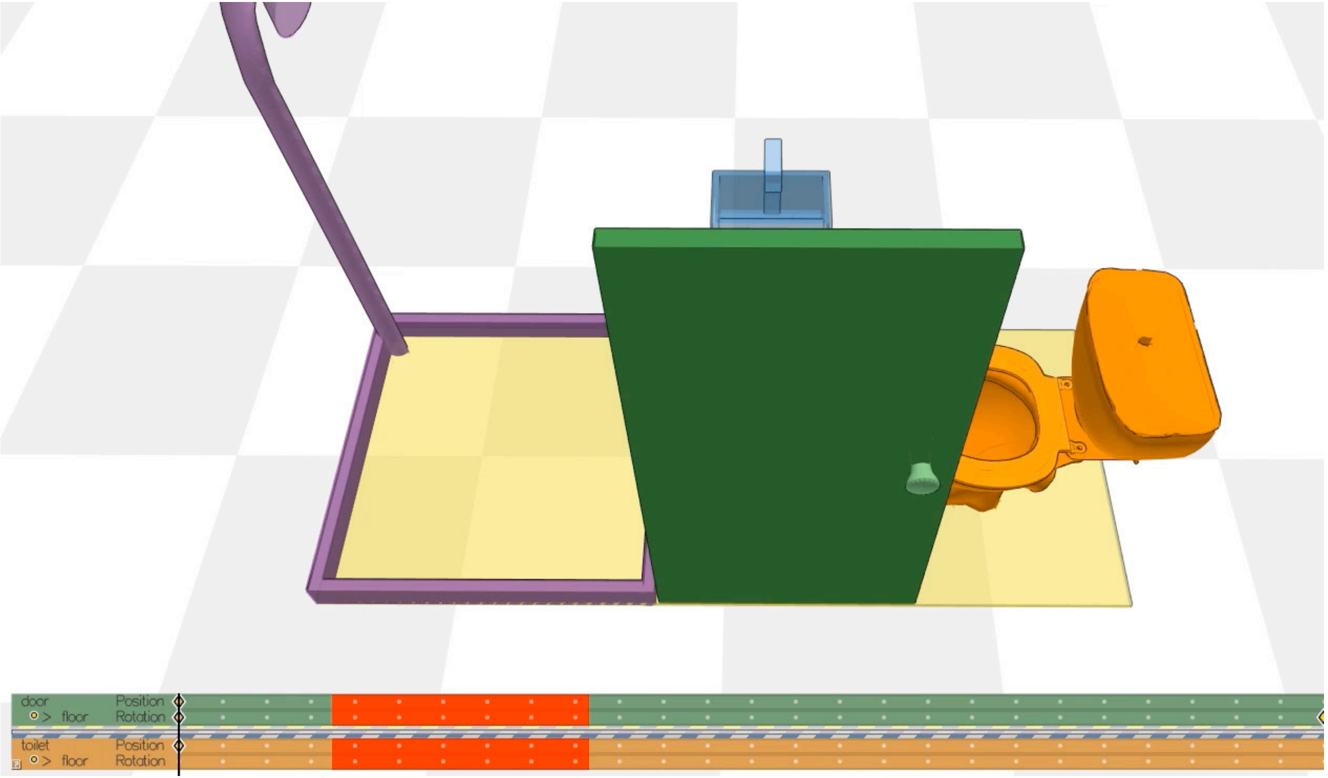
Generalized Matryoshka
Computational Design of Nesting Objects

Alec Jacobson

University of Toronto

Previous work enables computational design of *reconfigurables*



[Garg et al. 2016]

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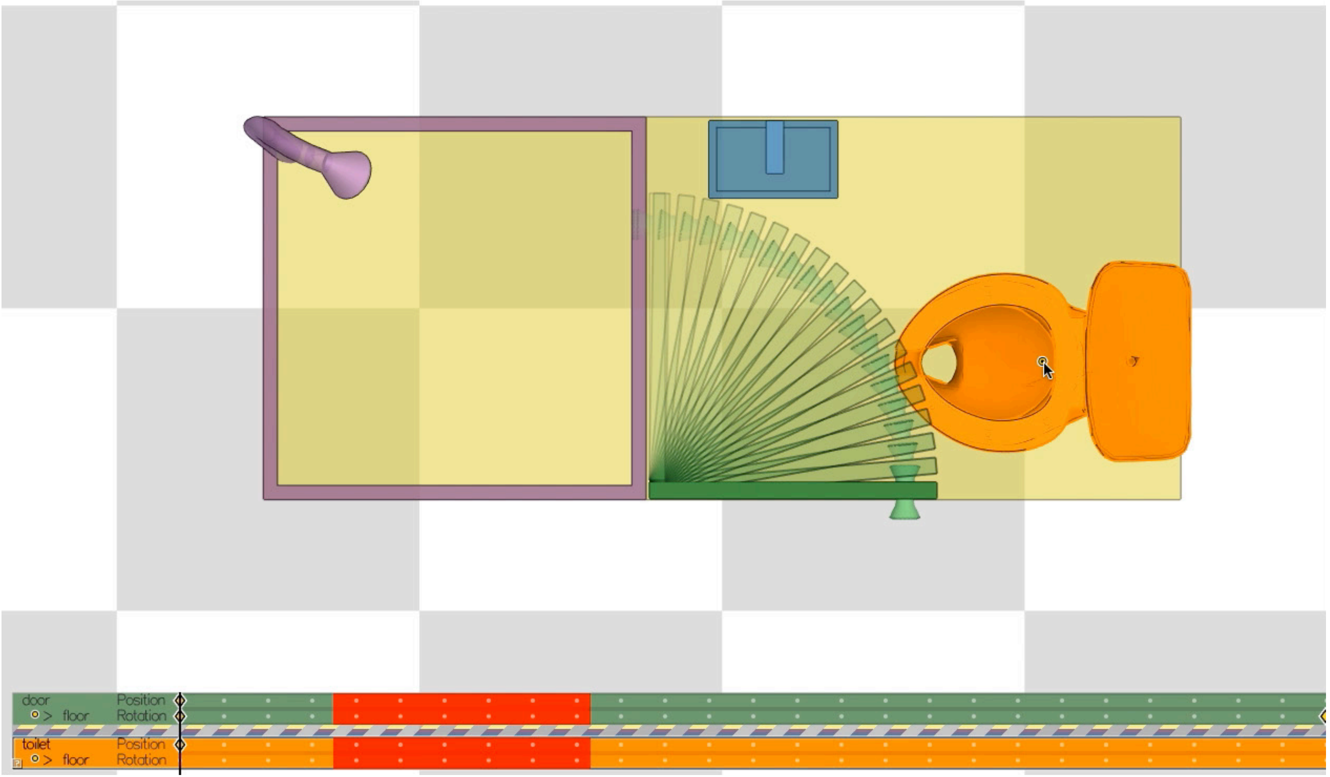

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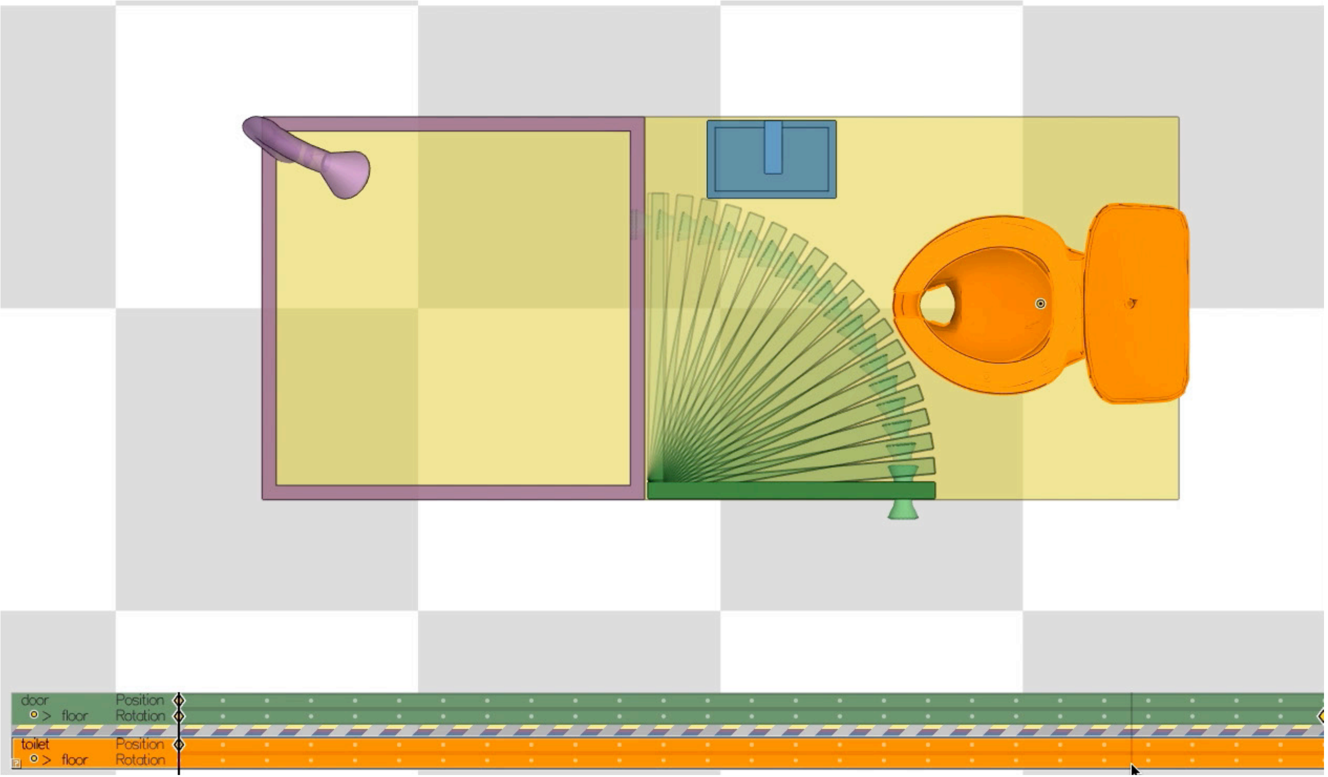

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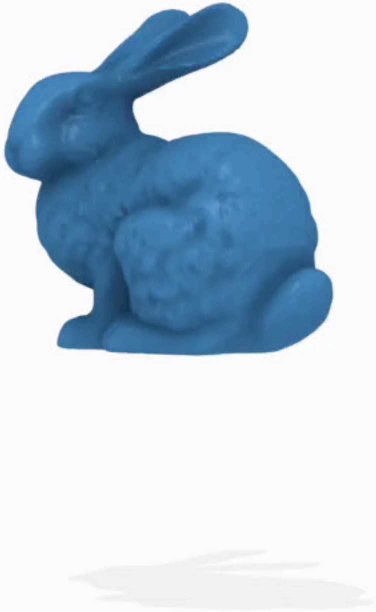
[Zvyozdochkin & Malyutin 1890]

We present a method to generalize Matryoshka to arbitrary shapes



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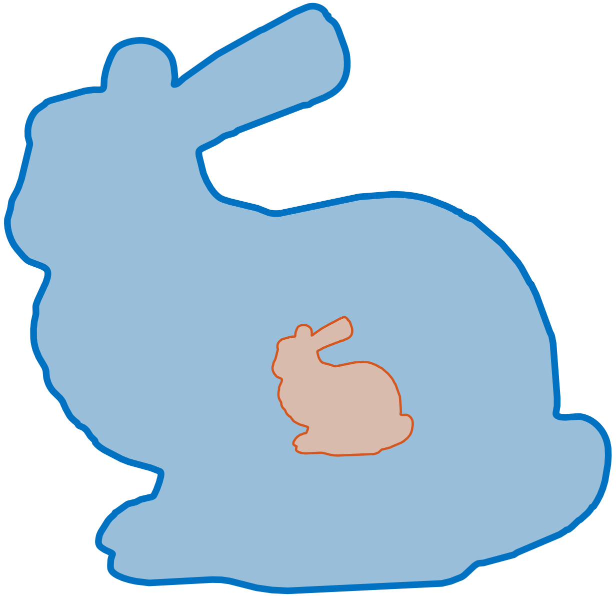

loose enclosure

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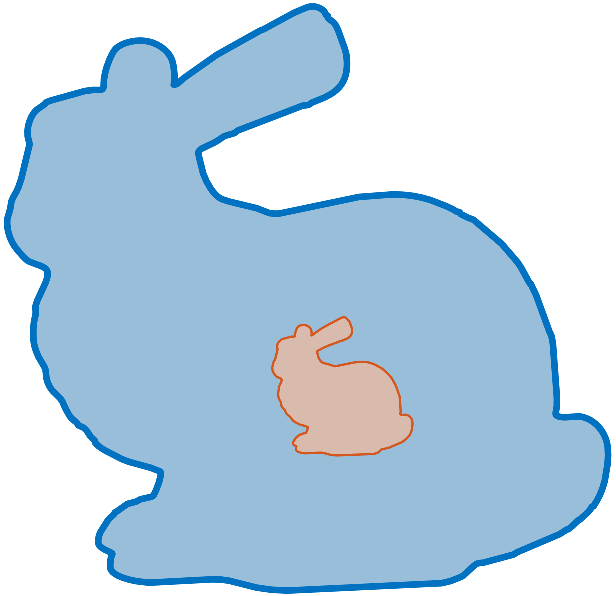

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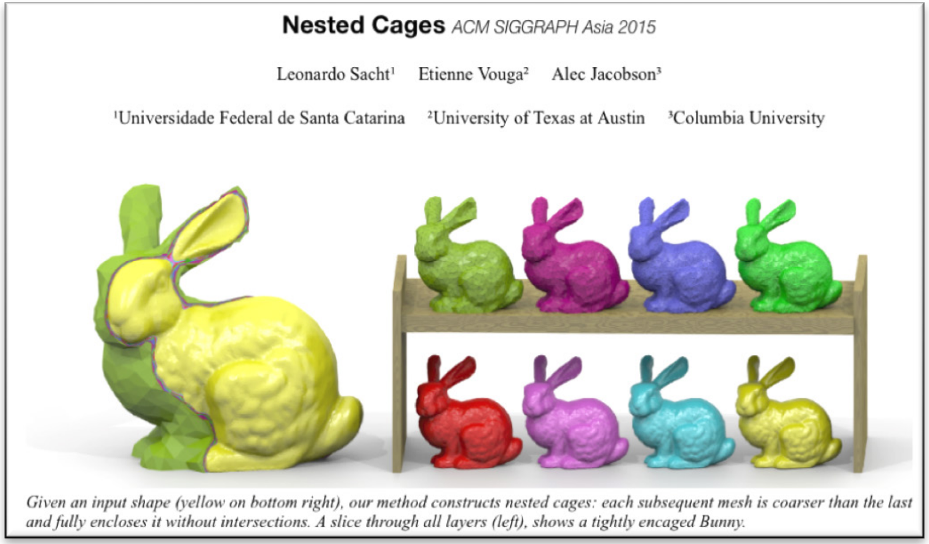
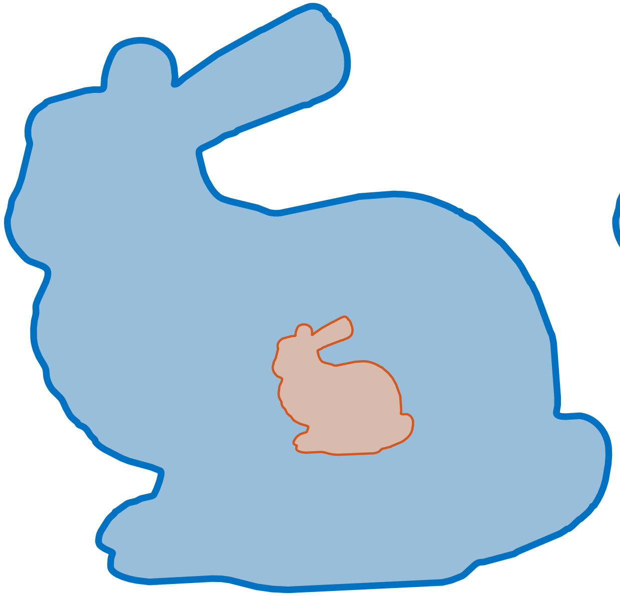


Nesting also requires *removal*



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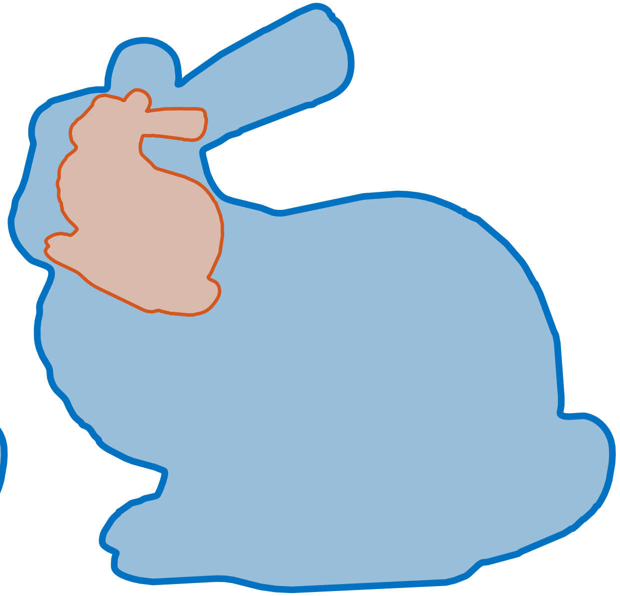

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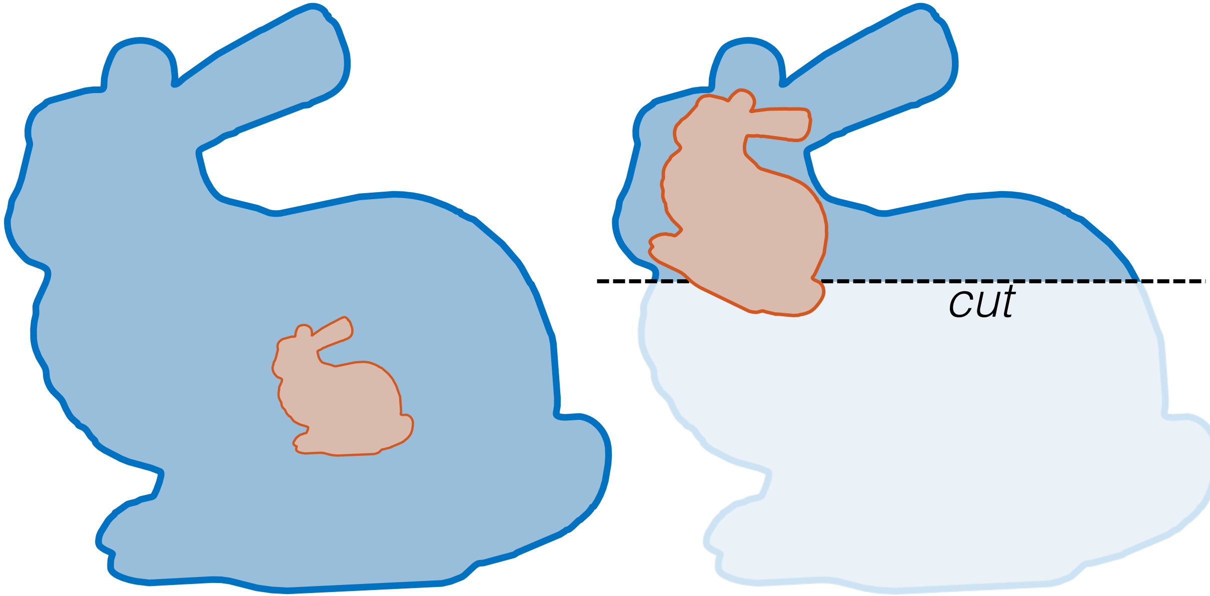

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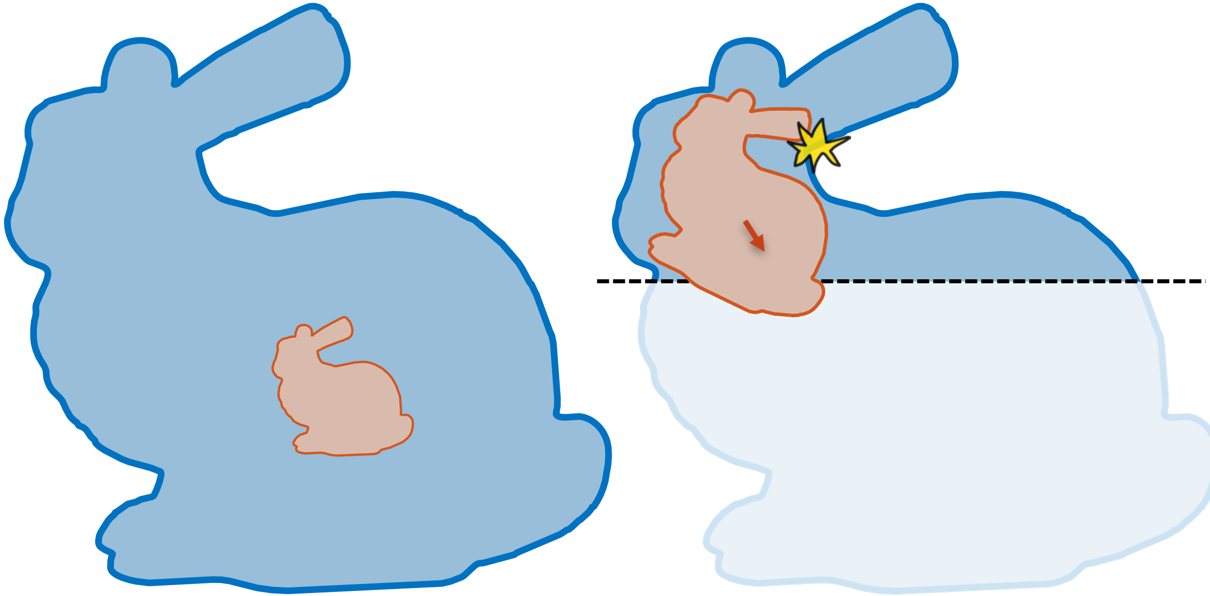

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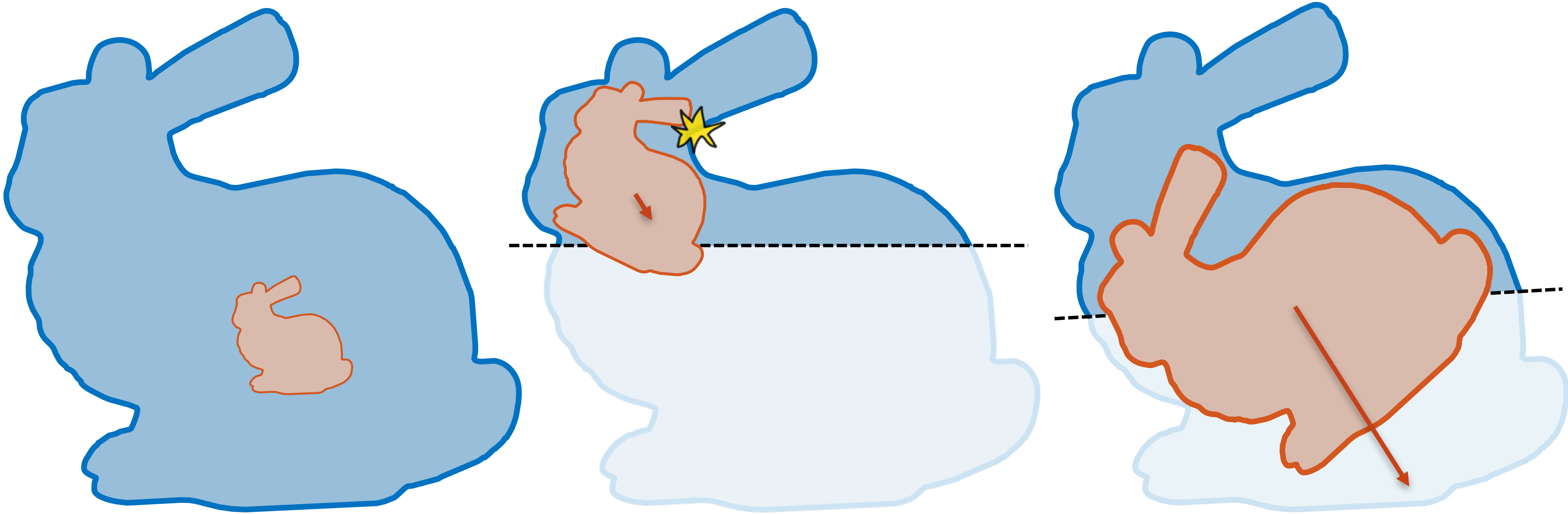

loose enclosure | enclosed, but not removable | enclosed and removable

We present highly parallelizable methods to…

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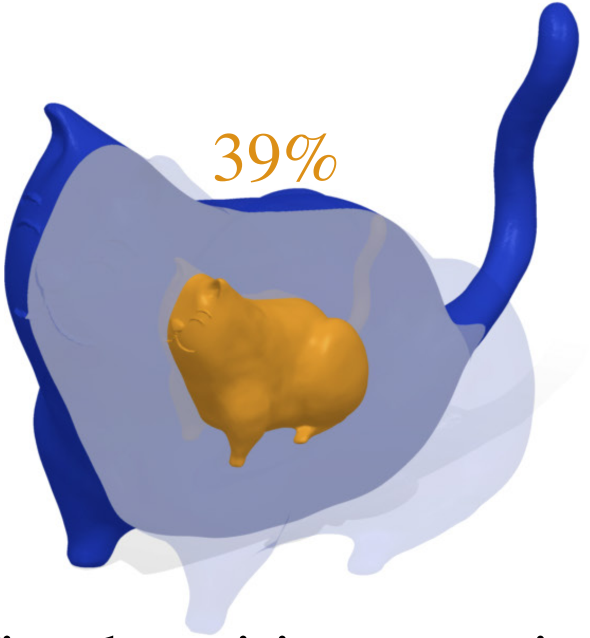
- determine feasibility of nesting,
- find maximum scale, and
- optimize nesting scale
  over some or all parameters

