CSC260S: Intro to Scientific, Graphical and Symbolic Computing
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Closed book. The test questions continue on the other side of this page.

1. [10] Sketch an (x,y) plot of each of the following parametrically defined functions given by $F(t) = (x(t), y(t))$. In each case, $t \in [-1,1]$.

   (a) [3] $F(t) = (t^2, t)$.

   (b) [3] $F(t) = (t^2, -t^2)$.

   (b) [4] $F(t) = (t^2, t^3)$.

2. [11] Suppose we wish to represent parametric piecewise polynomial curve segments over $[0,1]$. We have seen that a particularly useful way of representing such curves is by an equation of the form

$$\mathbf{p}(t) = T\,M\,V.$$

   (a) [3] For a polynomial of degree n, explain what each of T, M, and V represent and write down their specific form.

   (b) [2] Suppose we are interested in the second derivative of $\mathbf{p}$ with respect to t, which we shall write as $\mathbf{p}''(t)$. Assuming $\mathbf{p}(t)$ is in the form TMV, there is a faster way to compute $\mathbf{p}''(t)$ than to fully evaluate TMV before differentiating. What is this way?

   (c) [6] Let $\mathbf{P}$ be a space curve composed of two cubic curve segments $\mathbf{p}_0(t)$ followed by $\mathbf{p}_1(t)$, each parameterised over $[0,1]$. Suppose $\mathbf{p}_0$ is defined over the points A, B, C, D, and that $\mathbf{p}_1$ is defined over the points B, C, D, E. All points are in $\mathbf{R}^3$. Thus the curve segments share three points. It is often desirable to ensure that the overall curve $\mathbf{P}$ has a continuous second derivative everywhere (i.e., that $\mathbf{P}''$ can be drawn in one stroke without lifting your pen, roughly speaking). Now, since we are dealing with cubic polynomials, it is clear that $\mathbf{p}_0$ and $\mathbf{p}_1$ separately have this property. Assume furthermore that both $\mathbf{P}$ and $\mathbf{P}'$ are continuous for all t. There is only position in t at which there is any question as to whether or not $\mathbf{P}$ has a continuous second derivative. What is this value of t with respect to $\mathbf{p}_0$ and $\mathbf{p}_1$?
   If $\mathbf{p}_0(t)$ and $\mathbf{p}_1(t)$ are curves defined using the basis matrix

$$\frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix},$$

   then does $\mathbf{P}$ have second-derivative continuity at this point in question? What if the matrix is

$$\frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}?$$

3. [14] At one point in the derivation of an efficient line rendering program, we had the following code.

```
proc DrawLine(x,y: int)
    var ysf, m, m̄: real
    xi, ysi: int
    m = y/x; m̄ = m - 1
    ysi = 0;  ysf = ½
    for xi 0..x
        SetPixel(xi,ysi)
        if (ysf+m ≥ 1) then          % ysf overflow?
            ysi += 1
            ysf += m̄
        else
            ysf += m
        end if
    end for
end DrawLine
```

We converted the above into a program using only integer arithmetic by employing the following program transformation:

$$\texttt{r} \;=\; \texttt{2y + 2(ysf-1)x} \;.$$

(a) [4] Asking if `ysf+m` $\geq$ `1` is the same as asking if `r` $\geq$ `n` for what value of `n`? Derive this.

(b) [10] In the above program, apply the program transformation by replacing every expression involving `ysf` into one involving `r`. Show all of your derivations, and then write out a new, efficient version of the program, including your derivation in (a). The bulk of the marks for this question will be for the derivations.

5. [5] Many beautiful curves and surfaces are best described in *polar form*, namely

$$r \;=\; f(\theta).$$

For example, the definition $r \;=\; \cos 2\theta,\ \theta \in [0, 2\pi]$ defines a four petalled flower. The relationship between polar co-ordinates of the form (r, θ) and points (x,y) in standard rectangular co-ordinates is

$$x \;=\; r\cos\theta,$$

$$y \;=\; r\sin\theta.$$

Now suppose we would like to write down the equation of a curve defined in polar form as an implicit equation of the form $F(x,y) = 0$. The implicit form for a line segment, for example, is $Ax + By + C = 0$. Show how to convert from a polar representation to an implicit representation, and give the general form of an implicit equation corresponding to a polar equation.