```
    |\^/|      MAPLE V
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#
# we need functions defined in the linear algebra library
#
> with(linalg):
Warning: new definition for   norm
Warning: new definition for   trace
```

```
#
# create the power basis
#
> T:= transpose(vector([t^3,t^2,t,1]));
                                           3   2
                        T := transpose([ t , t , t, 1 ])
```

```
#
# create a change of basis matrix for an interpolating polynomial
# satisfying P(0) = P0, P(1/3) = P1, P(2/3) = P2, P(1) = P3.
#
> M := matrix( [[  -9/2,  27/2, -27/2,  9/2],
>               [     9, -45/2,    18, -9/2],
>               [ -11/2,     9,  -9/2,    1],
>               [     1,     0,     0,    0]]);
                              [  -9/2   27/2  -27/2   9/2 ]
                              [                           ]
                              [   9    -45/2    18   -9/2 ]
                         M := [                           ]
                              [ -11/2    9     -9/2    1  ]
                              [                           ]
                              [   1      0      0      0  ]
```

```
#
# create the vector of constraints/points to interpolate, but leave
# them undefined.
#
> V := vector([P0,P1,P2,P3]);
                            V := [ P0, P1, P2, P3 ]
```

```
#
# this is how you would create a polynomial using just the power
# basis; this is not what we want, though.
#
> multiply(T,V);
                            3       2
                           t  P0 + t  P1 + t P2 + P3
```

```
#
# use the change of basis matrix and create the interpolating polynomial
#
> P := multiply(T,M,V);
             3             2                             3         2
 P := - 9/2 t  P0 + 9 P0 t  - 11/2 P0 t + P0 + 27/2 P1 t  - 45/2 t  P1 + 9 P1 t

                 3           2                            3            2
      - 27/2 P2 t  + 18 P2 t  - 9/2 t P2 + 9/2 P3 t  - 9/2 P3 t  + P3 t
```

```
#
# collect terms on the same power of t together
#
> P := collect(P,t);
                                                3
 P := (9/2 P3 - 27/2 P2 + 27/2 P1 - 9/2 P0) t

                                              2
      + (18 P2 + 9 P0 - 9/2 P3 - 45/2 P1) t

      + (P3 - 9/2 P2 - 11/2 P0 + 9 P1) t

      + P0
```

```
#
# try it out; try different values of t, and then evaluate P.  Note that
# at t = 0, 1/3, 2/3, and 1, it interpolates the points P0, P1, P2, and P3.
# at t=1/2, a combination of these four points in returned, with
# P2 and P3 more heavily weighted.
#
> t := 0;
                                     t := 0
```

```
> P;
                                       P0
```

```
> t := 1/3;
```

$$t := 1/3$$

```
> P;
```

$$P1$$

```
> t := 1/2;
```

$$t := 1/2$$

```
> P;
```

$$-1/16\ P3 + 9/16\ P2 + 9/16\ P1 - 1/16\ P0$$

```
> t := 2/3;
```

$$t := 2/3$$

```
> P;
```

$$P2$$

```
> t := 1;
```

$$t := 1$$

```
#
# turn t back into a mathematical variable
#
> t := 't';
                                     t := t
```

```
#
# compute the basis functions by just multiplying T and M
#
> B := multiply(T,M);
                             3      2
    B := transpose([ - 9/2 t  + 9 t  - 11/2 t + 1,
                             3         2
                       27/2 t  - 45/2 t  + 9 t,
                             3       2
                     - 27/2 t  + 18 t  - 9/2 t,
                             3        2
                        9/2 t  - 9/2 t  + t ])
```

```
#
# plot the basis functions
#
> interface(plotdevice=postscript,plotoutput=`basis.ps`);
```

```
> Bt := transpose(B);
```

$$Bt := \left[-\frac{9}{2} t^3 + 9 t^2 - \frac{11}{2} t + 1,\ \frac{27}{2} t^3 - \frac{45}{2} t^2 + 9 t,\ -\frac{27}{2} t^3 + 18 t^2 - \frac{9}{2} t,\ \frac{9}{2} t^3 - \frac{9}{2} t^2 + t \right]$$

```
> Bt[1];
```

$$-\frac{9}{2} t^3 + 9 t^2 - \frac{11}{2} t + 1$$

```
> plot({Bt[1],Bt[2],Bt[3],Bt[4]},t=0..1);
```

```
#
# try to interpolate the same points using Lagrange interpolation
#
> Q := interp([0,1/3,2/3,1],[P0,P1,P2,P3],t);
                                                 3
 Q := (9/2 P3 - 27/2 P2 + 27/2 P1 - 9/2 P0) t

                                               2
      + (18 P2 + 9 P0 - 9/2 P3 - 45/2 P1) t

          + (P3 - 9/2 P2 - 11/2 P0 + 9 P1) t

      + P0
```

```
#
# the answer turns out to be what we have already done.
#
> Q - P;
                                       0
```