Our optimization utilizes rigid motion for tighter nesting



fixed position+rotation

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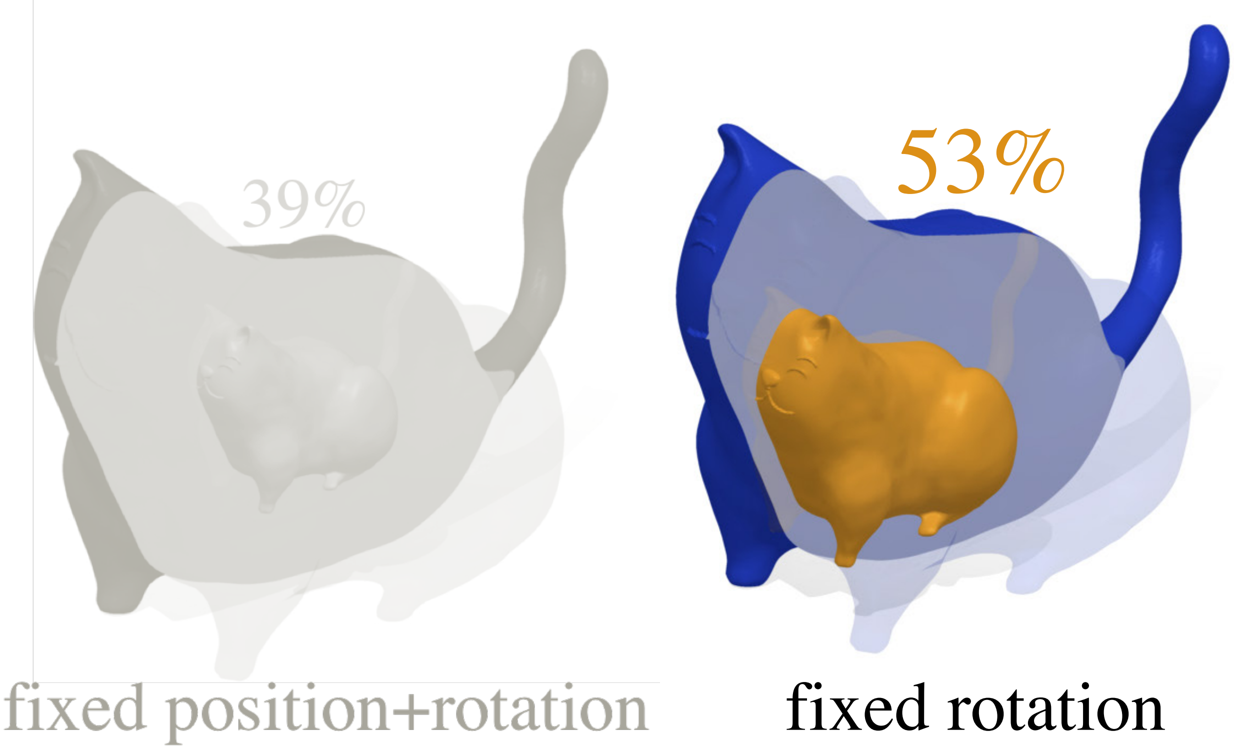

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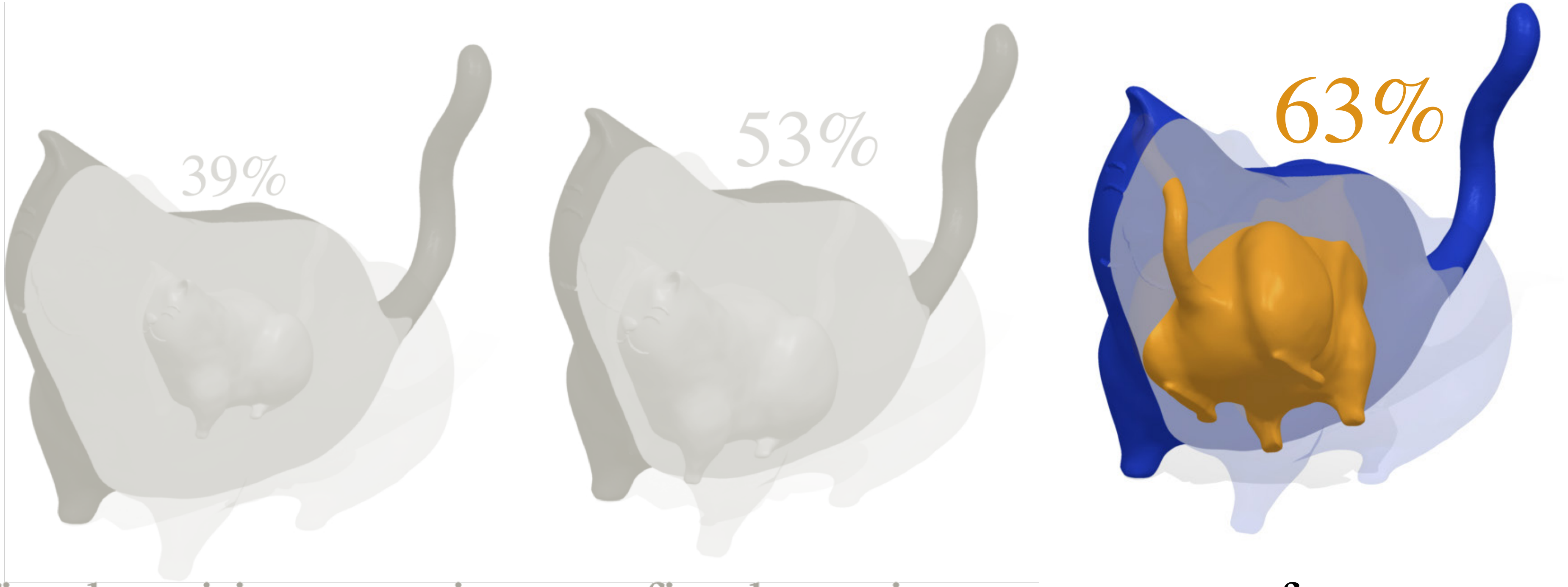
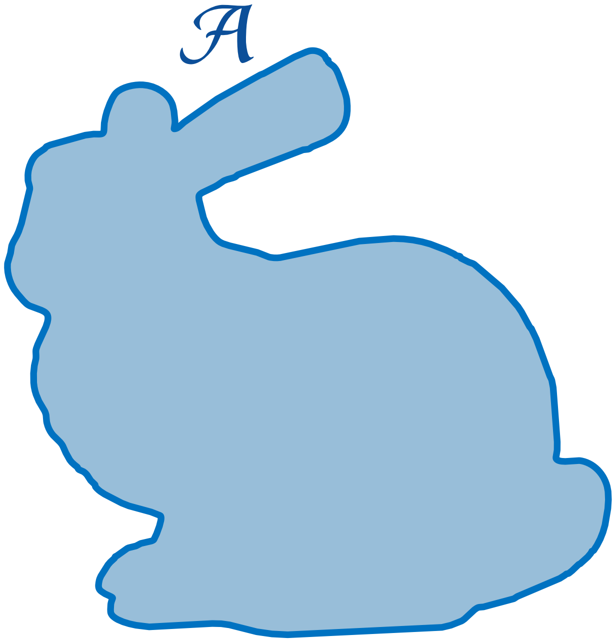


fixed position+rotation fixed rotation free

We define *valid self-nesting*

Given:

1. shape $\mathcal{A}$,



We define *valid self-nesting*

Given:

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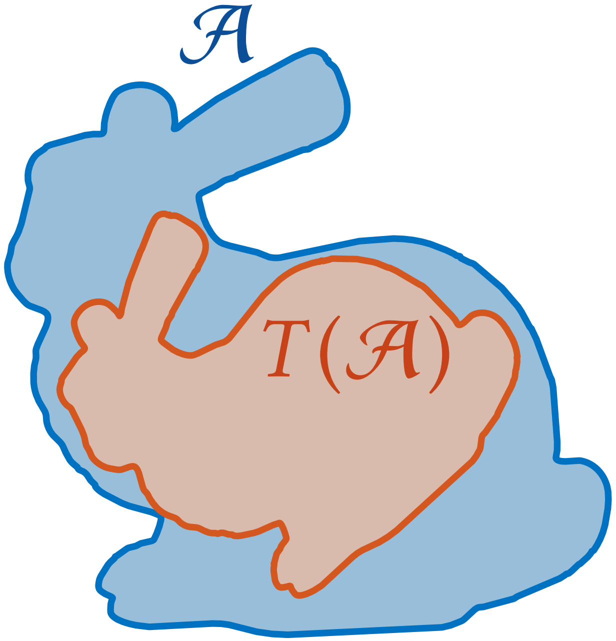

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Given:

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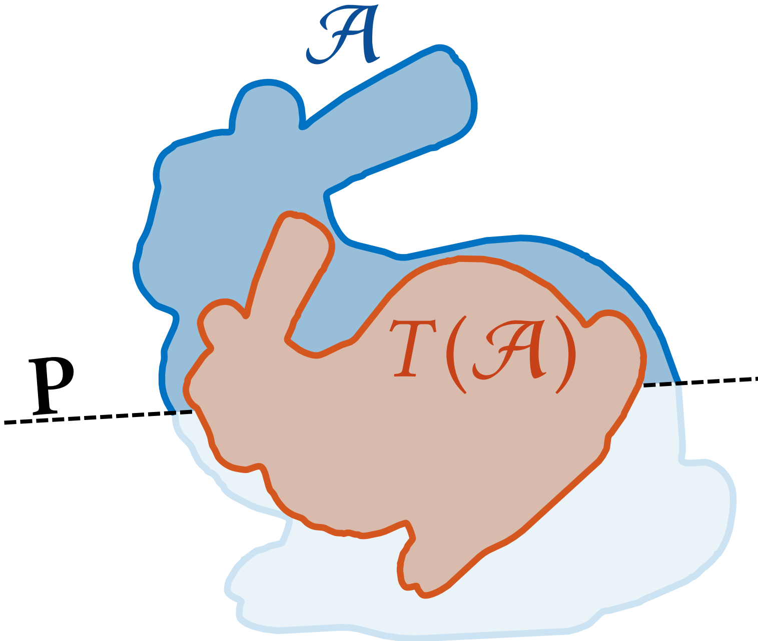

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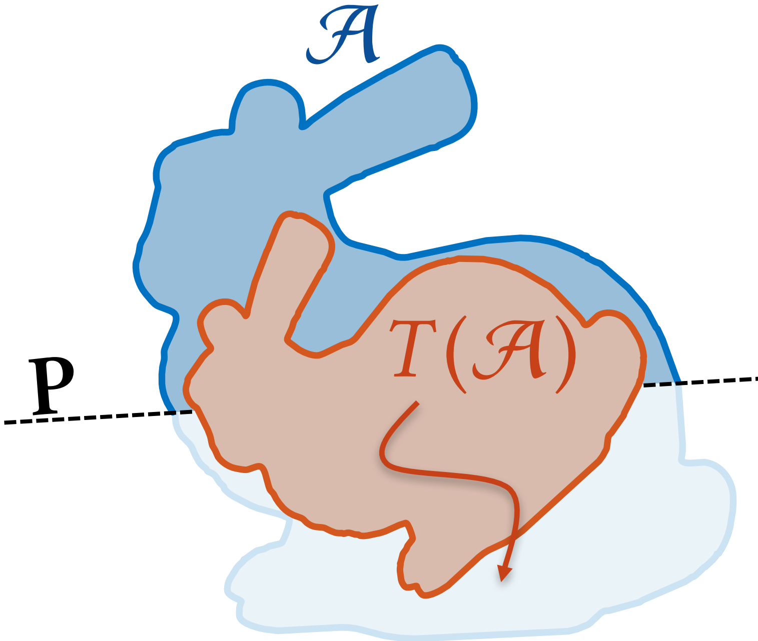

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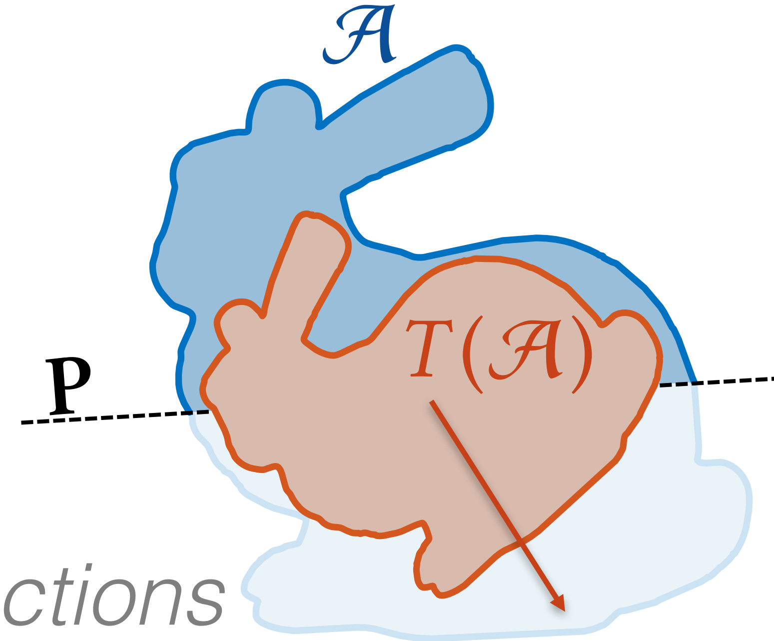

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Must have:

1. $T(\mathcal{A}) \subset \mathcal{A}$, and
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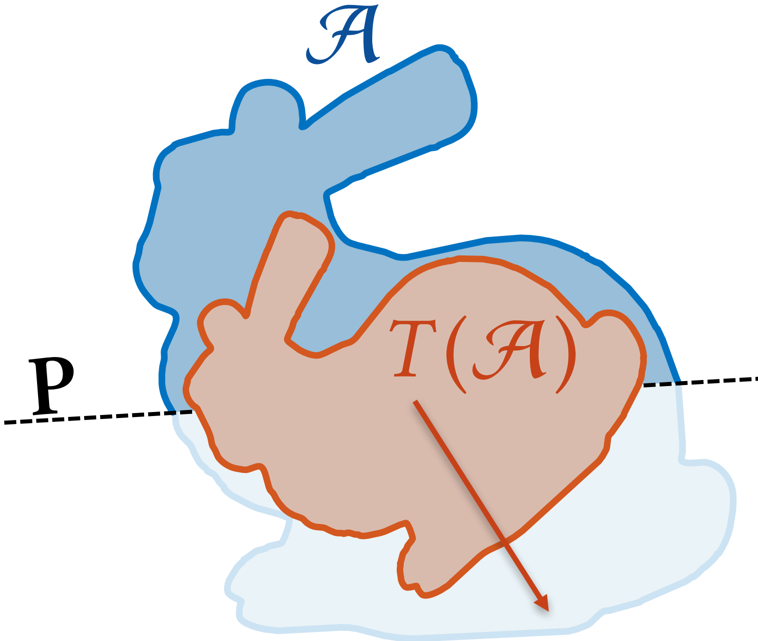

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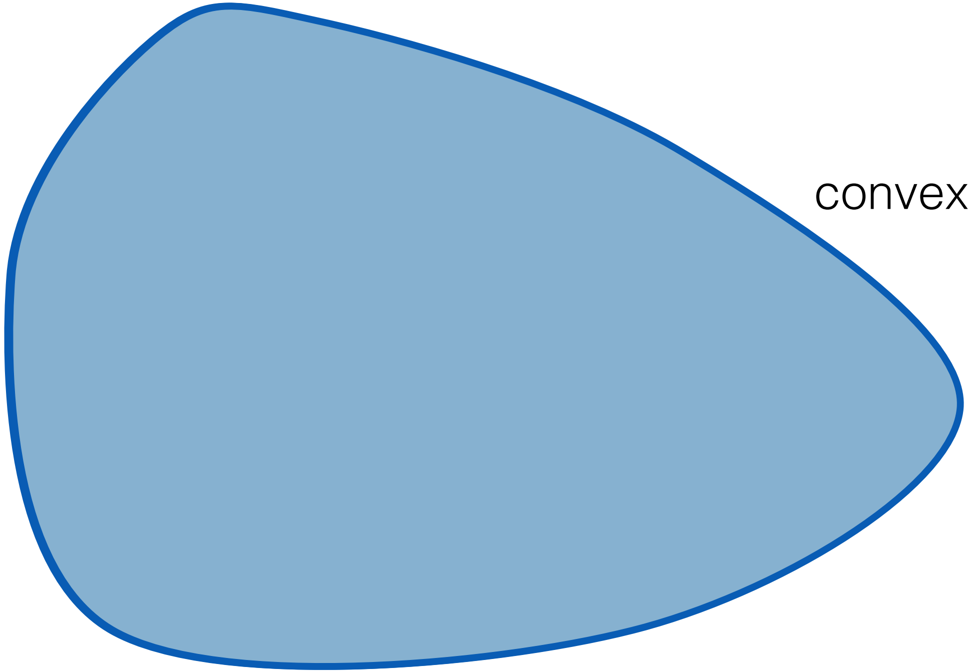
Definition depends on choice of cut plane and removal directions.

Some configurations admit *perfect self-nesting*



convex shapes?

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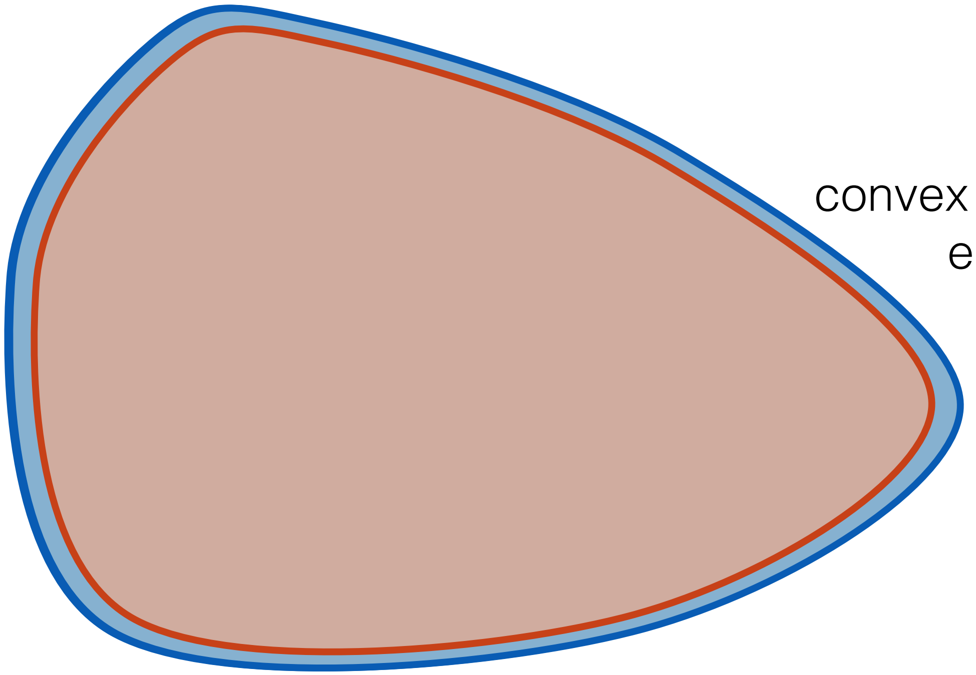

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enclosure is easy ….

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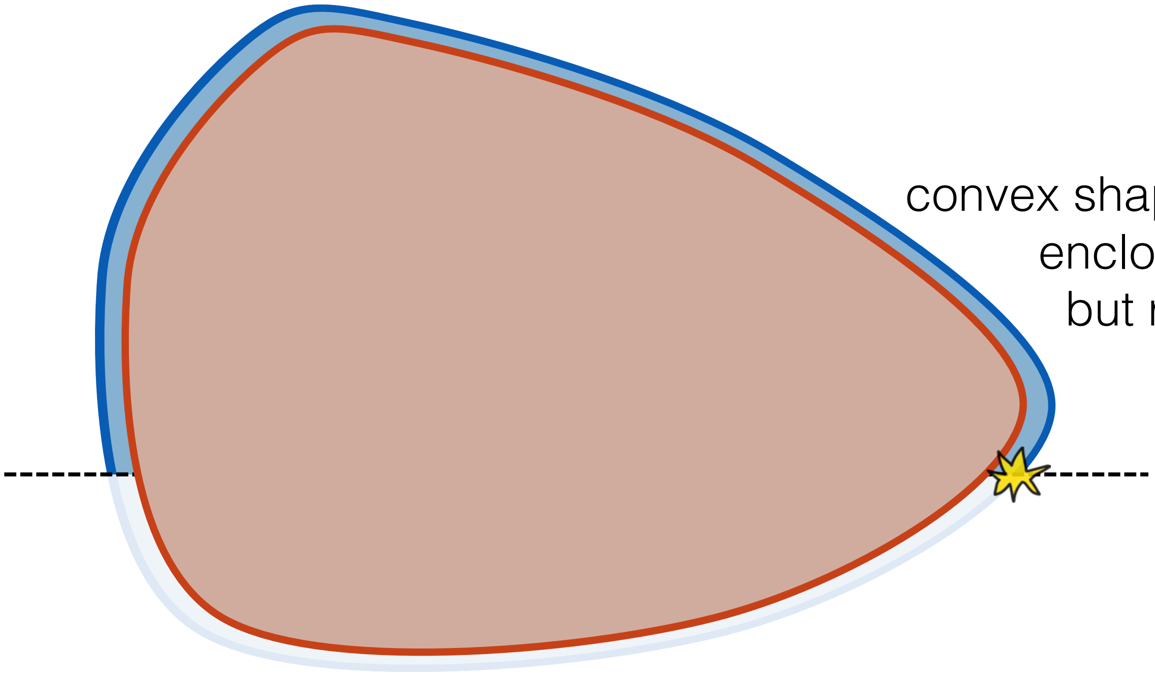
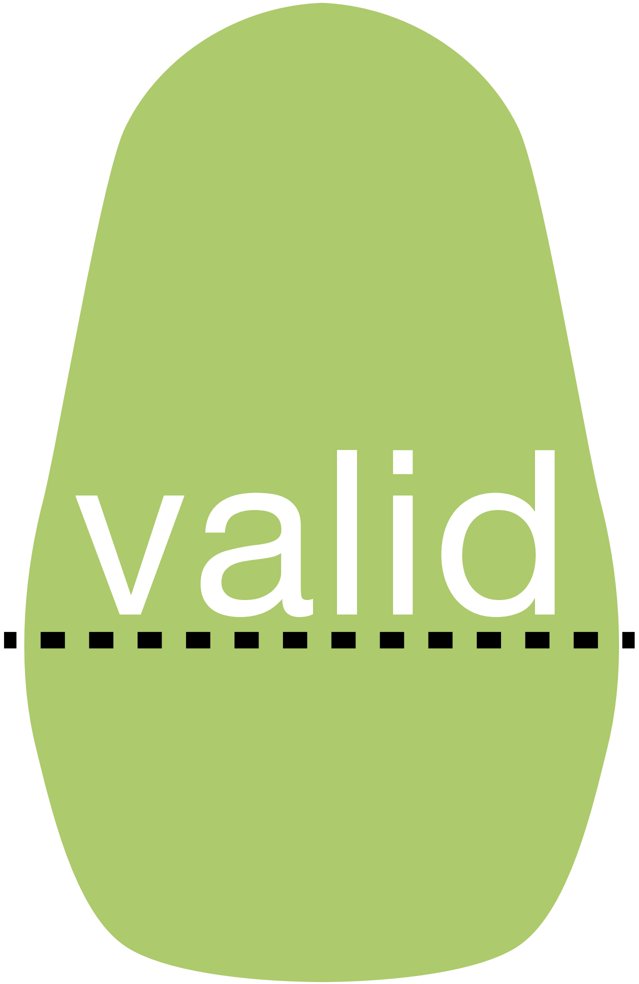


convex shapes?
enclosure is easy ….
but removal depends on cut plane!

[Zvyozdochkin & Malyutin 1890]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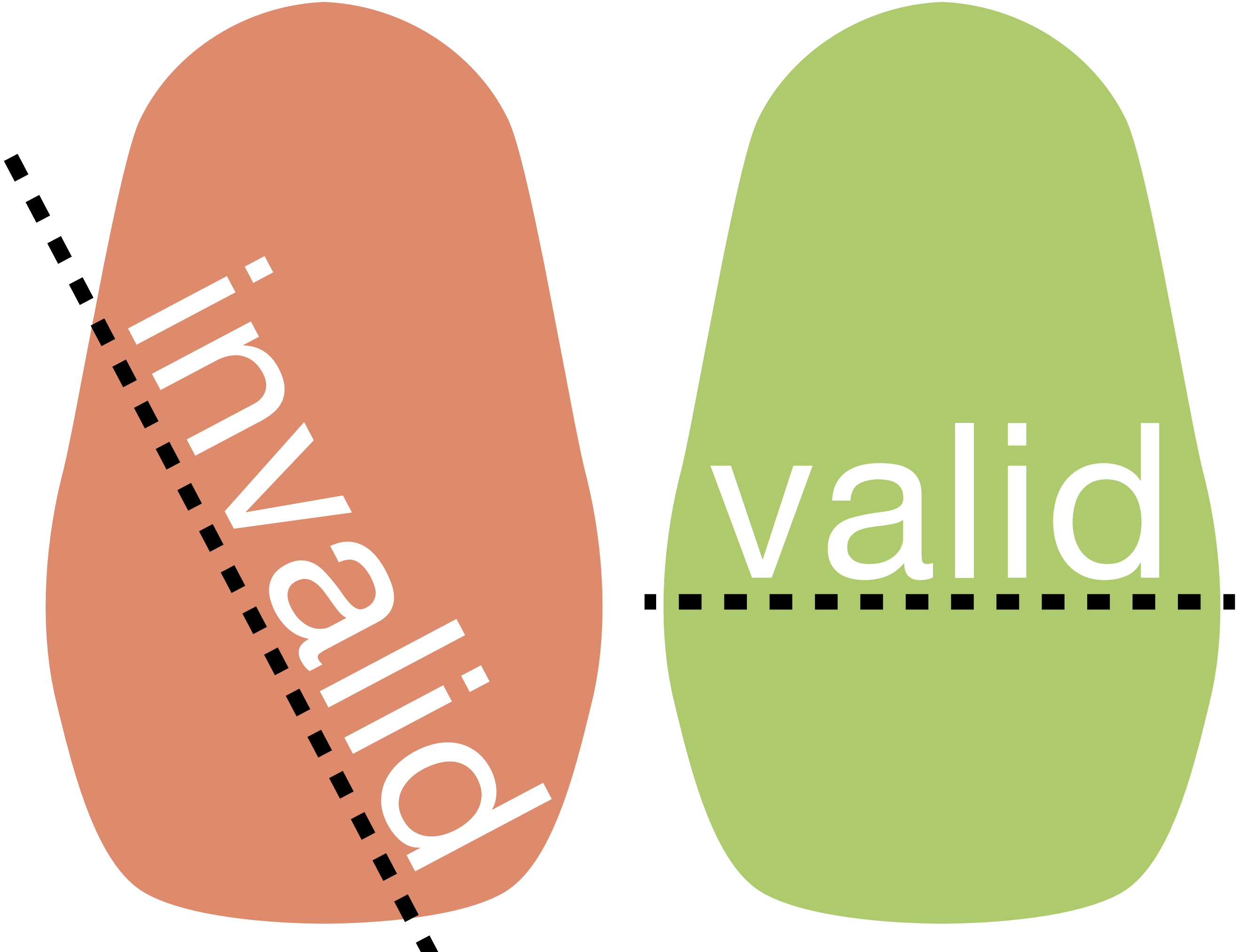
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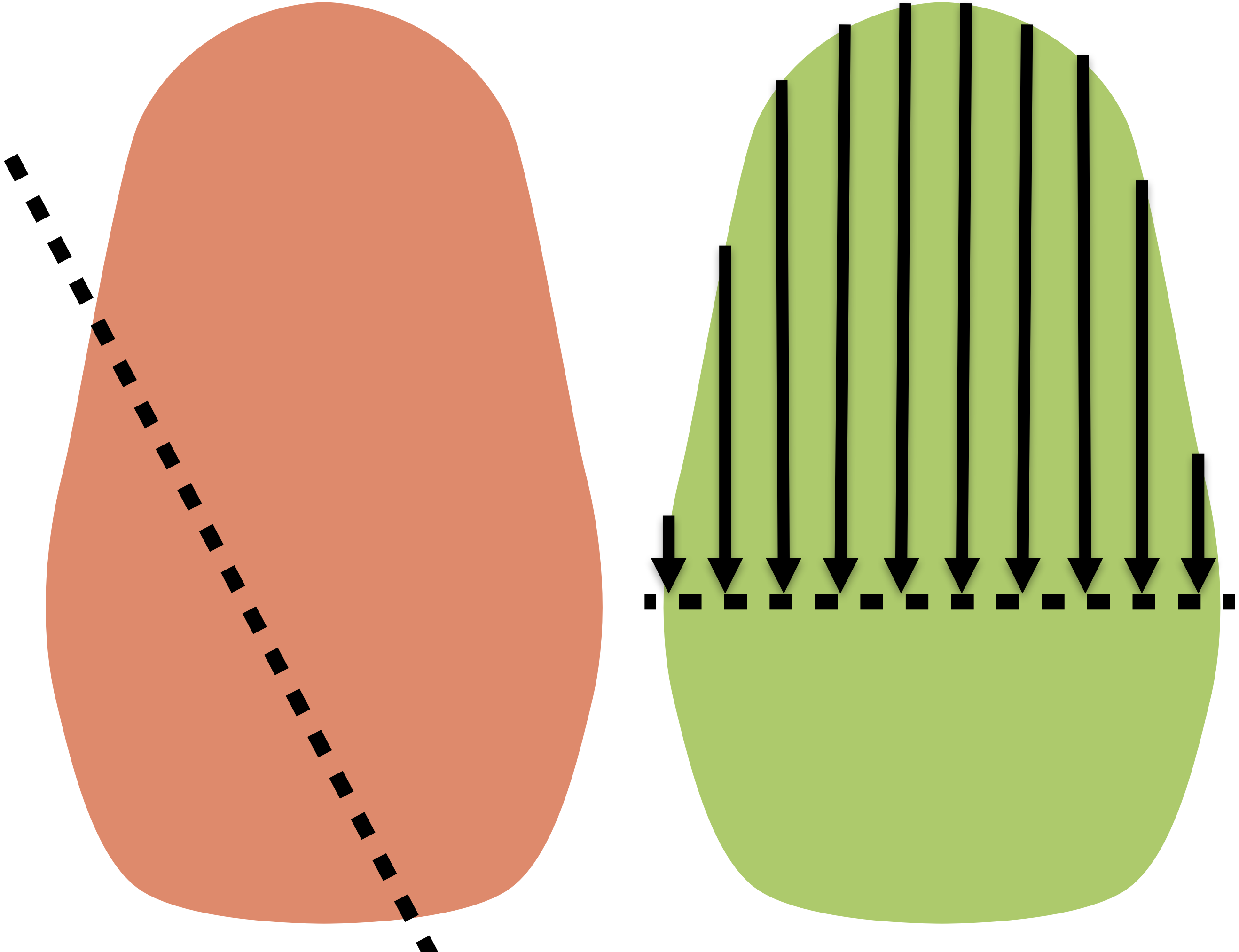
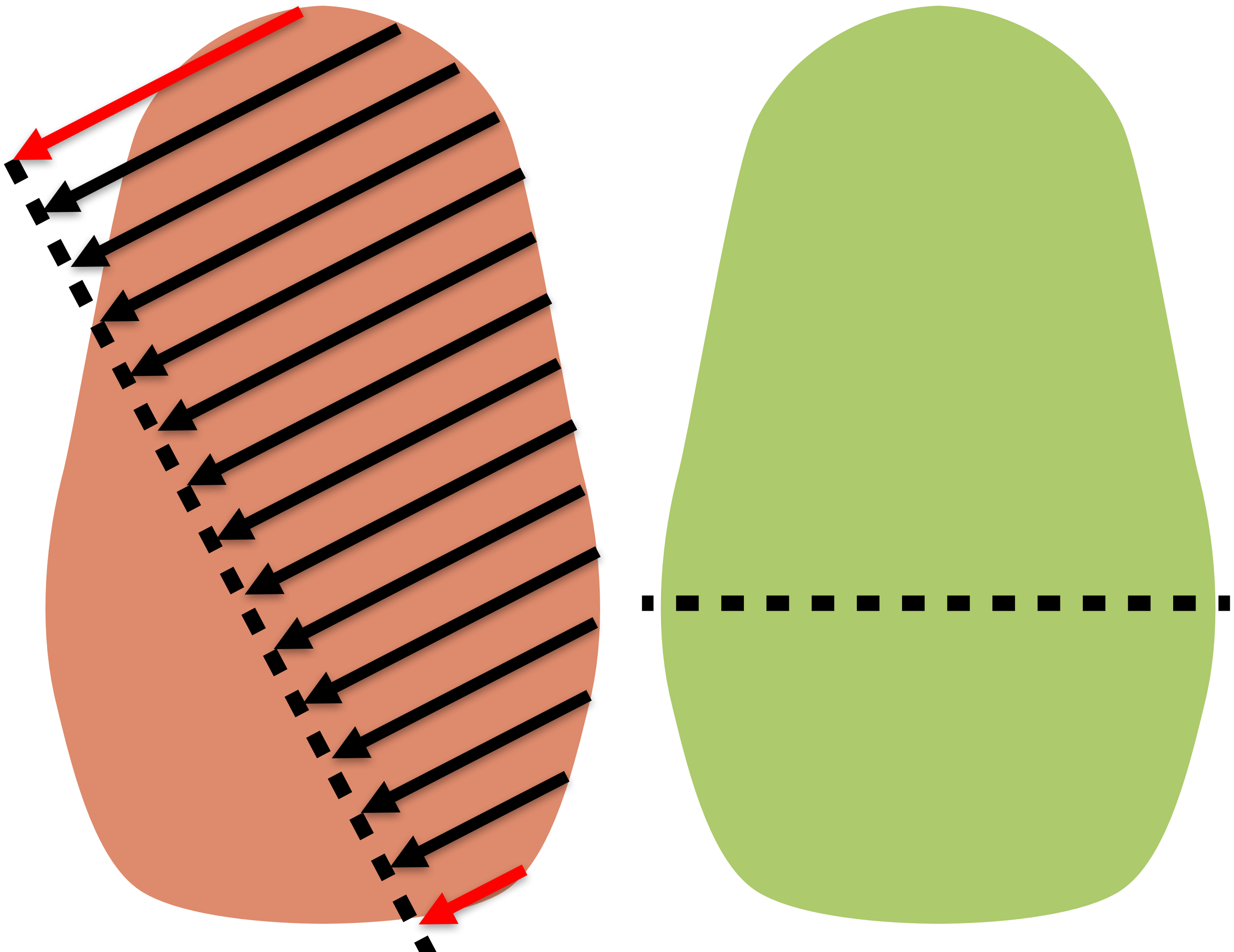
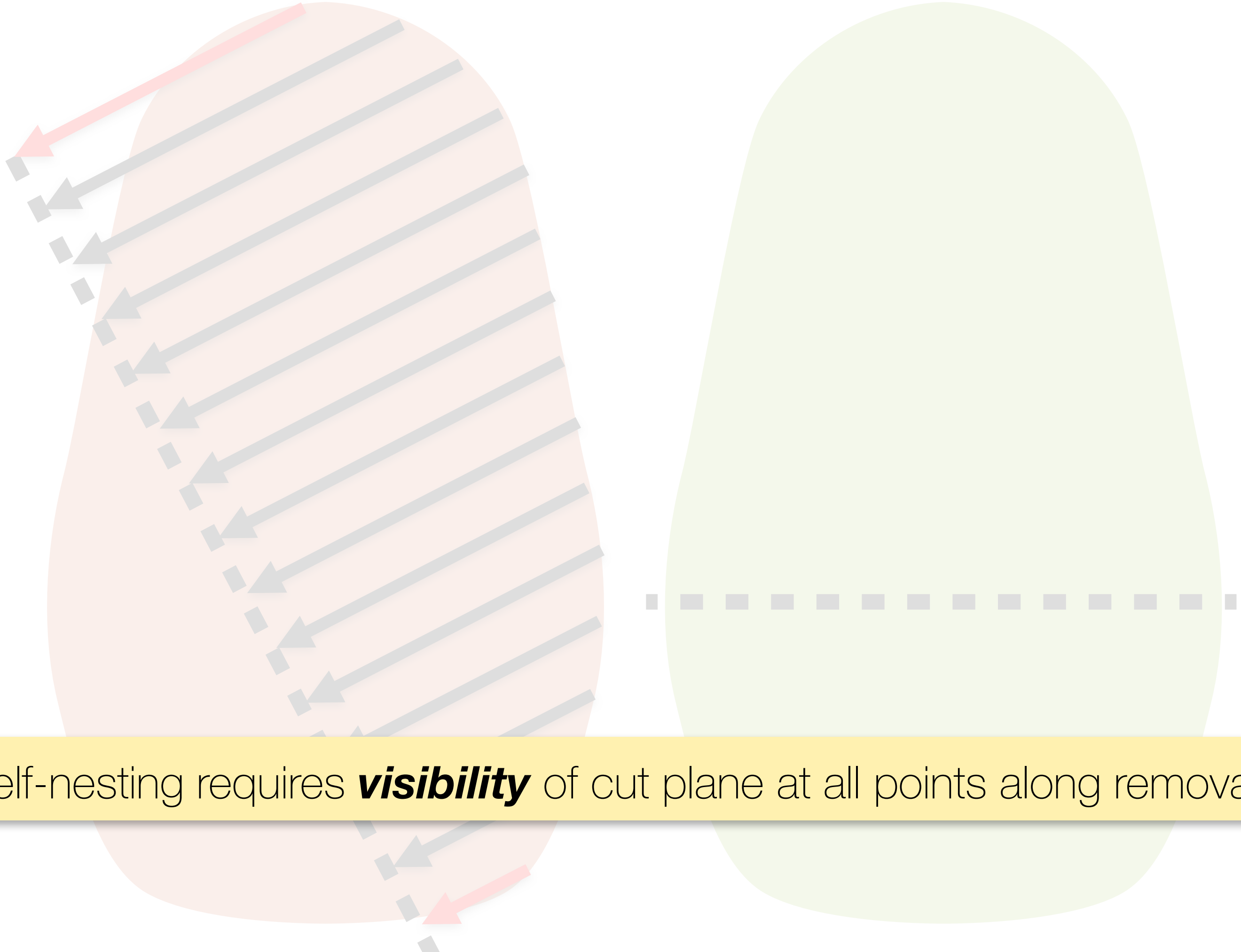
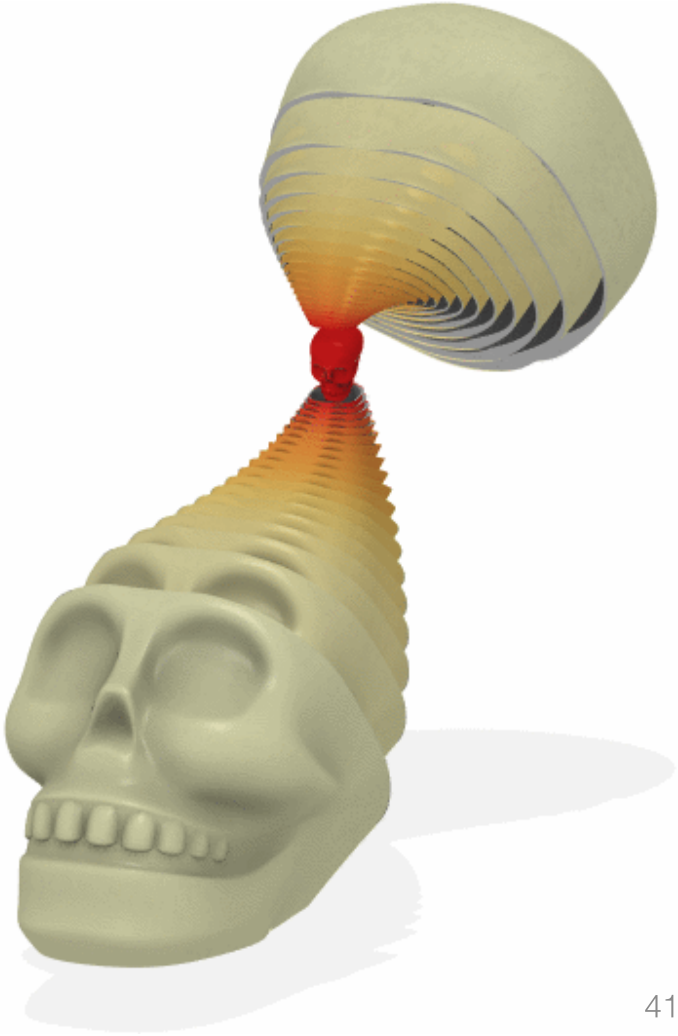






Perfect self-nesting requires ***visibility*** of cut plane at all points along removal directions

Our tool explores nesting of *arbitrary* solid 3D shapes



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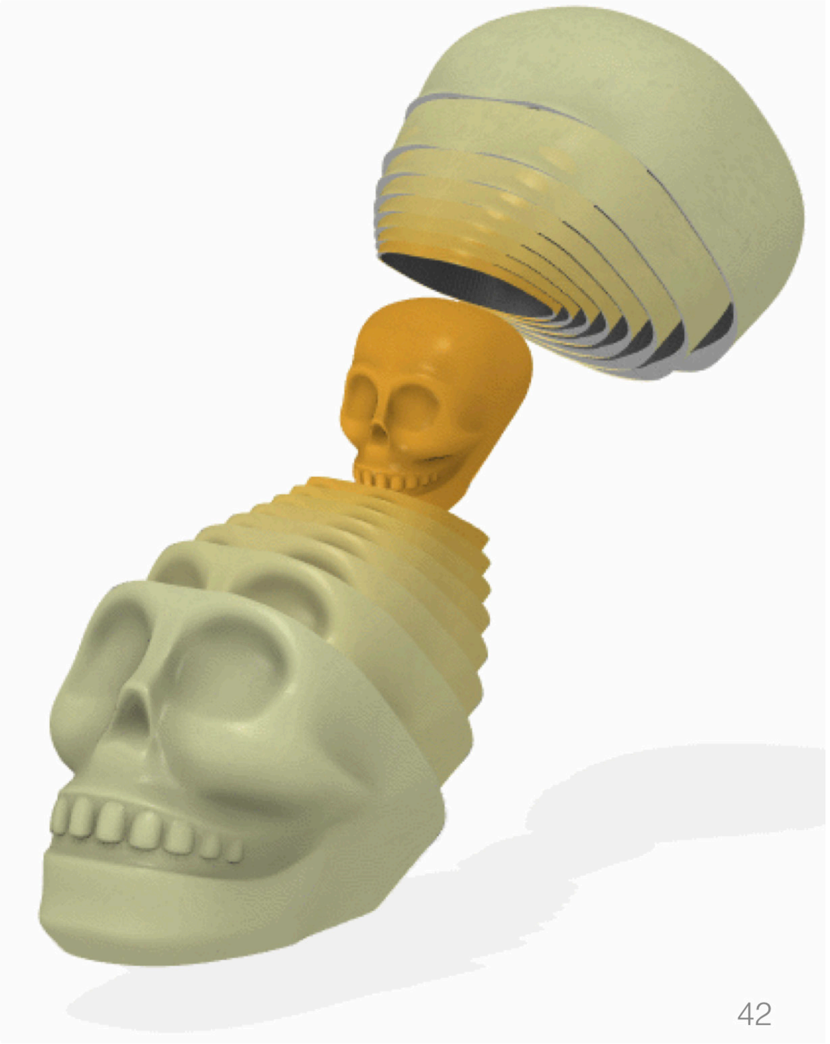

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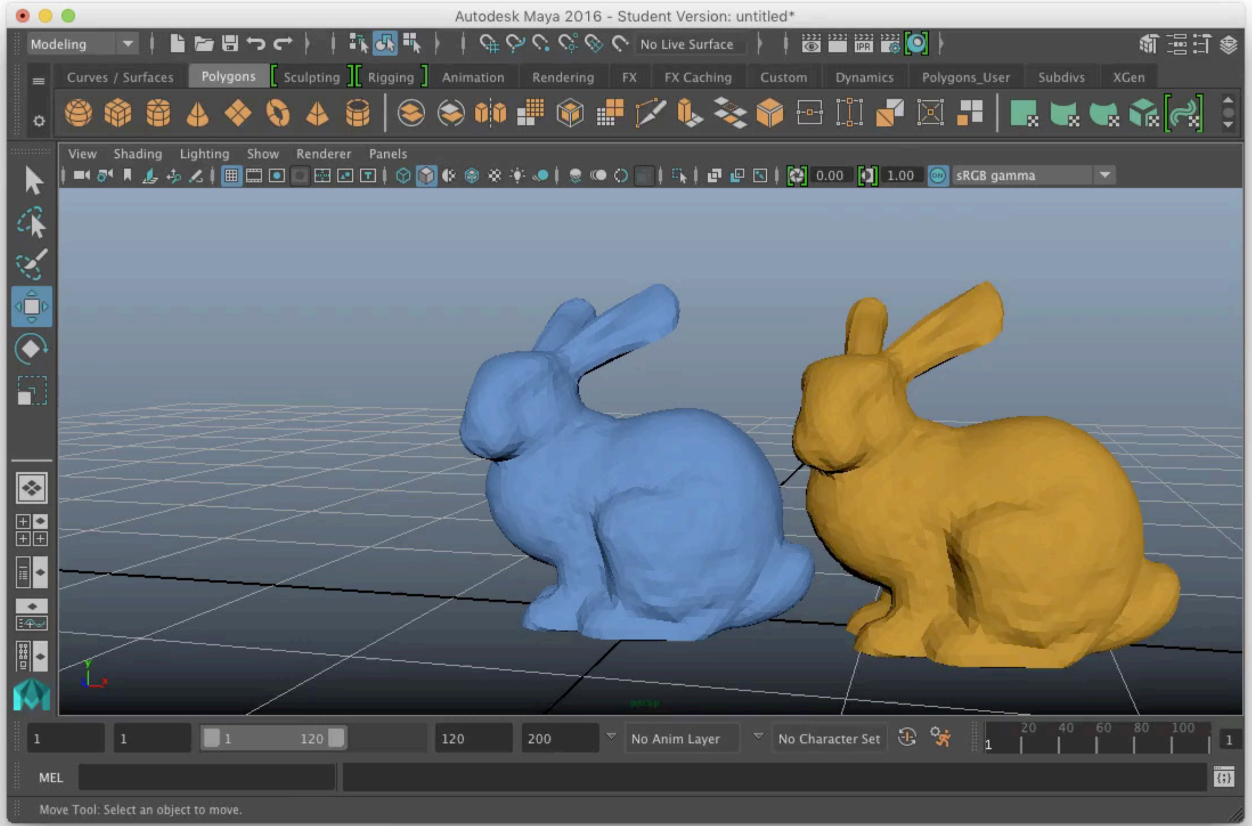
We cast this as a *computational design* problem



Manual design with traditional tools would be tortuous

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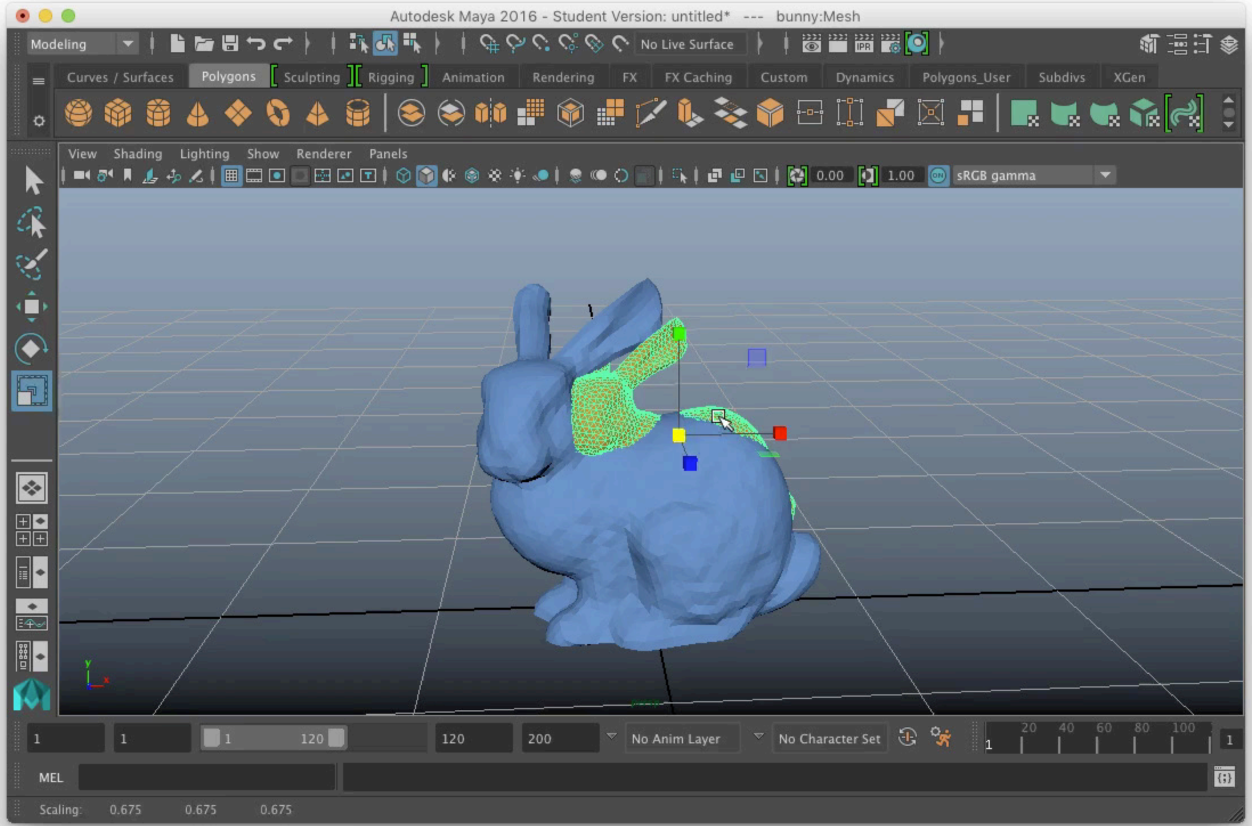

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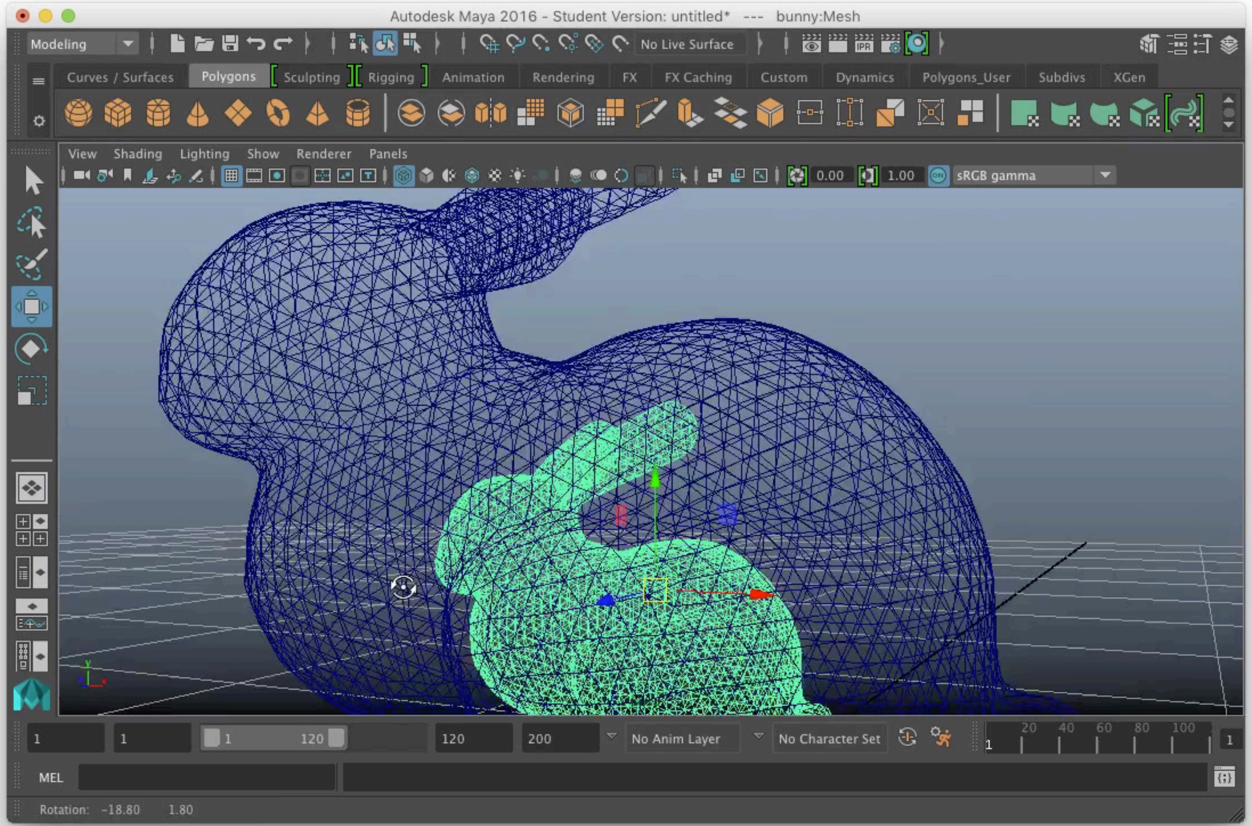

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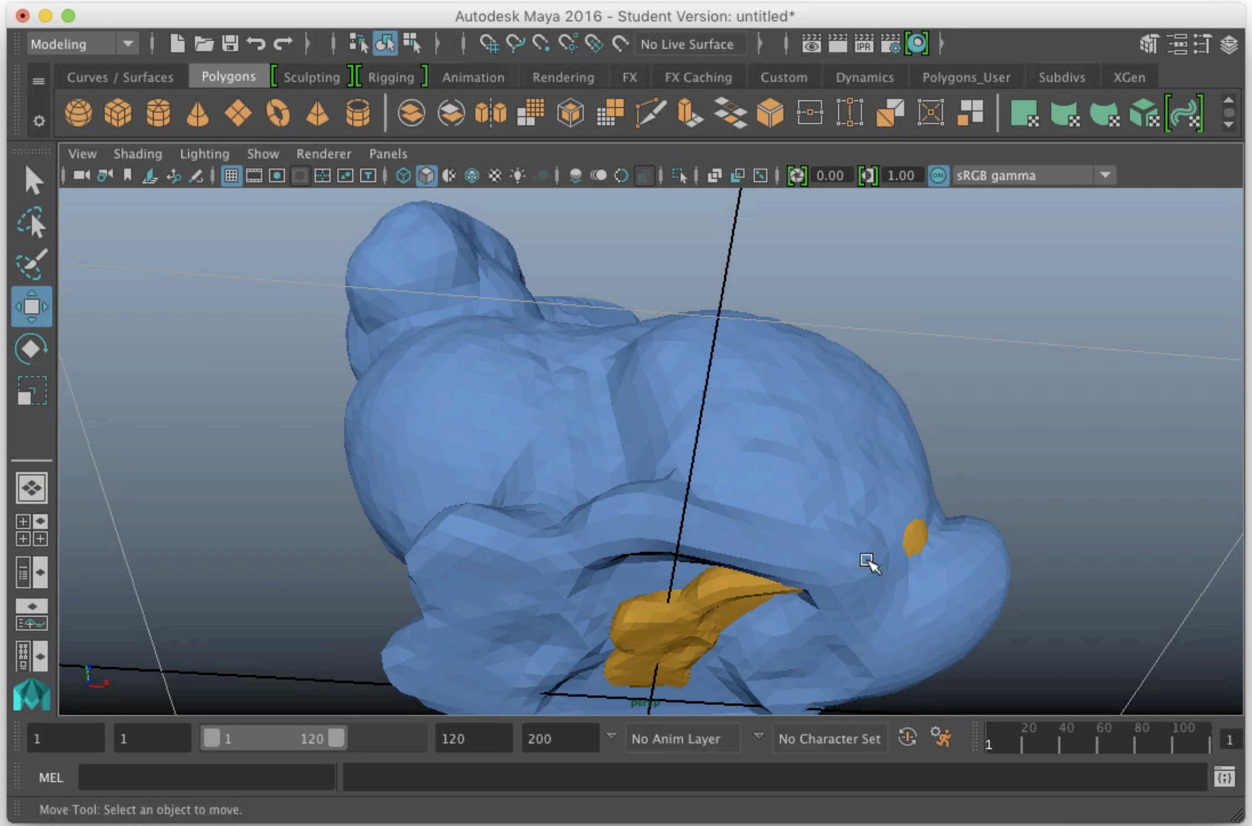

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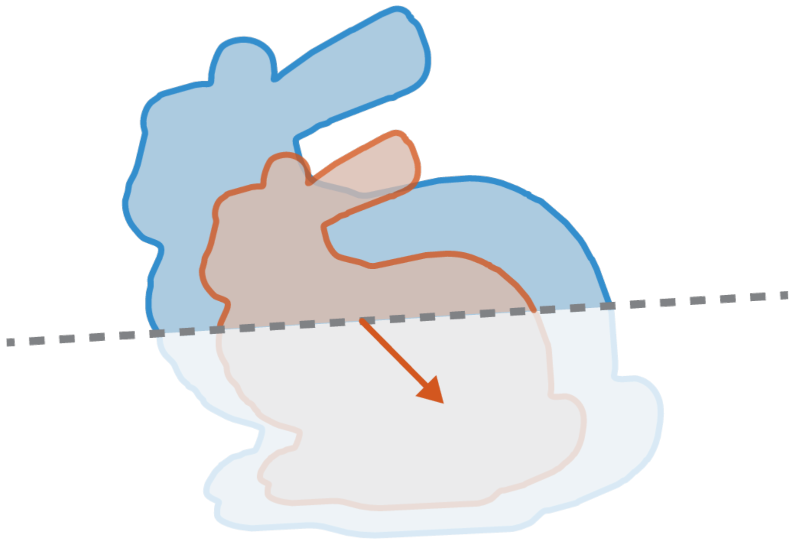
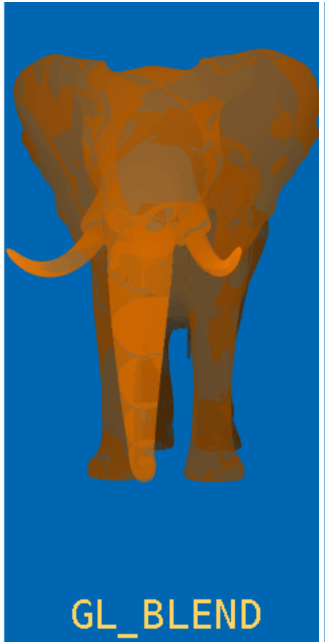
Step 1: we determine feasibility in real-time by exploiting orthographic rendering



Take a clue from order-independent transparency by "depth peeling"



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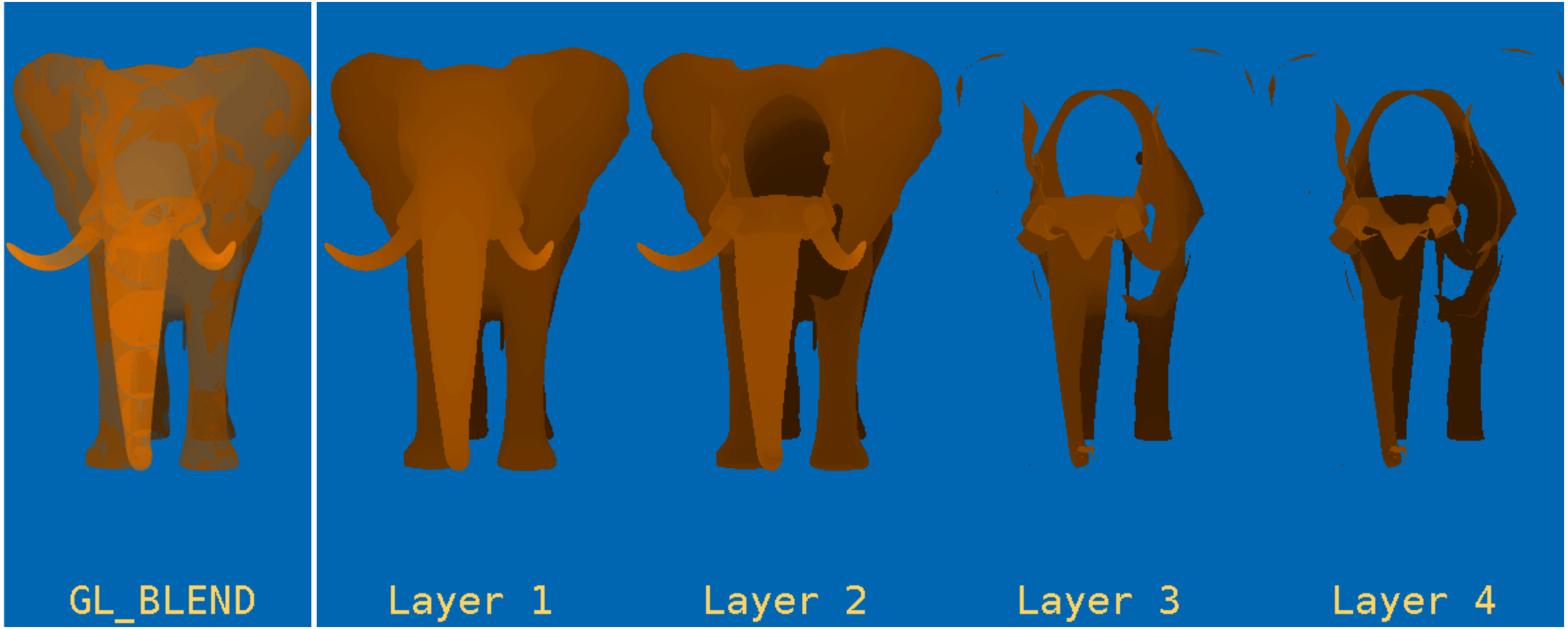

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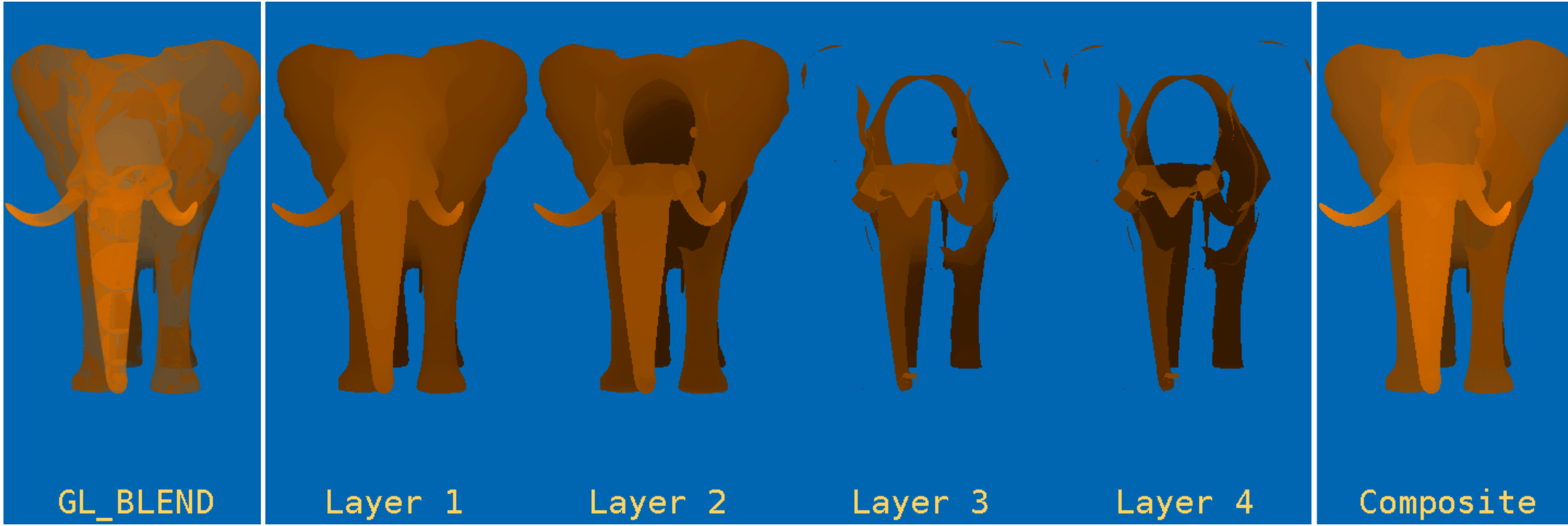

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a.k.a. K-Buffer, Layered Depth Images

transparency	[Everitt 2001, Bavoil et al. 2007]
shape diameter	[Baldacci et al. 2016]
image-based rendering	[Shade et al. 1998]
CNC milling	[Inui & Ohta 2007]
intersection volume	[Faure et al. 2008]
swept volumes	[Kim et al. 2002]
collision detection	[Myszkowski et al.1995, Knott & Pai 2003, Heidelberger et al. 2004]
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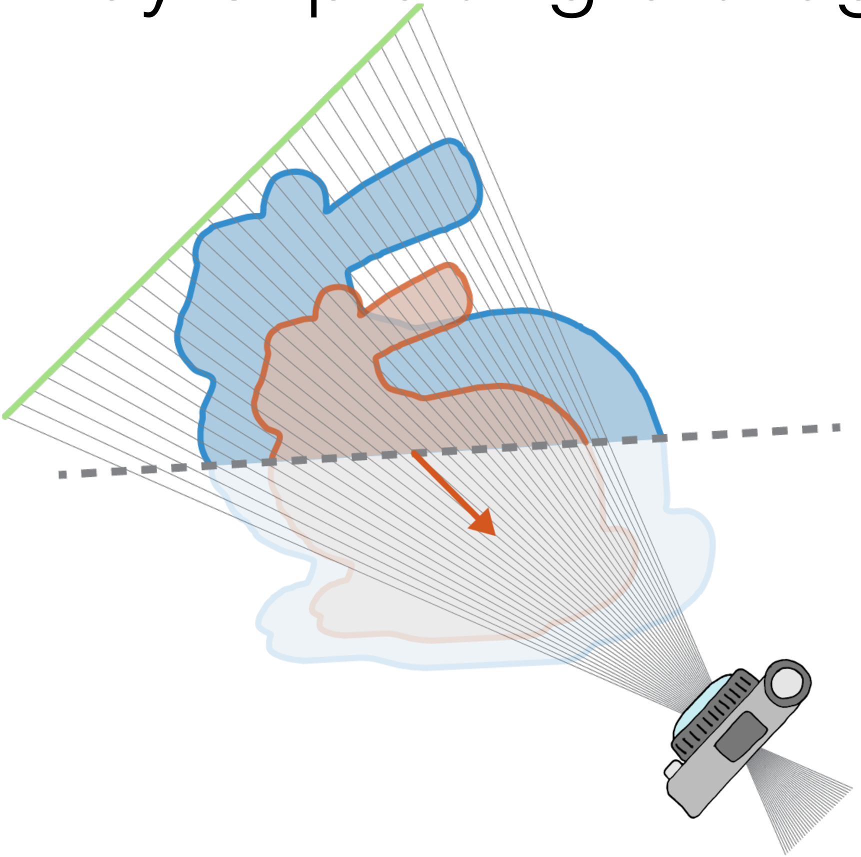

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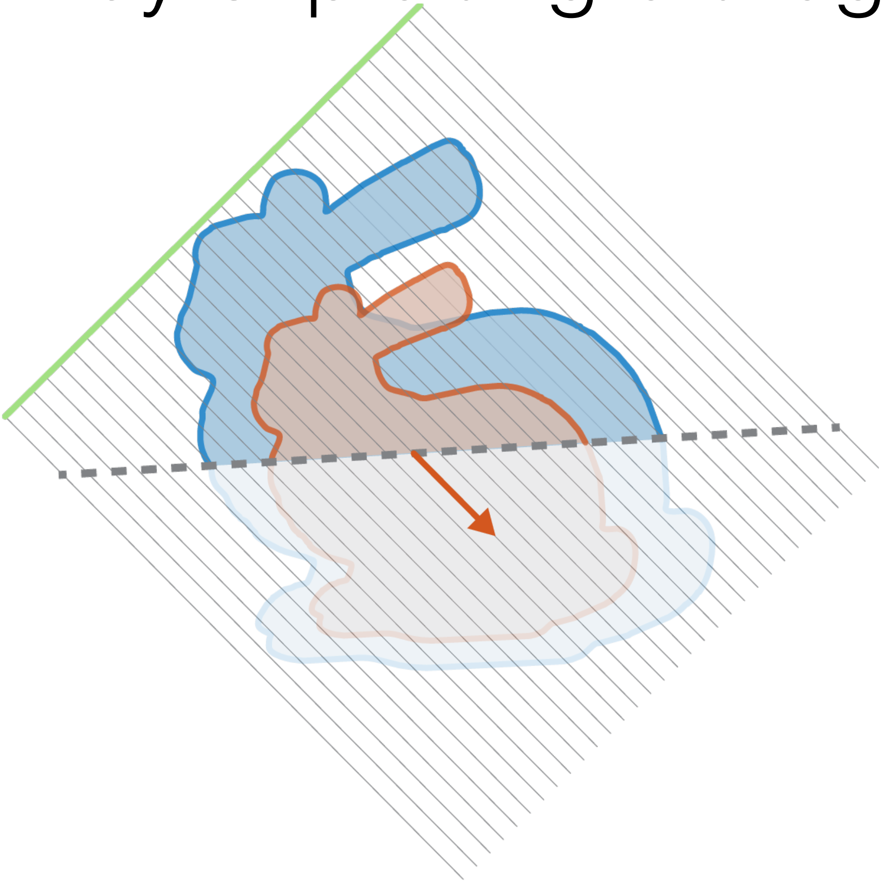

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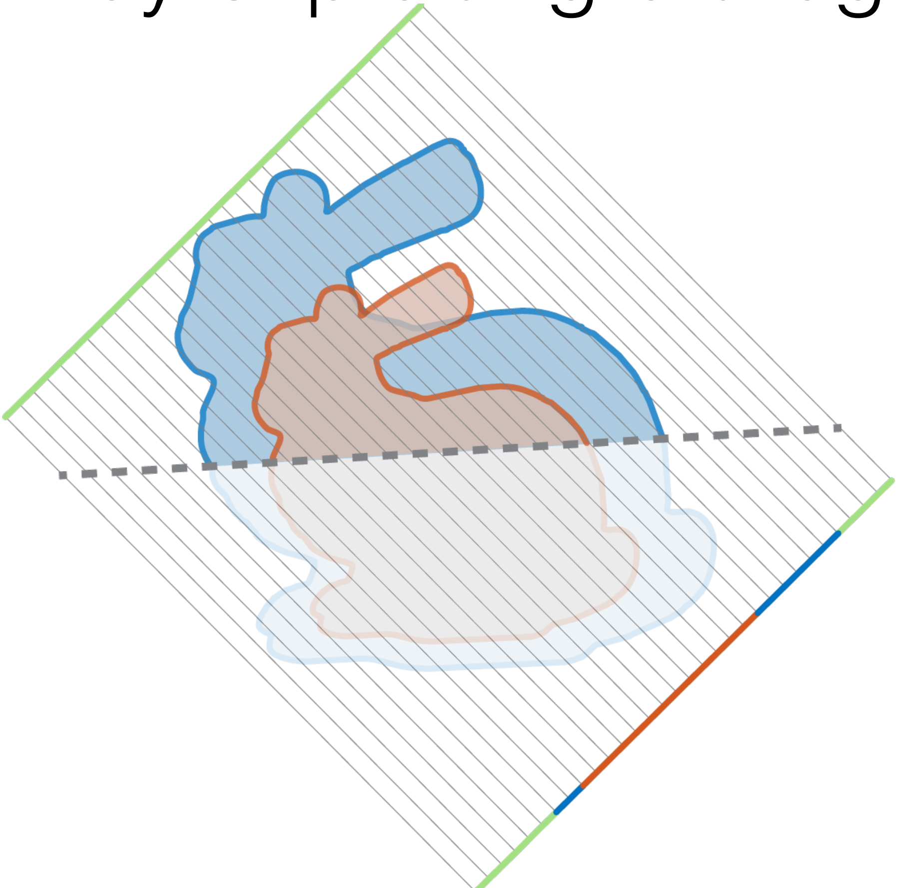

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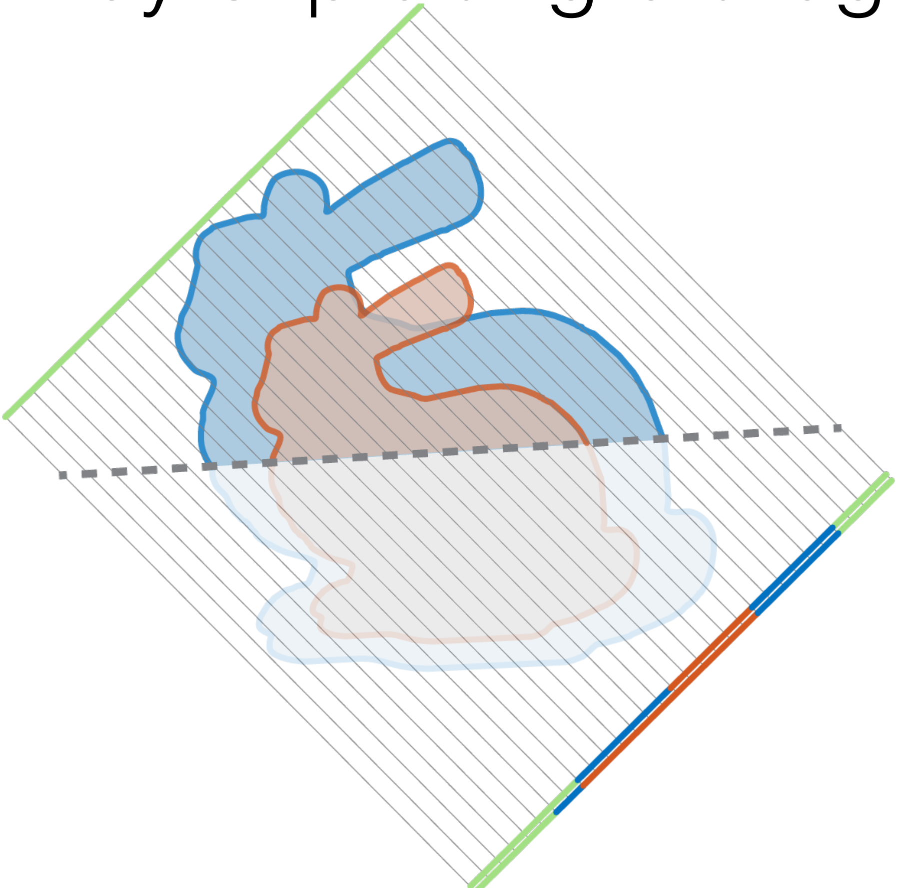

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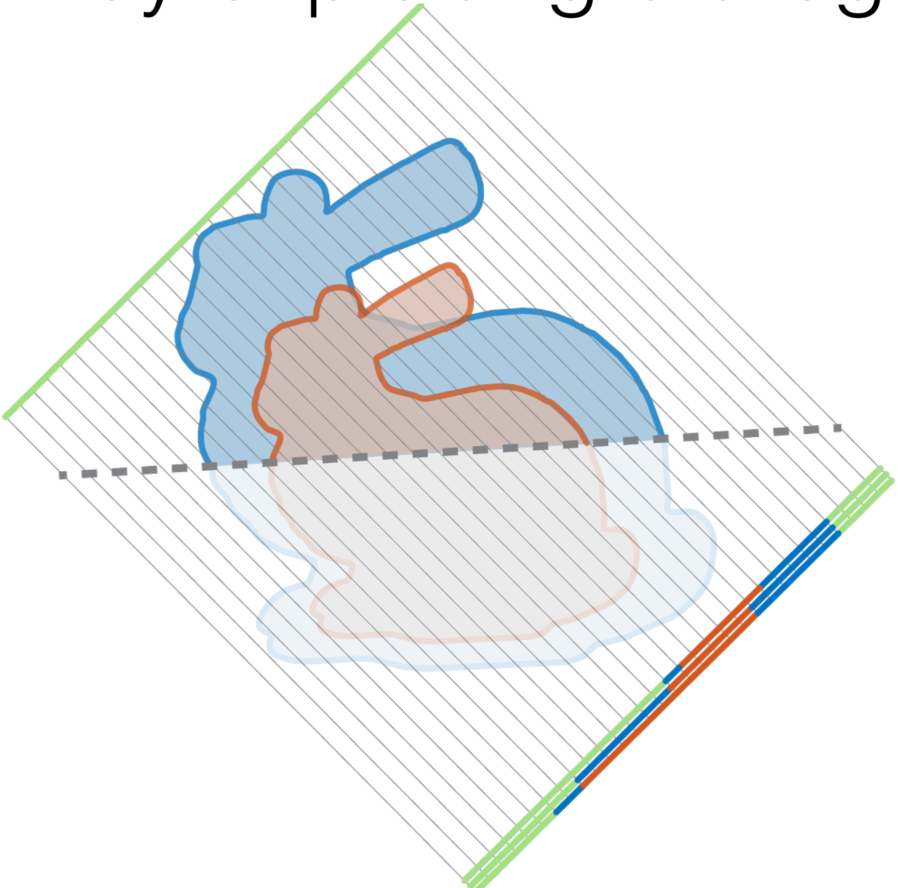

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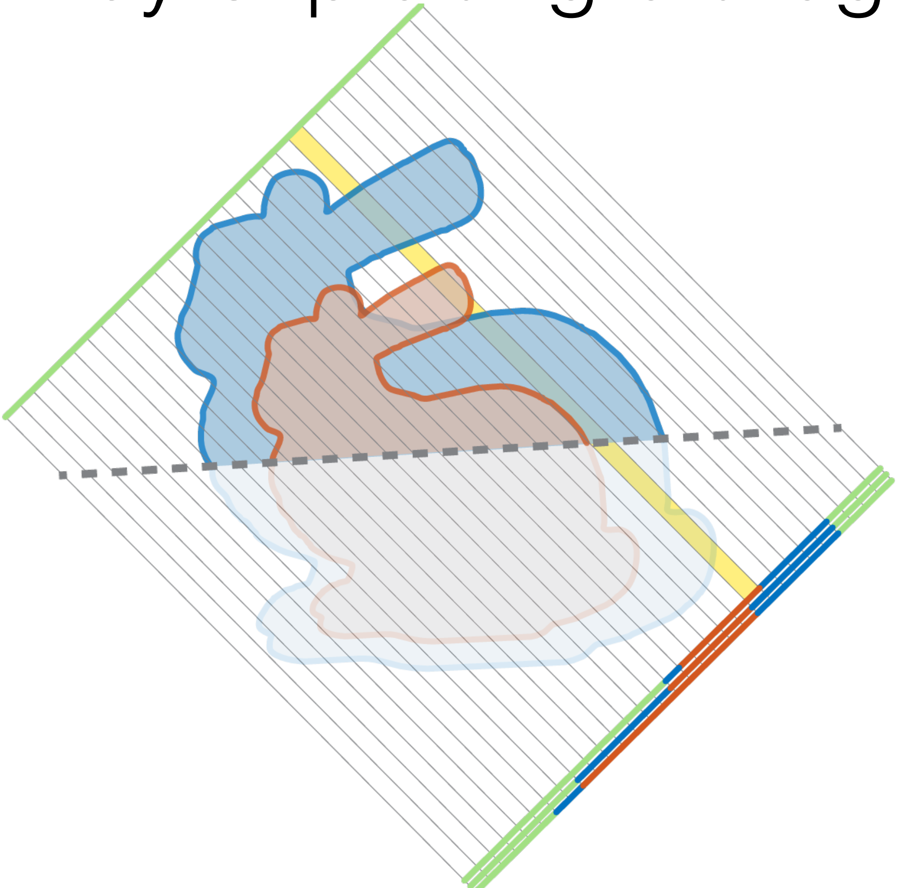

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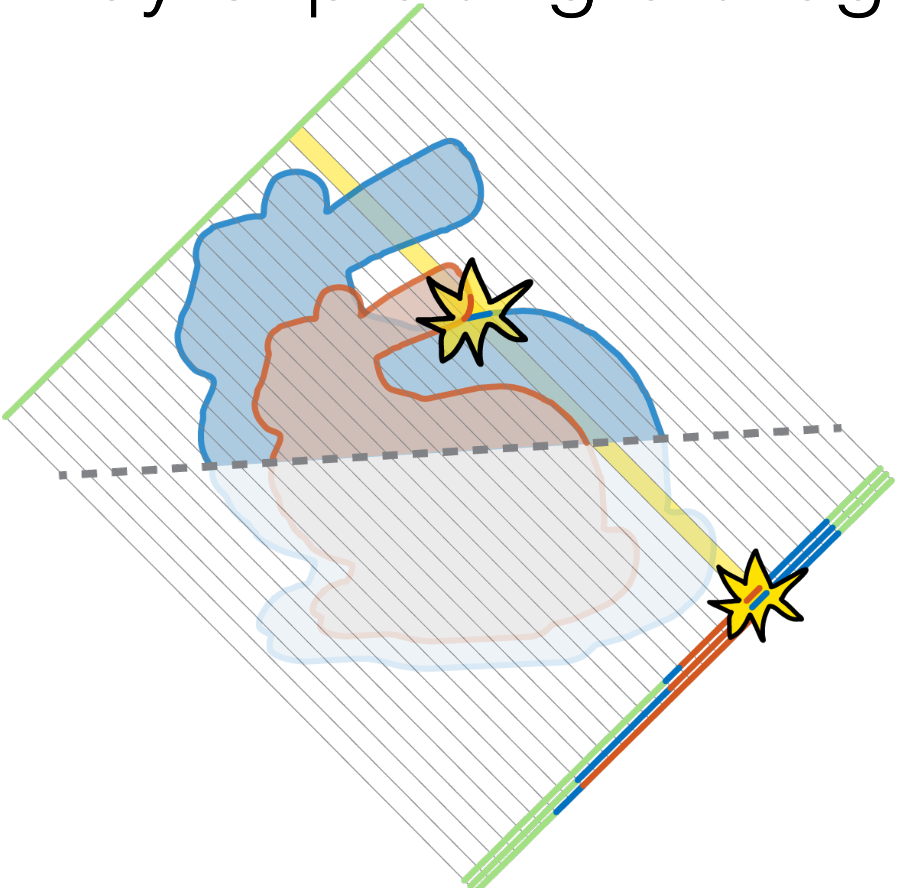

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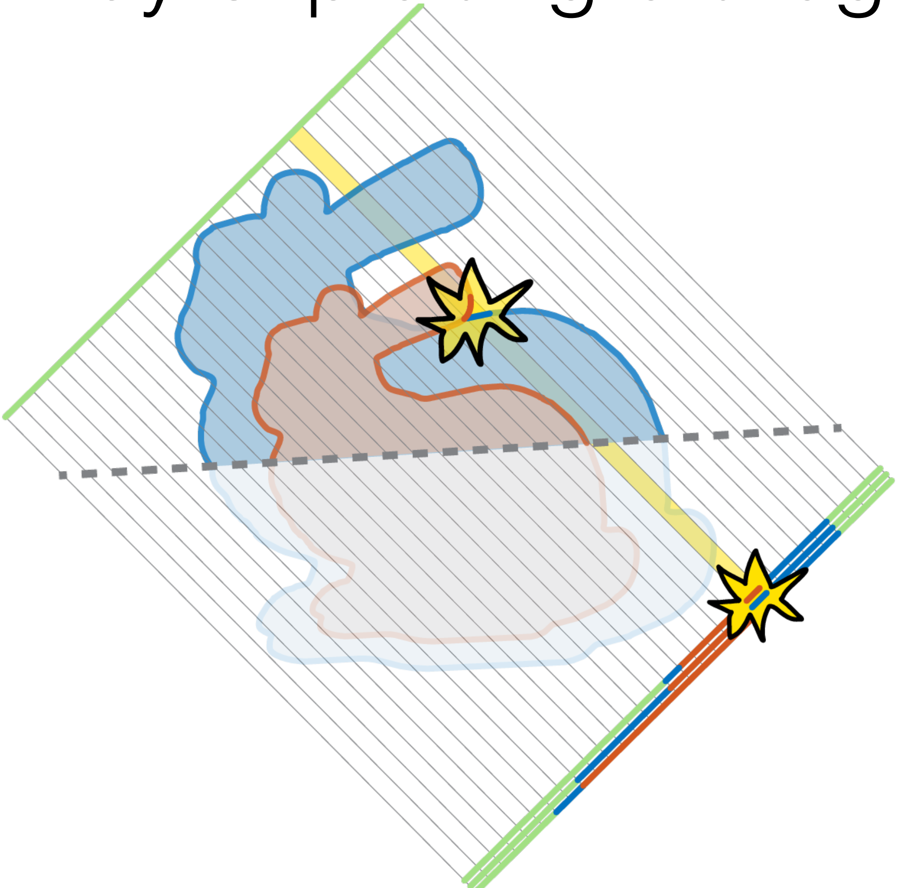

Bad “codes”:

- **blue** before **orange**
- **orange** before **green**
- **orange** before front-facing **blue**

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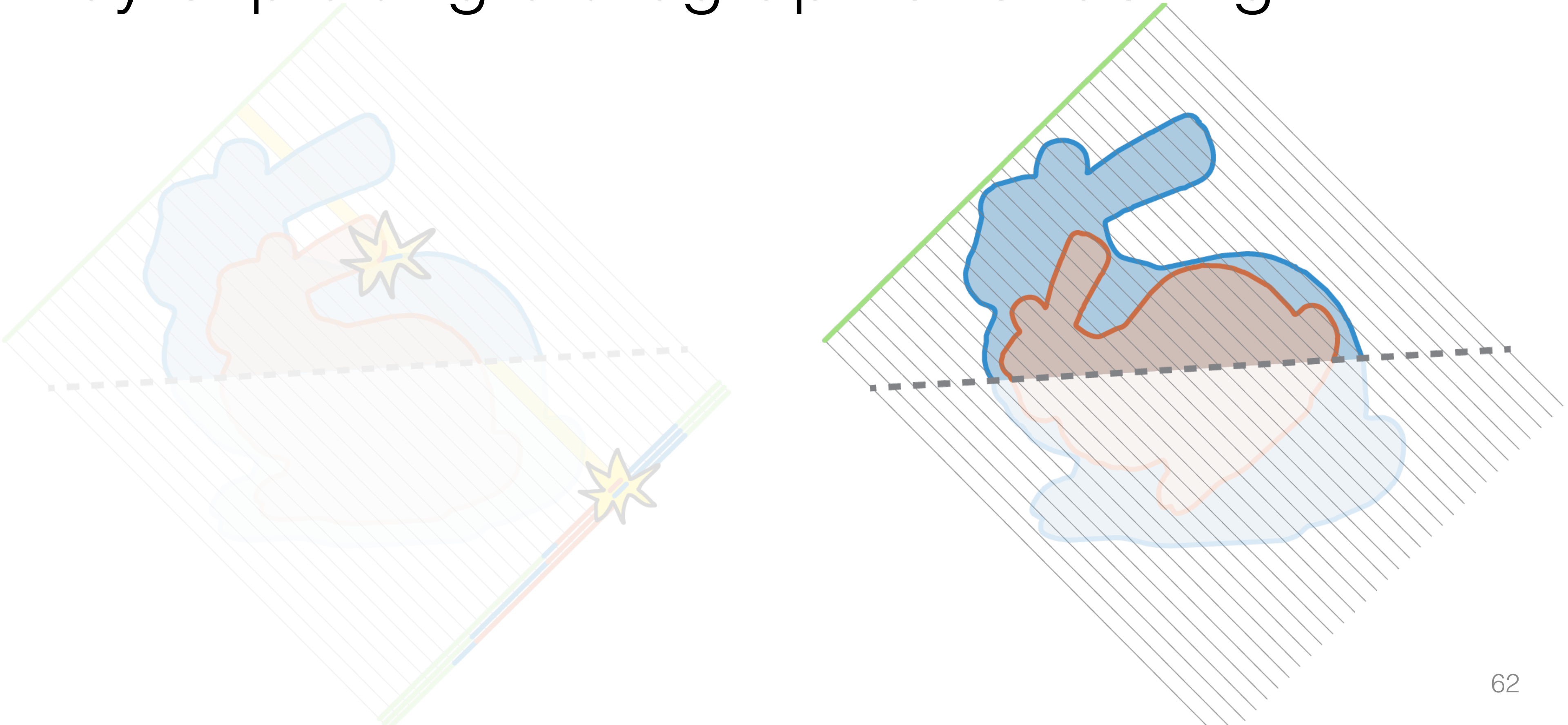

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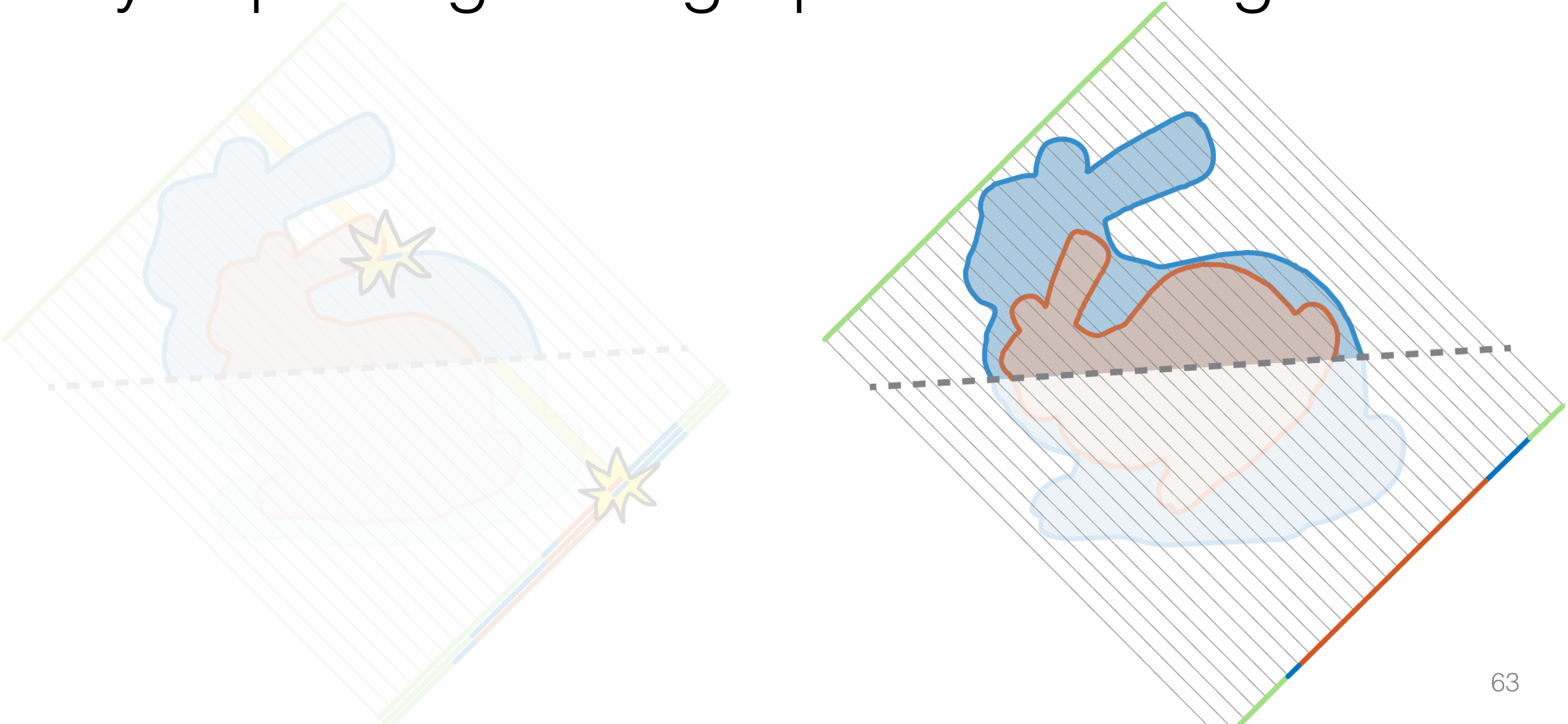

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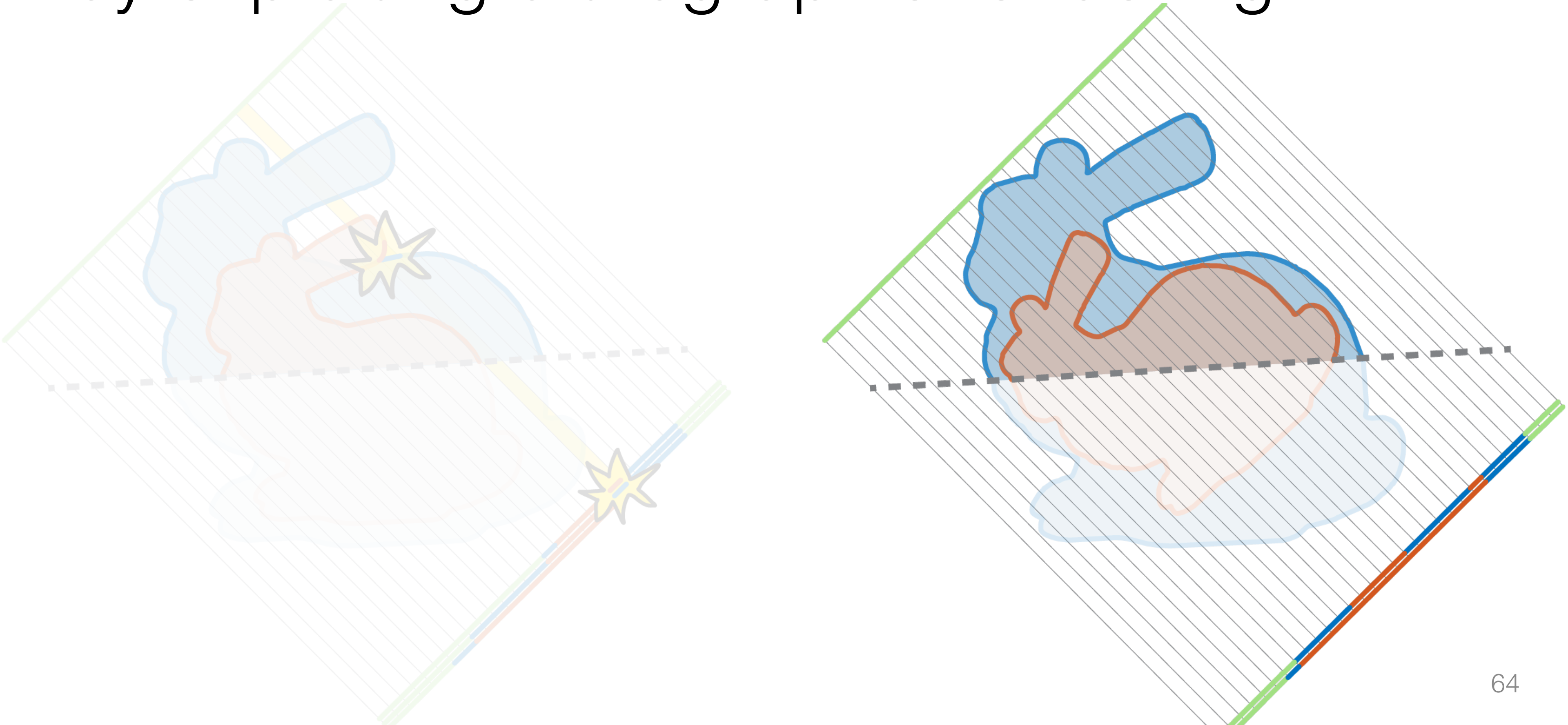

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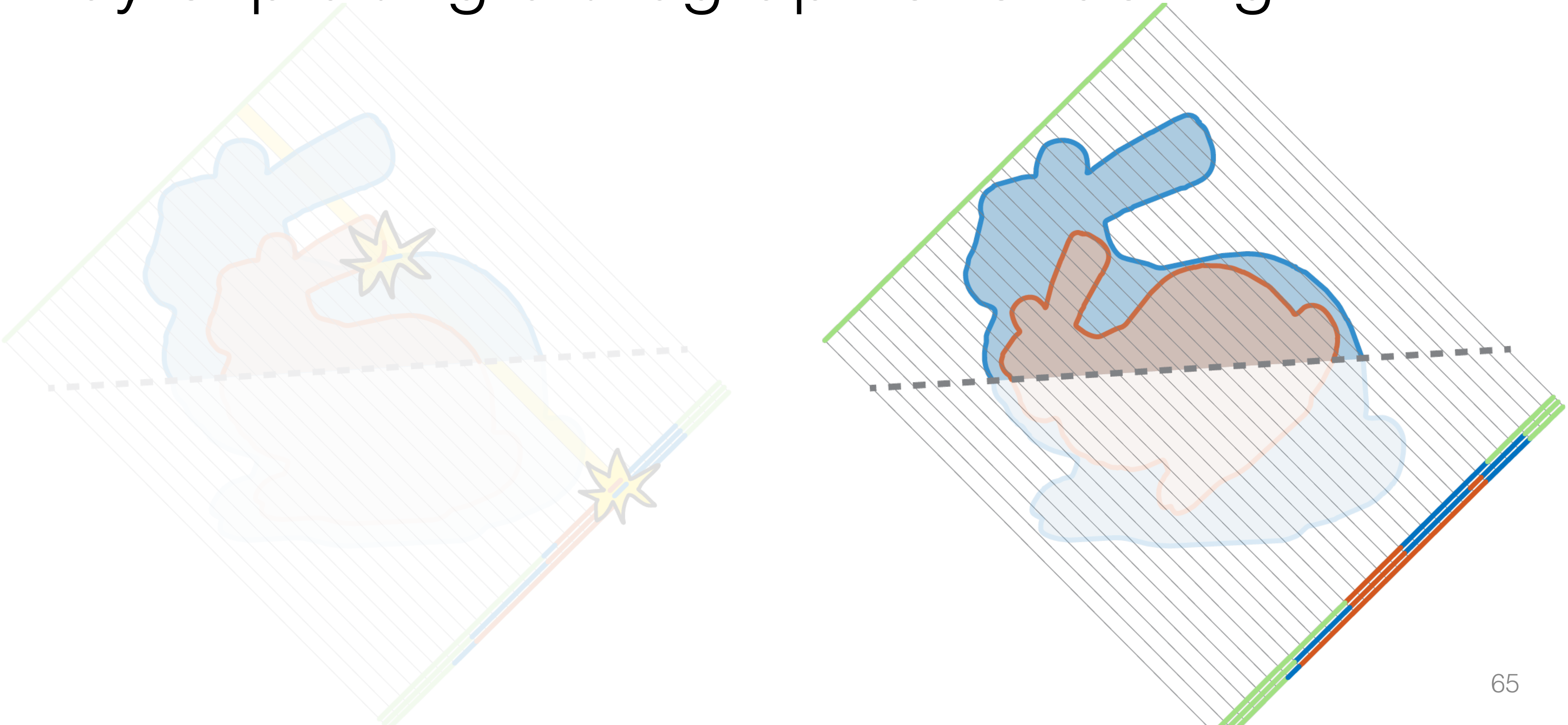

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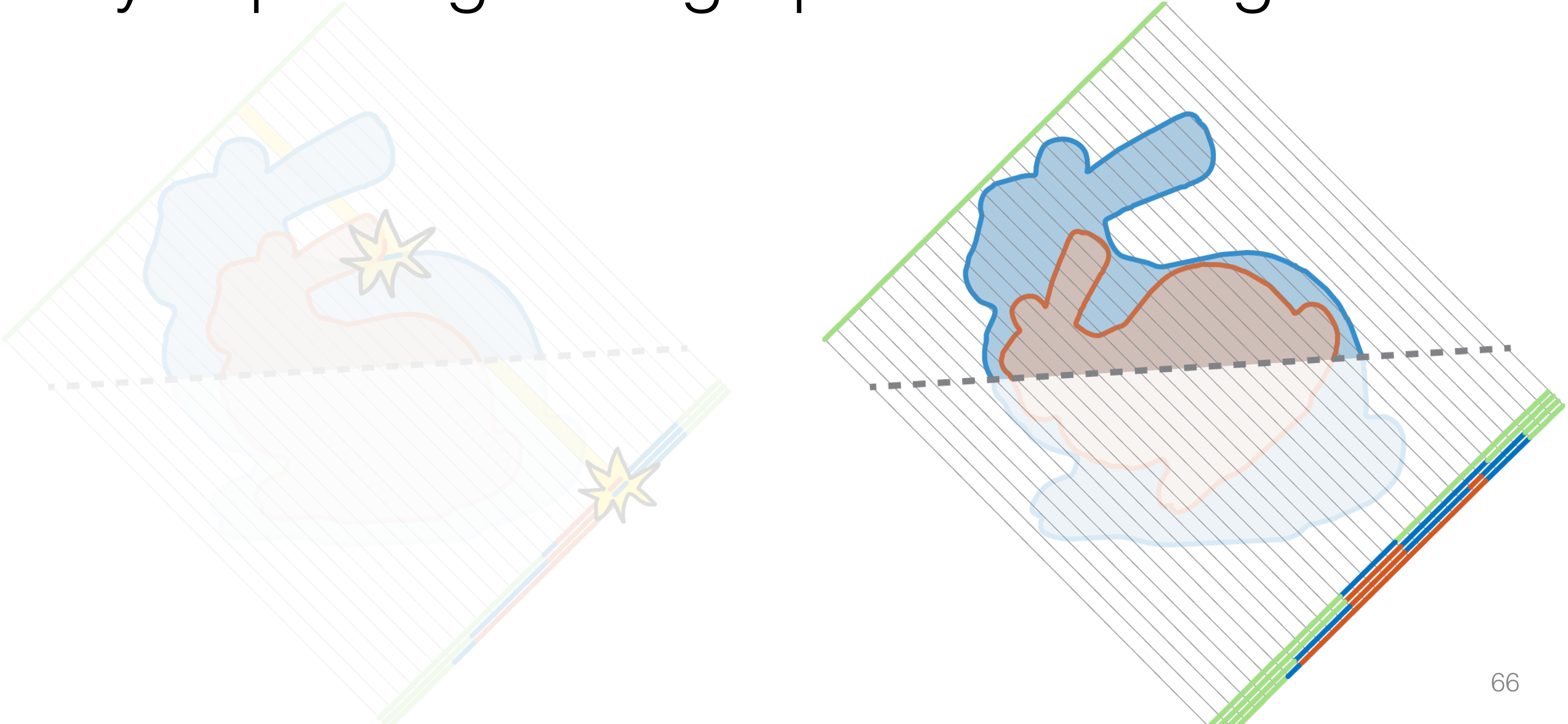

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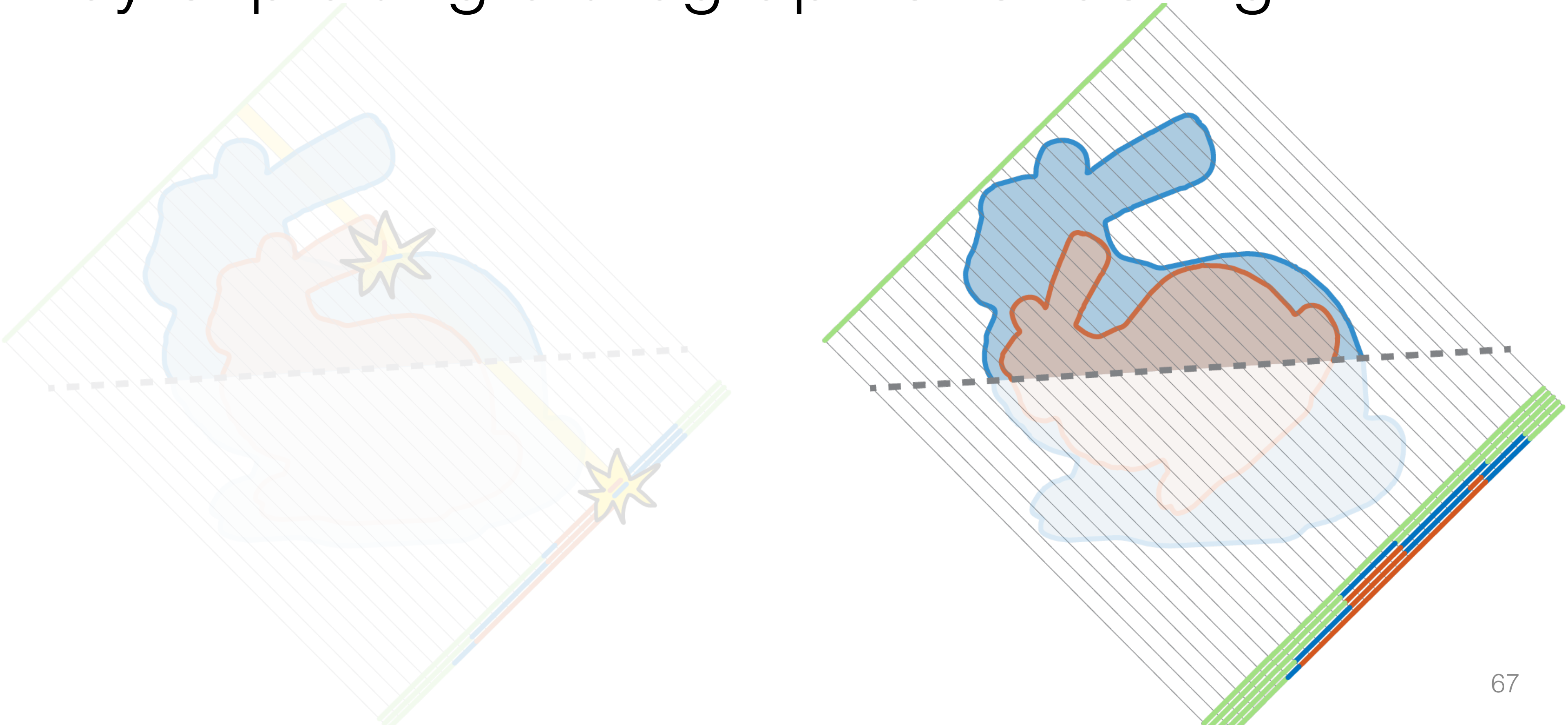

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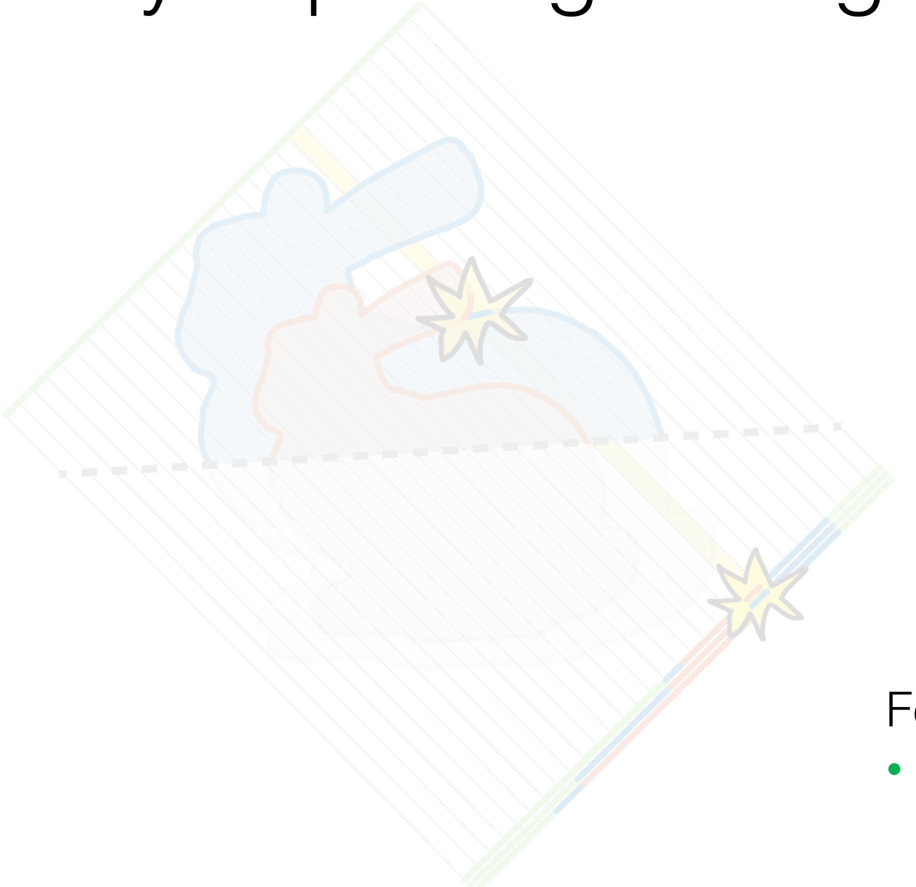

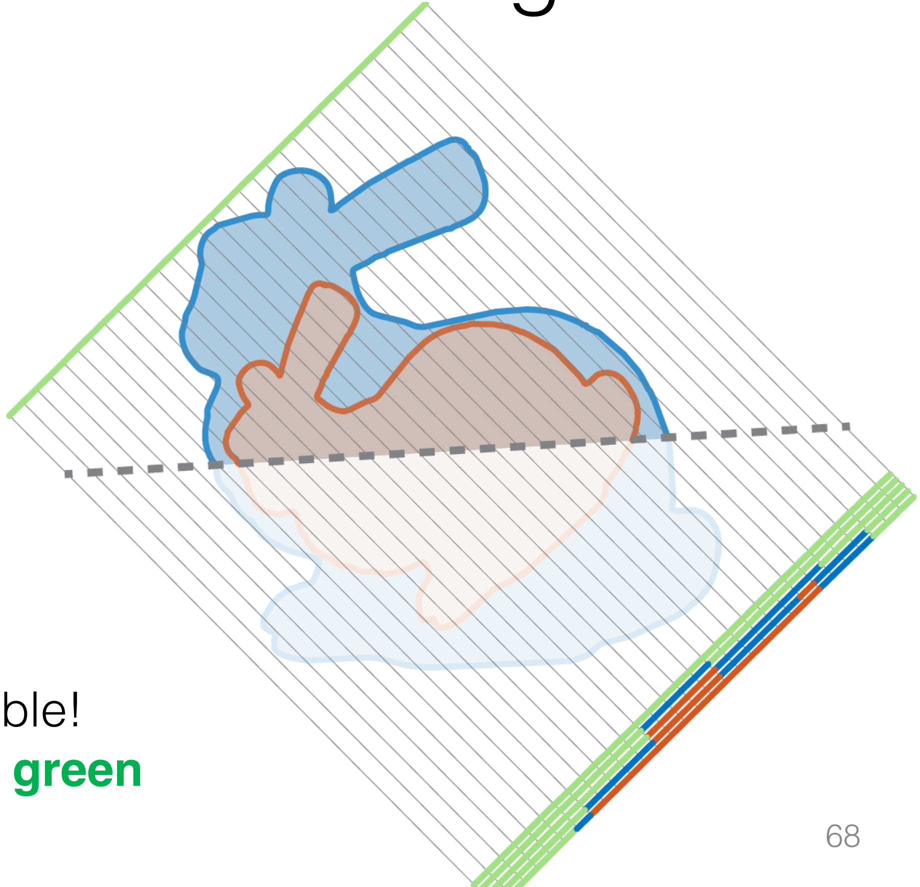

Feasible!

- **all green**

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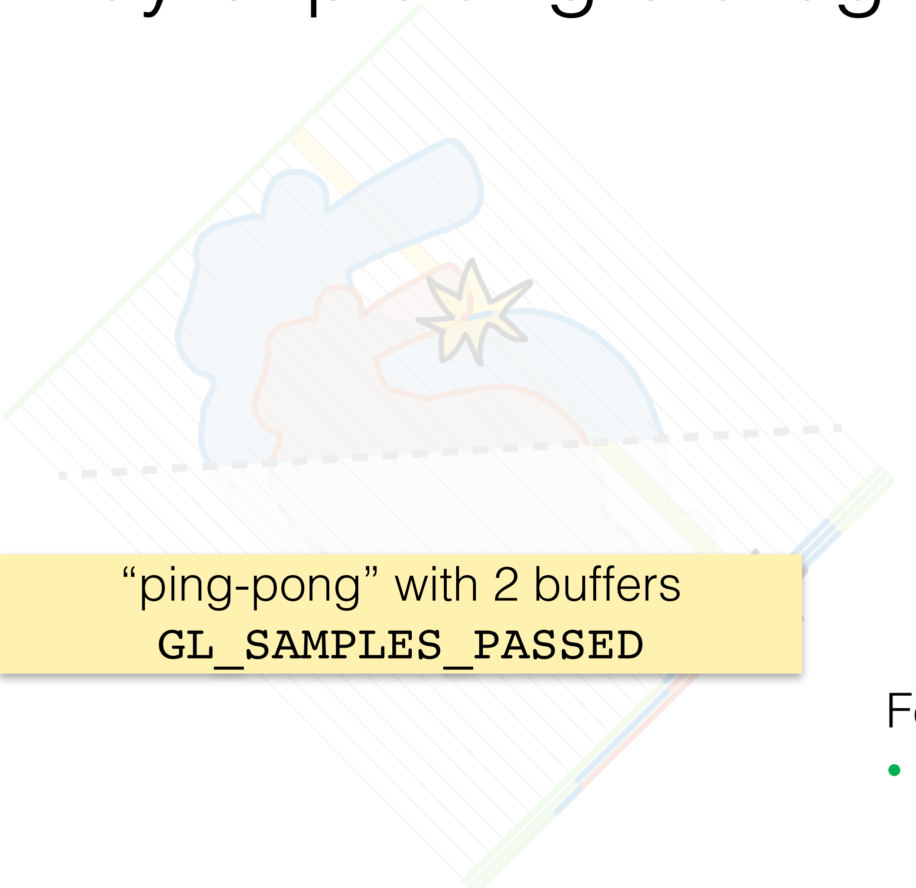
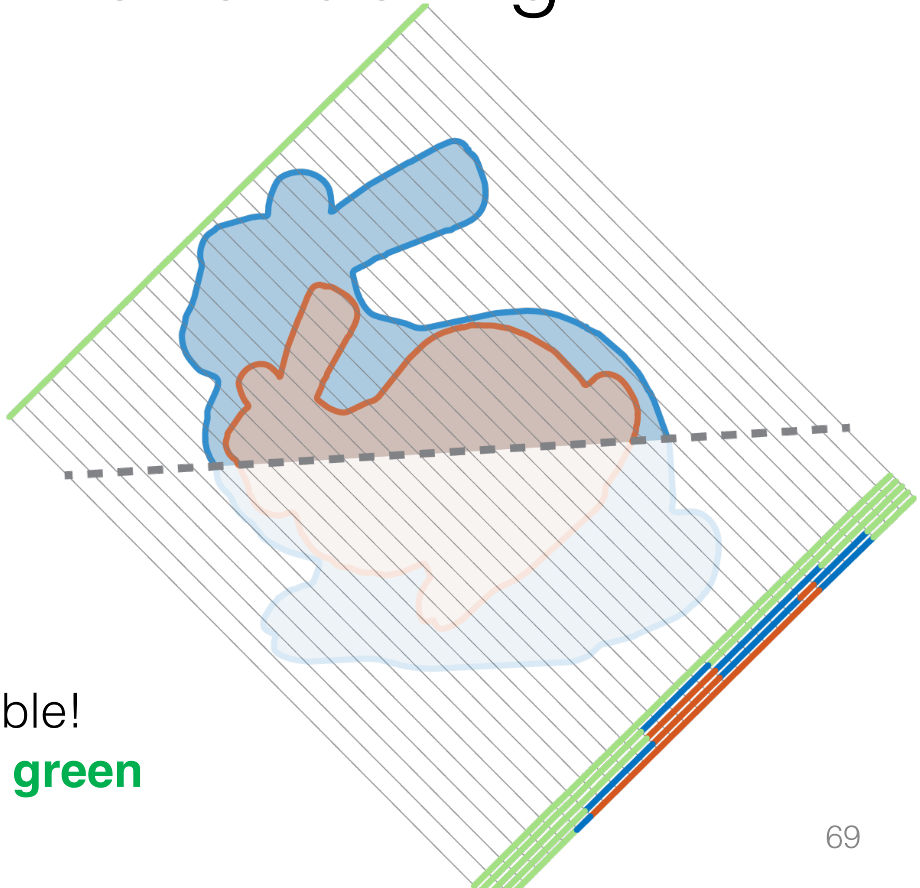


“ping-pong” with 2 buffers
`GL_SAMPLES_PASSED`



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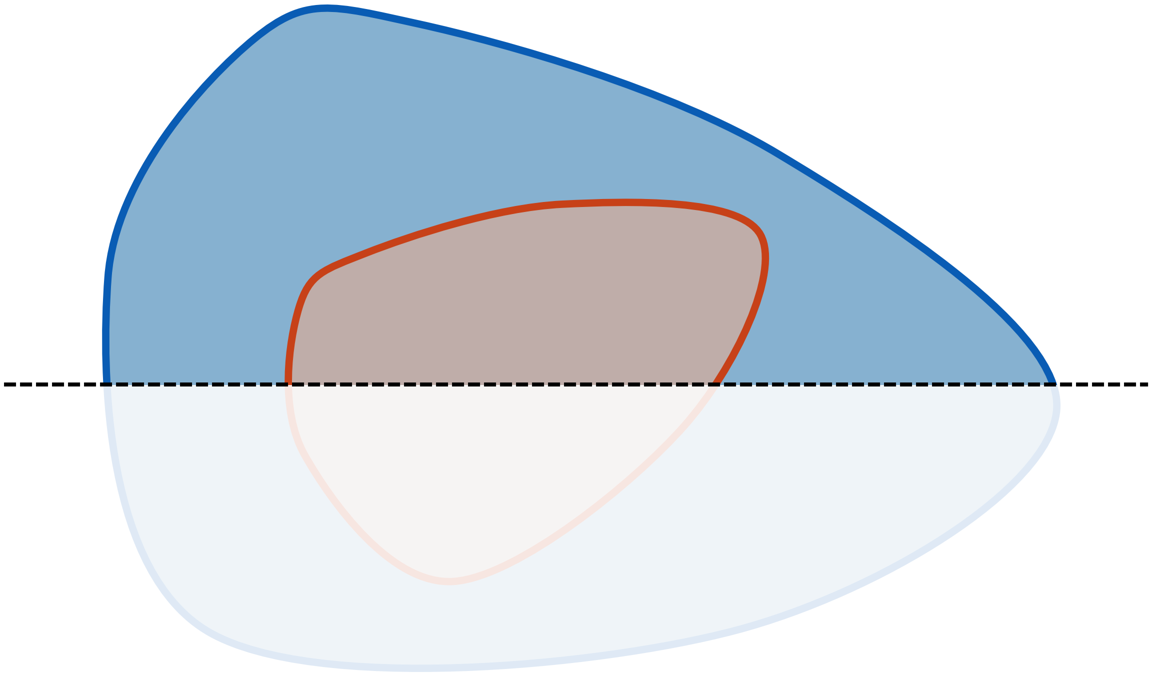
Step 2: binary search to maximize scale



Assume *momentarily* that shape is convex

Fix cut plane, center of mass, rotation

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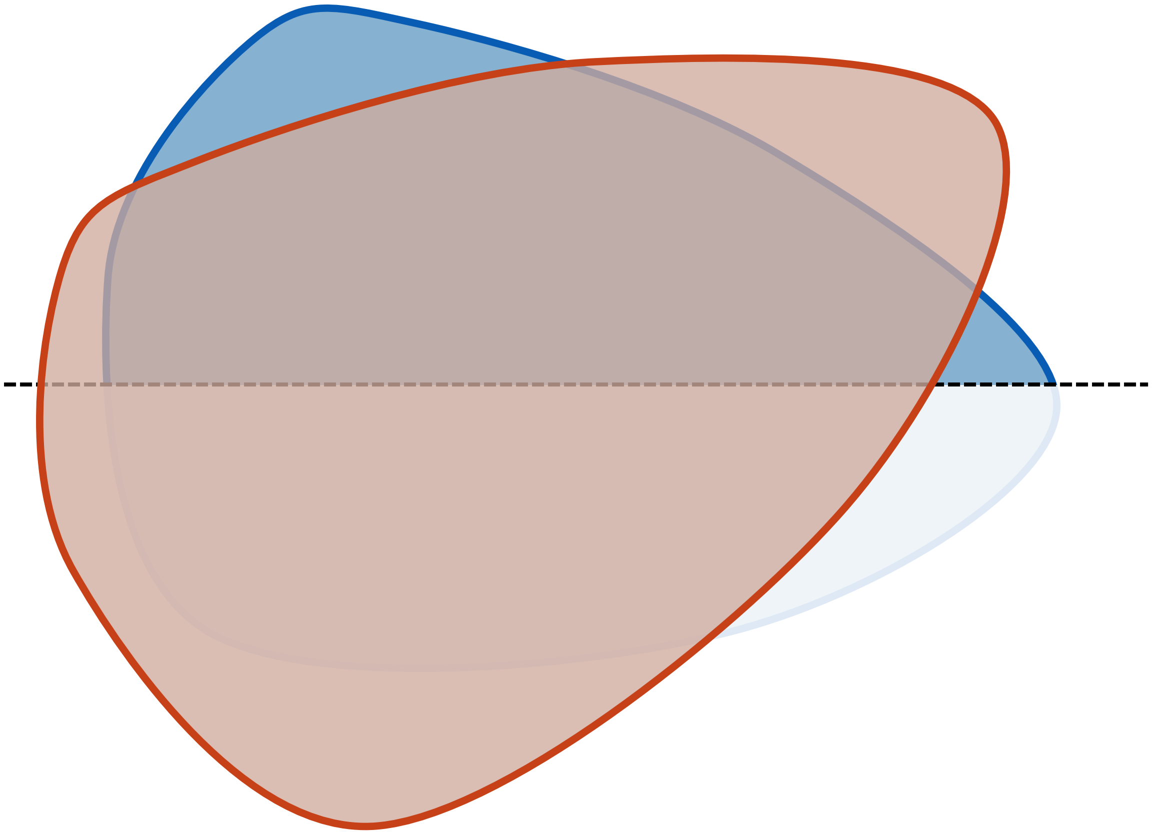

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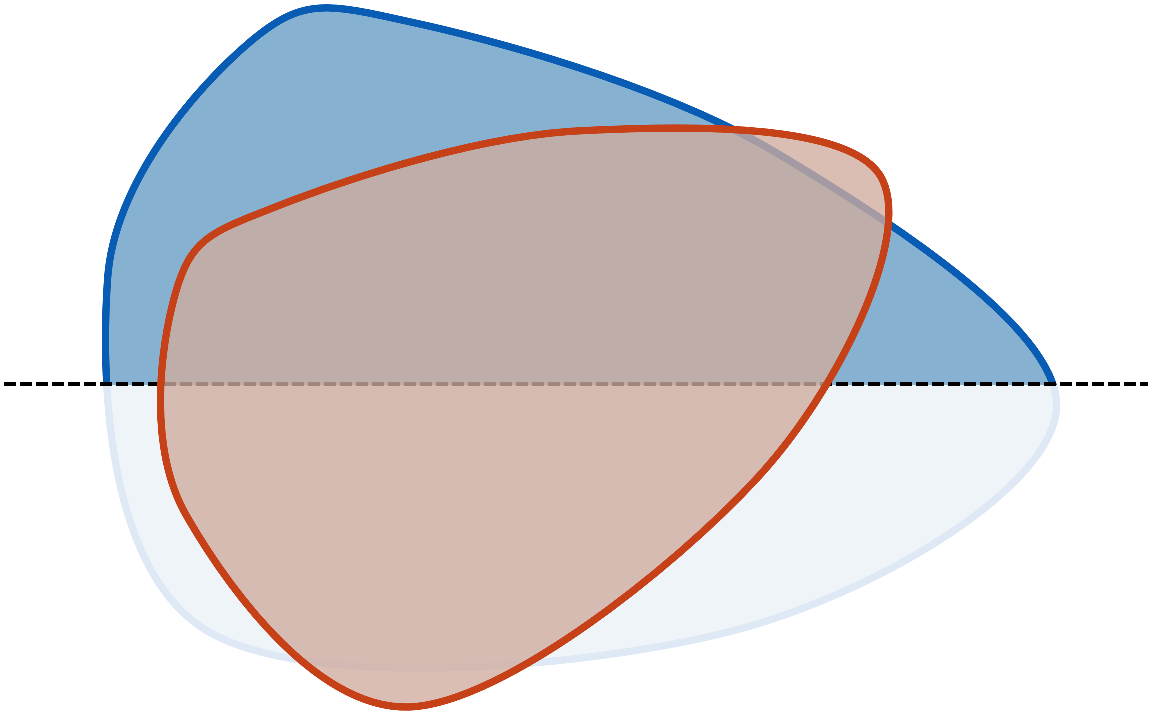

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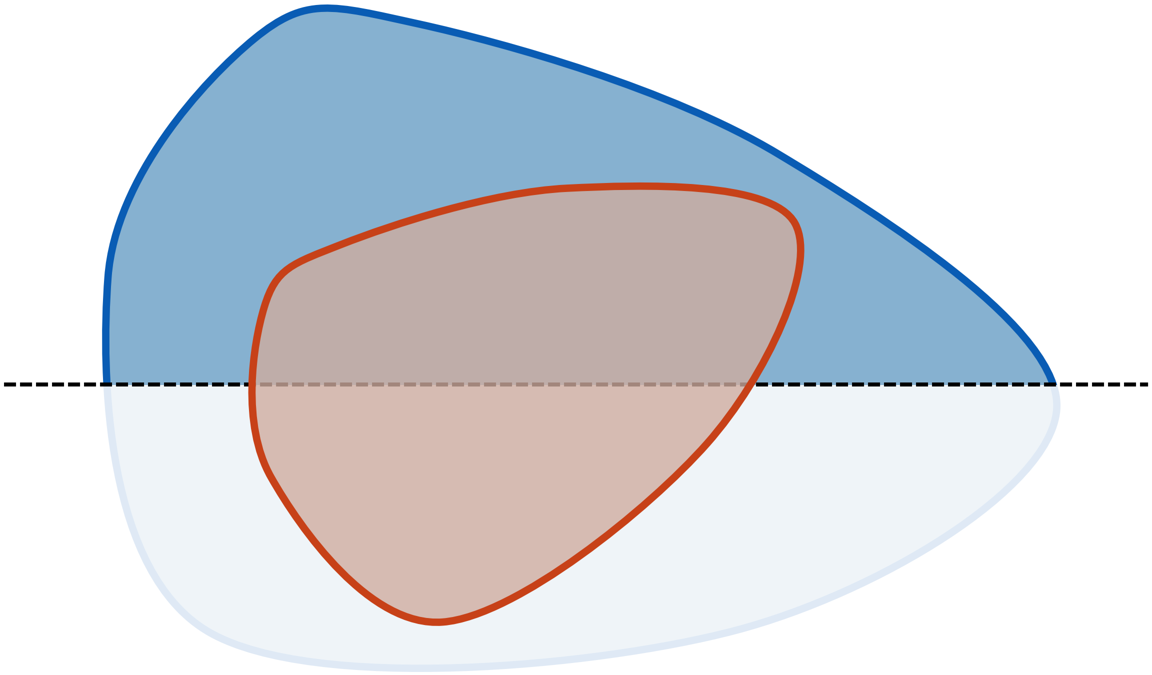

Assume *momentarily* that shape is convex

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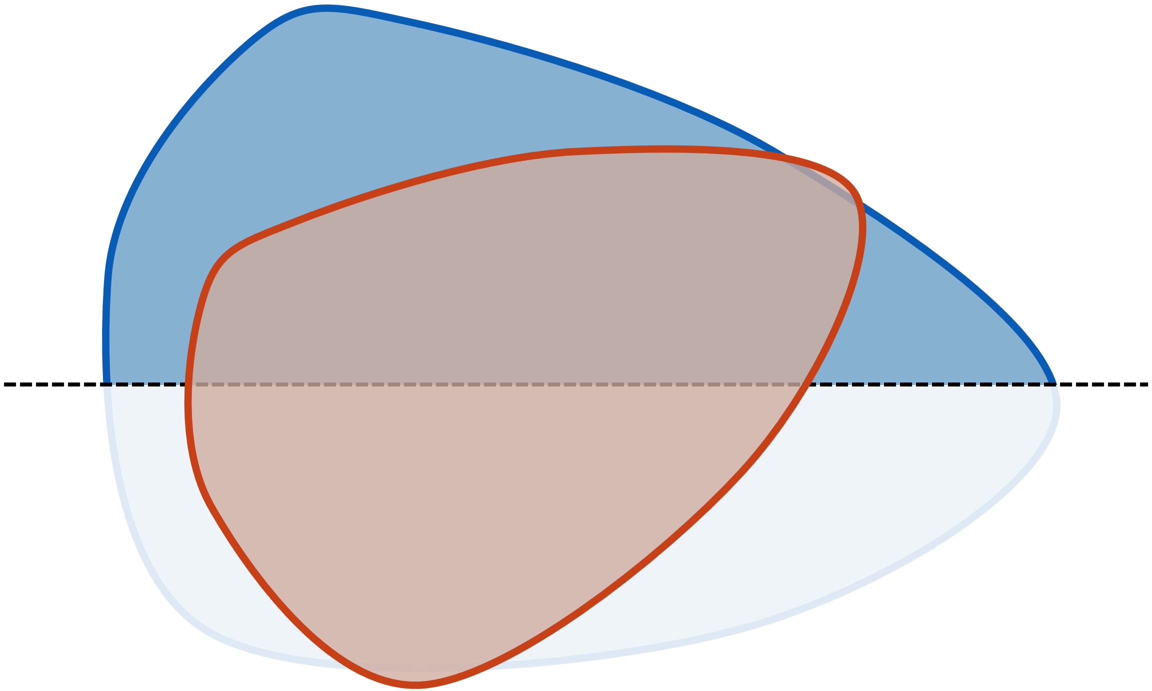

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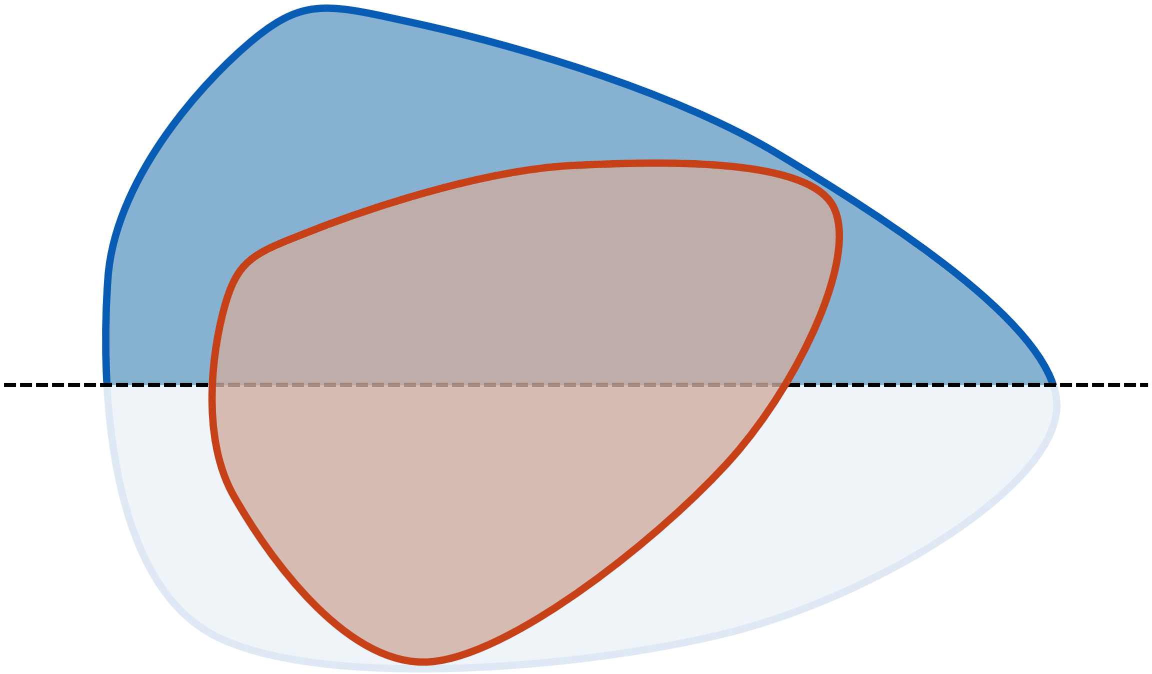

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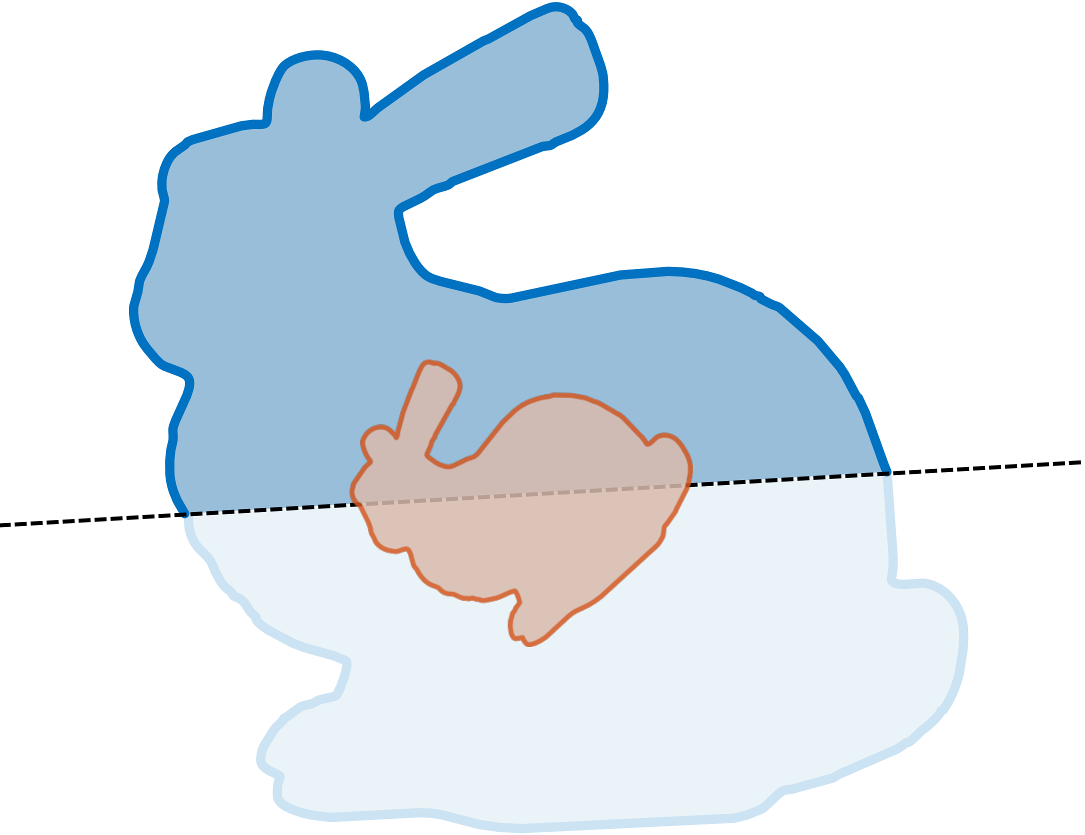

For non-convex shapes
binary search is conservative,

but in practice optimal

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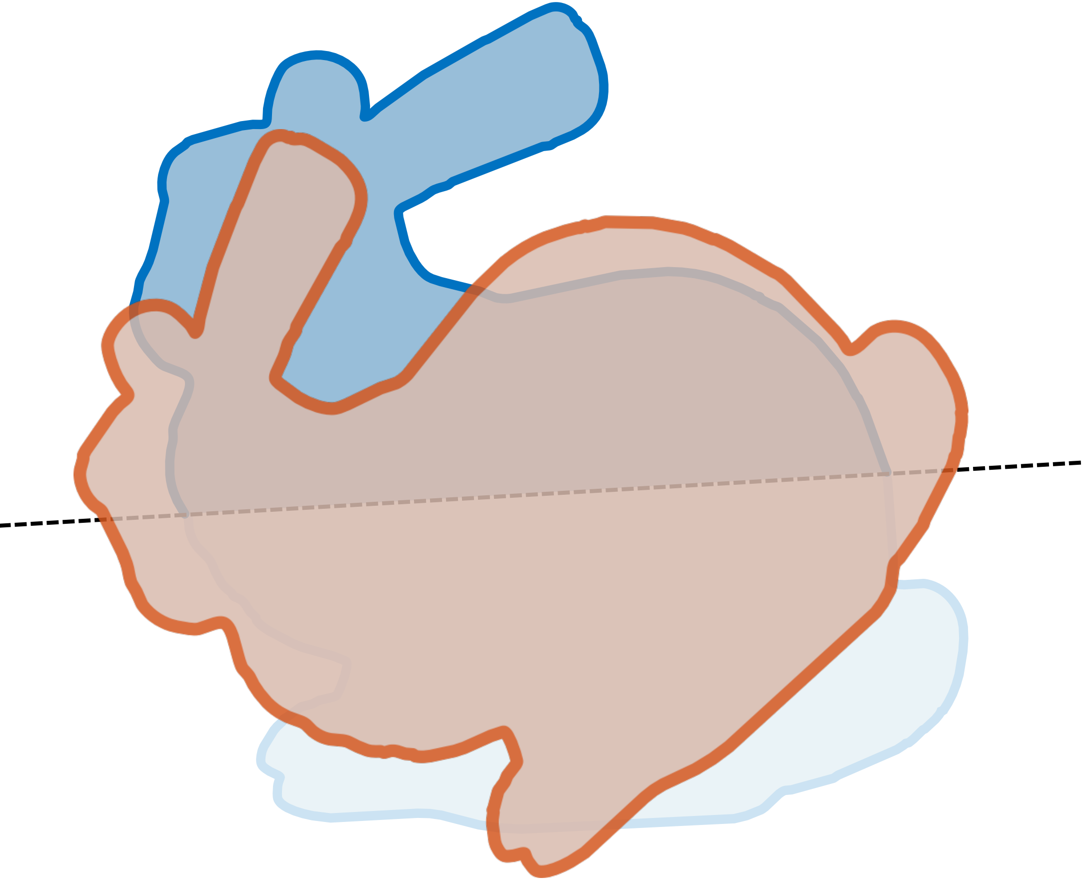

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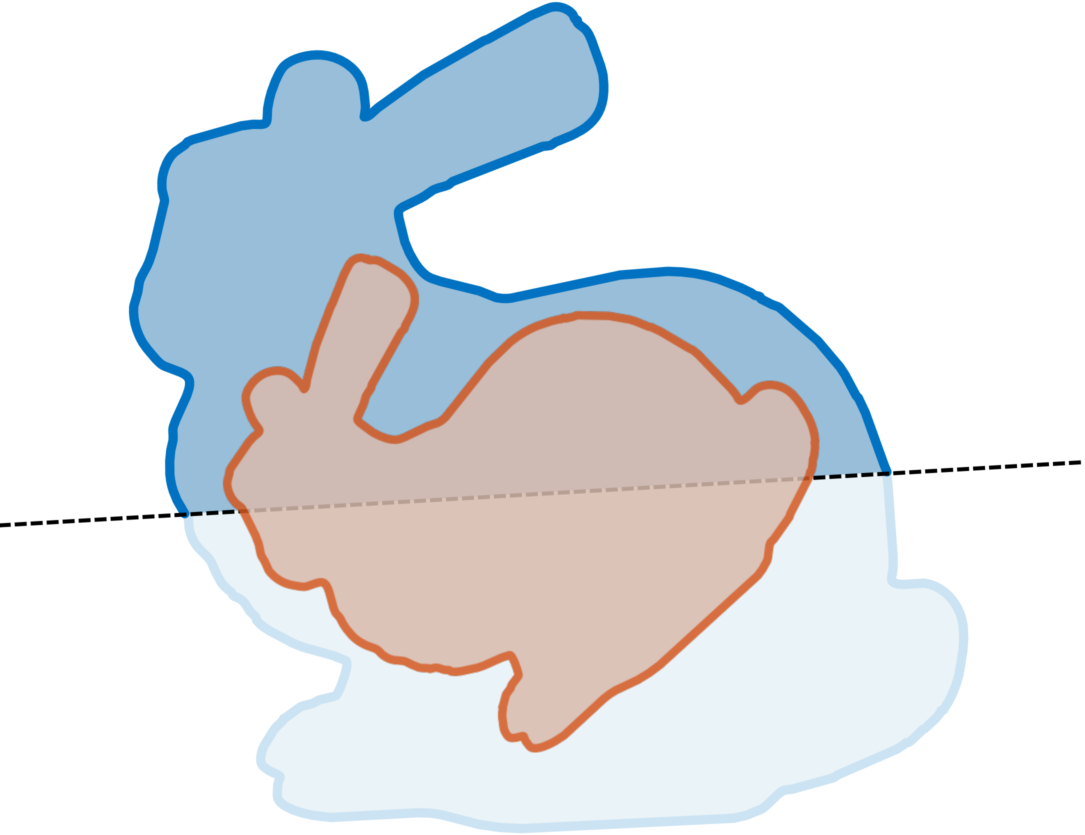

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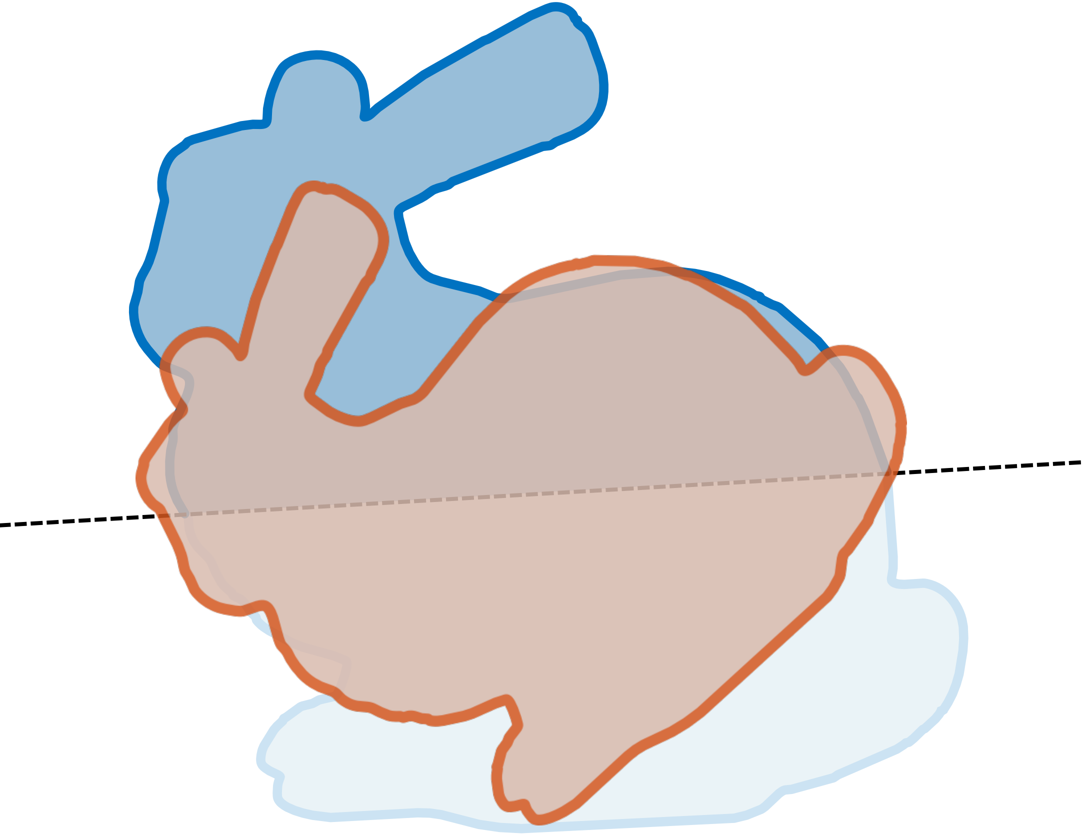

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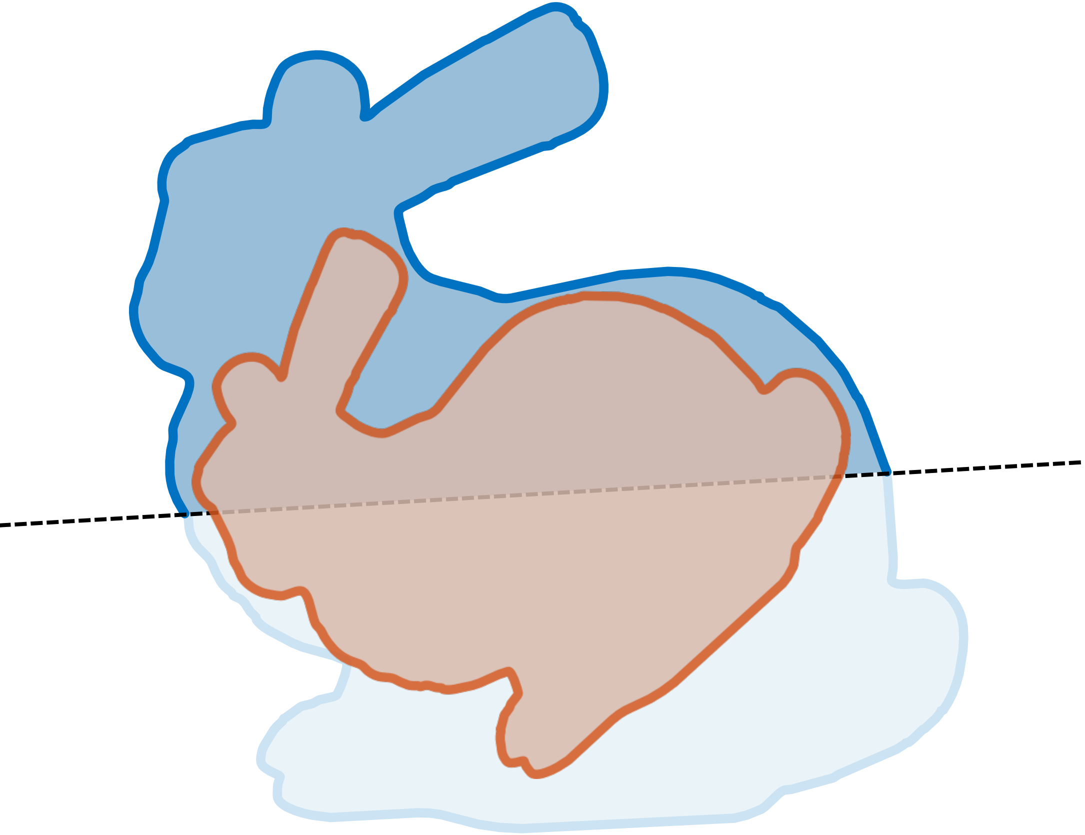

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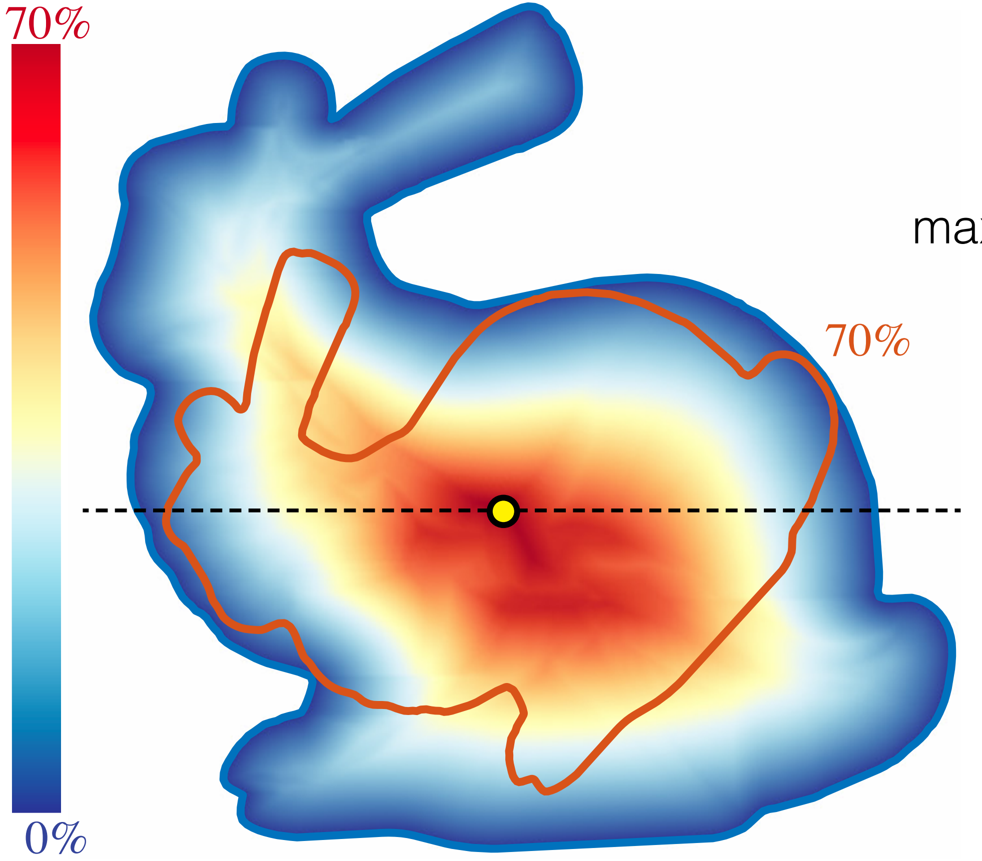
Step 3: optimize over all parameters



maximize *scale* subject to *nesting constraint*

non-convex energy landscape

Step 3: optimize over all parameters *via particle swarm optimization*

k parameter vector as point in *n*D $\mathbf{x}_i \in \mathbb{R}^n$

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k parameter vector as point in *n*D $\mathbf{x}_i \in \mathbb{R}^n$

update each iteration according to "velocity"

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i,$$

Step 3: optimize over all parameters *via particle swarm optimization*

k parameter vector as point in *n*D $\mathbf{x}_i \in \mathbb{R}^n$

pull velocity toward ***personal best*** *and* ***global best*** *of swarm*

$$\mathbf{v}_i \leftarrow \omega \mathbf{v}_i + \phi_p r_p \left(\boxed{\mathbf{x}_i^p} - \mathbf{x}_i \right) + \phi_g r_g \left(\boxed{\mathbf{x}^g} - \mathbf{x}_i \right),$$

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i,$$

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random perturbations

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i,$$

Naive P-Swarm would treat scale as just another parameter (coordinate)…

$$\begin{aligned} \underset{s,\mathbf{R},\mathbf{c},\mathbf{P},\mathbf{a}^+,\mathbf{a}^-}{\text{maximize}} \quad & s \\ \text{such that} \quad & \mathbf{T}(\mathcal{B}) \text{ nests in } \mathcal{A} \text{ w.r.t. } \mathbf{P}, \mathbf{a}^+, \mathbf{a}^- \end{aligned}$$

… instead optimize over all others,

$$\underset{\mathbf{R},\mathbf{c},\mathbf{P},\mathbf{a}^+,\mathbf{a}^-}{\text{maximize}} \quad f(\mathbf{R}, \mathbf{c}, \mathbf{P}, \mathbf{a}^+, \mathbf{a}^-)$$

all other parameters

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where

$$\begin{aligned} f(\mathbf{R},\mathbf{c},\mathbf{P},\mathbf{a}^+,\mathbf{a}^-) = & \boxed{\underset{s}{\text{maximize}} \quad s} \\ \text{such that} \quad & \mathbf{T}(\mathcal{B}) \text{ nests in } \mathcal{A} \text{ w.r.t. } \mathbf{P},\mathbf{a}^+,\mathbf{a}^- \end{aligned}$$

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where

$$f \approx \underset{s}{\text{search}}(\mathbf{R}, \mathbf{c}, \mathbf{P}, \mathbf{a}^+, \mathbf{a}^-)$$

abort search early if
upper bound < *best*

Our optimization enables fully automatic Matryoshka generation…

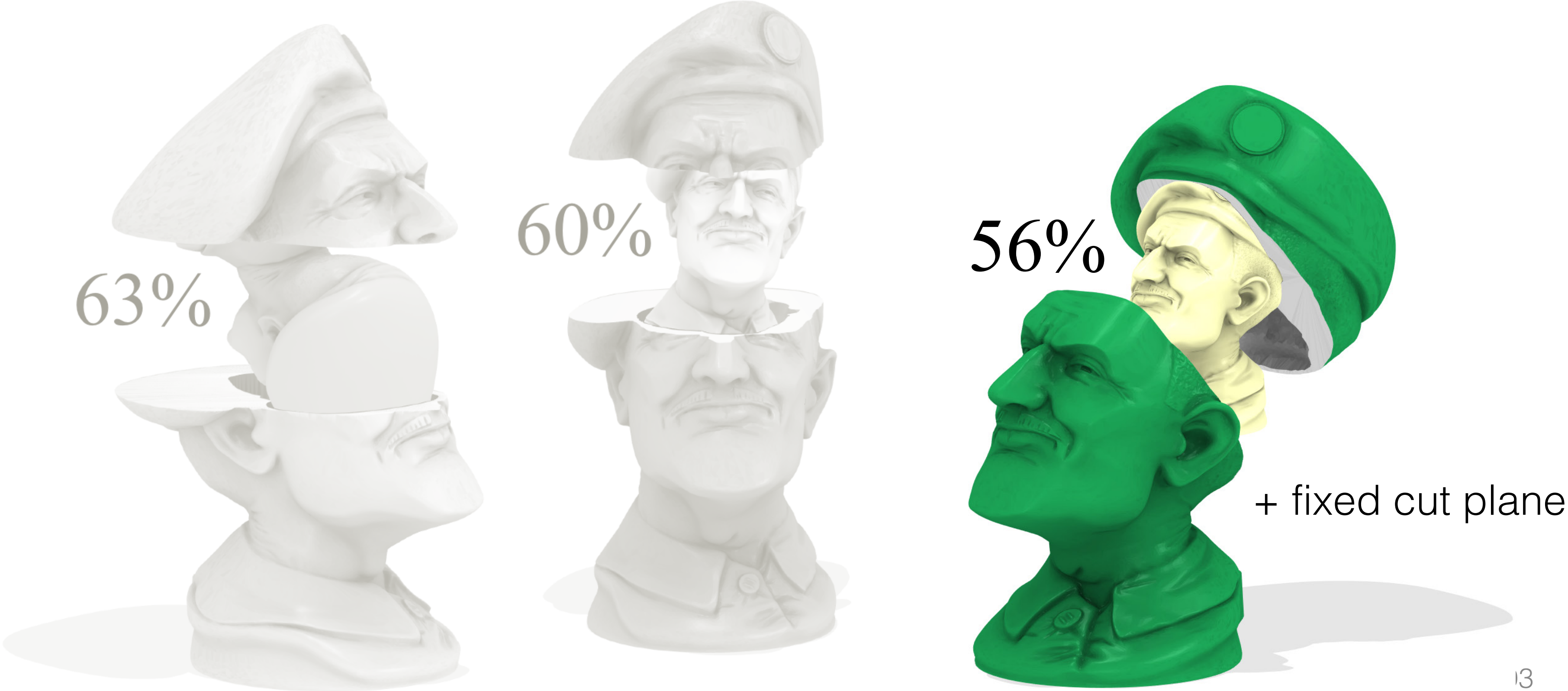
63%

fully optimized

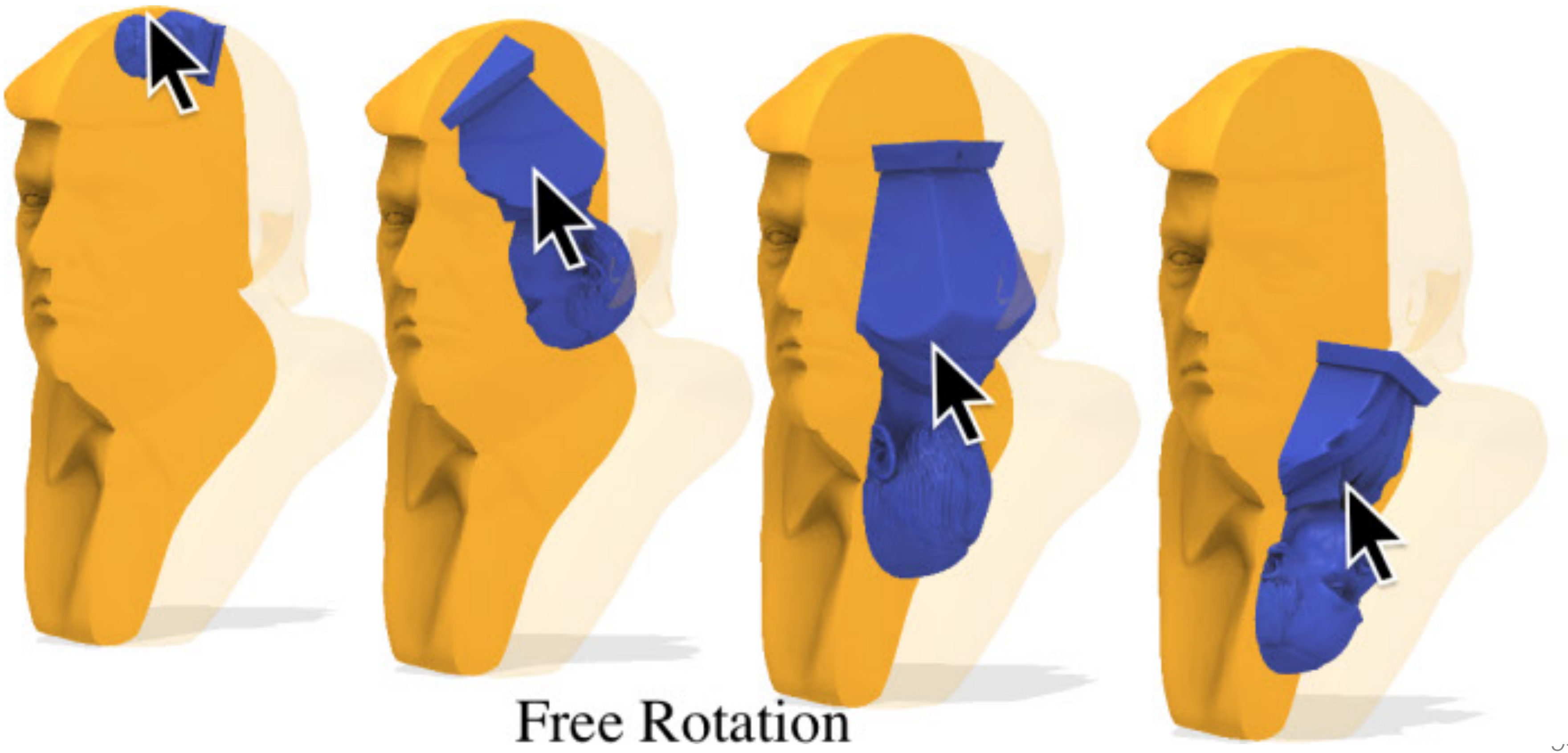
… or partially constrained interactive design

fixed upright orientation

… or partially constrained interactive design



Tool performs fast enough for interaction



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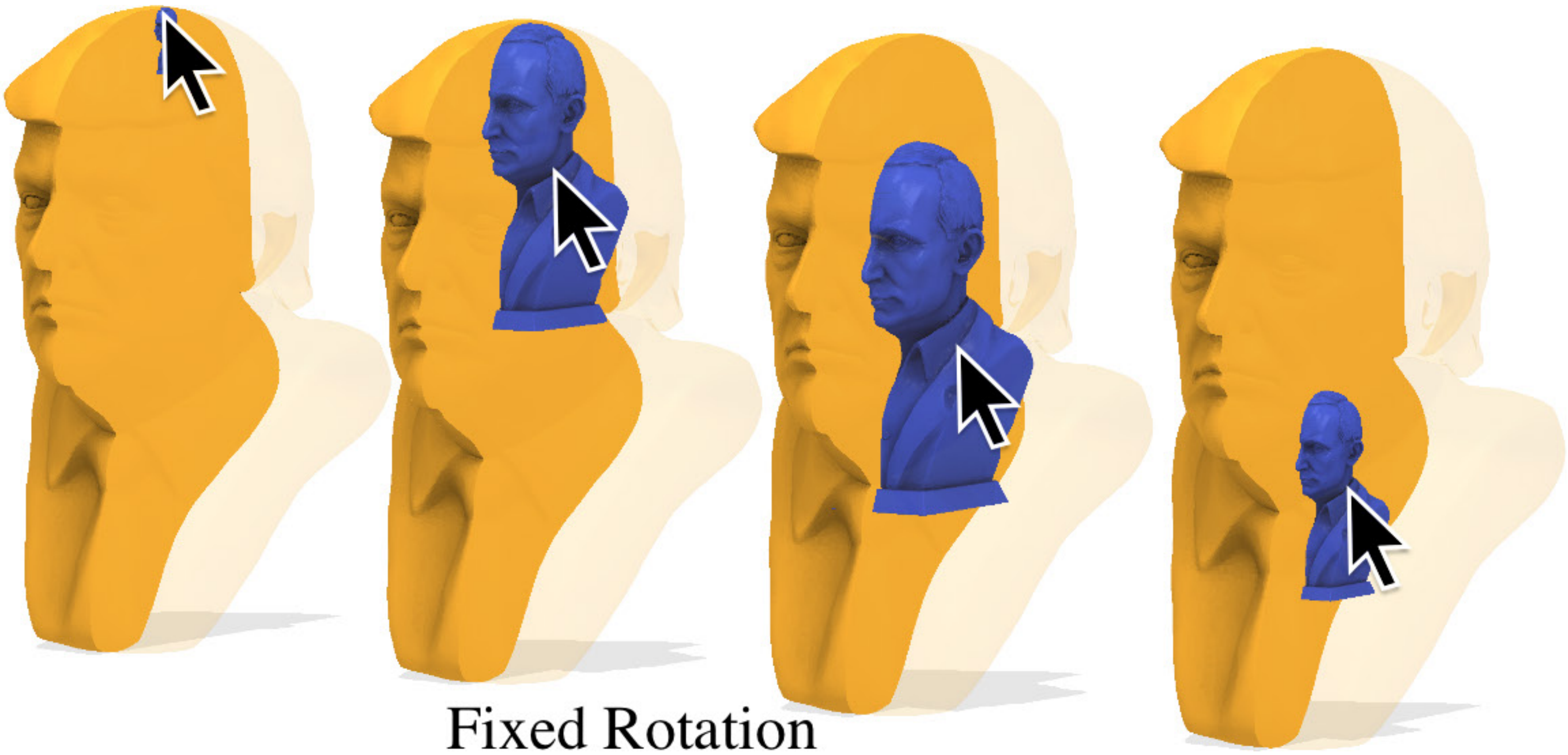

We validate our results via 3D printing

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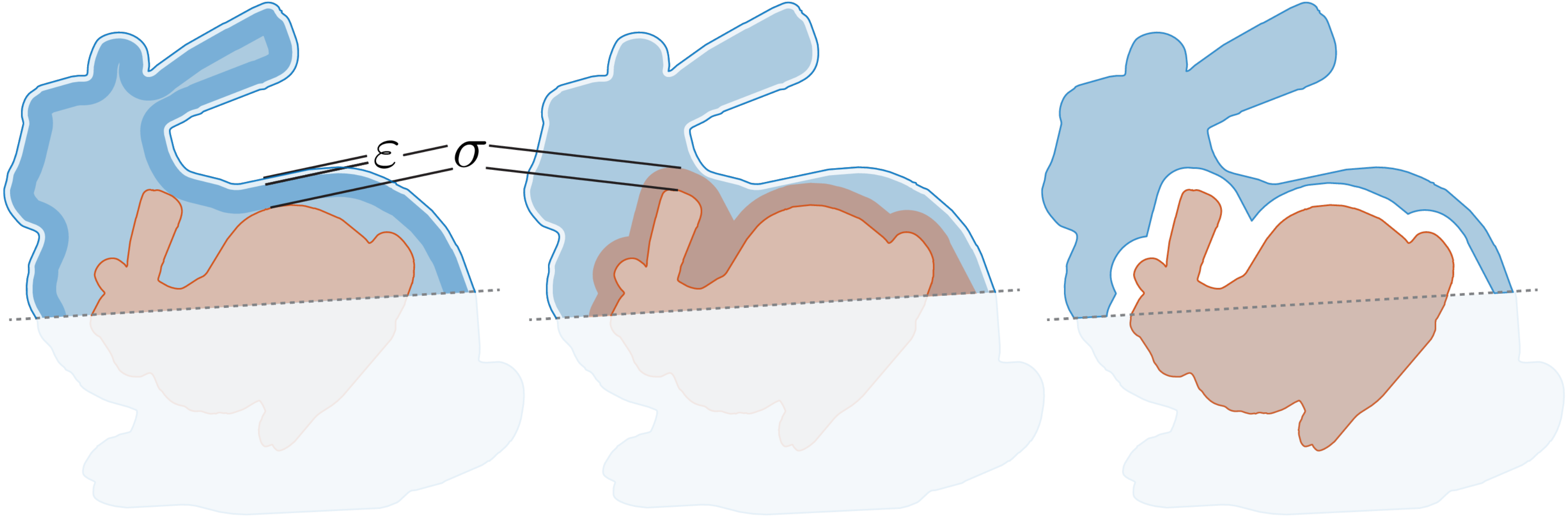
We accommodate printer tolerances by nesting *within* an offset surface



During validation During carving Result

Our tools trivially generalize to nesting disparate shapes

Limitations & Future Work

- no global optimum guarantee

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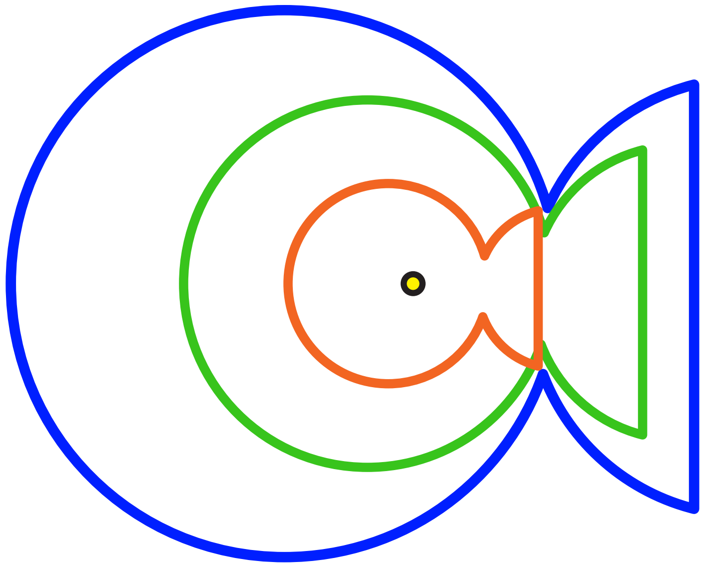

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Prevost et al.

Limitations & Future Work

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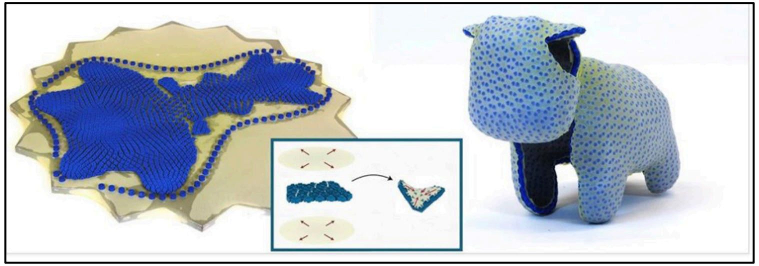
- thin shapes don't *rigidly* nest well
- deformable nesting?
    1. deform during design
    2. nest soft physical objects



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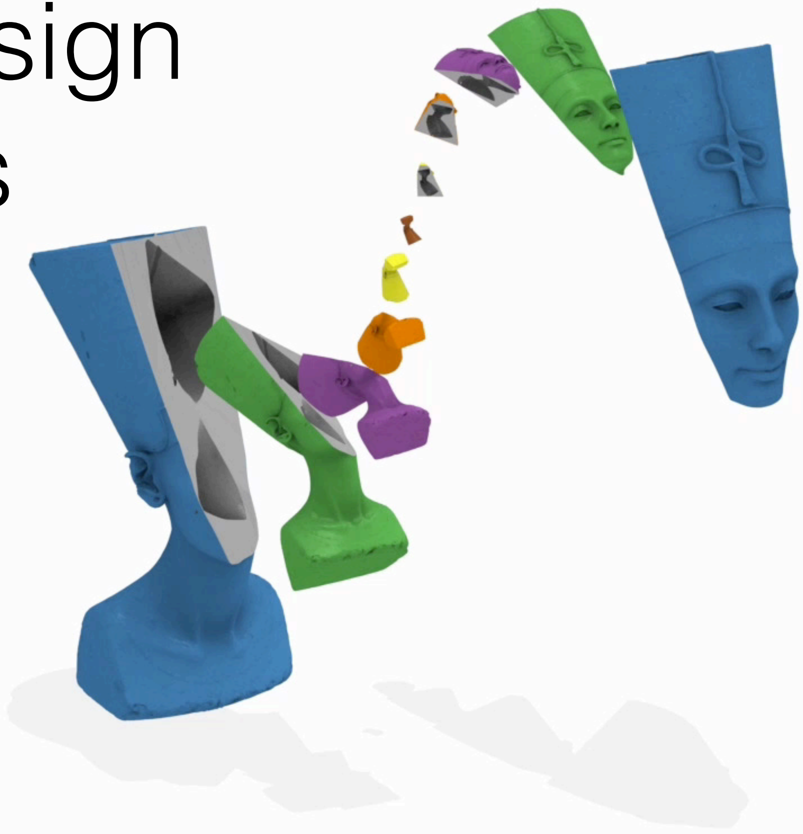
Generalized Matryoshka
Computational Design of Nesting Objects



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